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HANDBOOK

OF THE

STEAM-ENGINE.

CONTAINING ALL THE RULES REQUIRED FOR THE RIGHT CONSTRUCTION AND MANAGEMENT OF ENGINES OF EVERY CLASS, WITH THE EASY ARITHMETICAL SOLUTION OF THOSE RULES.

CONSTITUTING

A KEY

TO THE

'CATECHISM OF THE STEAM-ENGINE.'

ILLUSTRATED BY

SIXTY-SEVEN WOOD-CUTS, AND NUMEROUS TABLES AND EXAMPLES.

BY

JOHN BOURNE, C.E.,

AUTHOR OF 'A TREATISE ON THE STEAM-ENGINE,' 'A TREATISE ON THE SCREW-PROPELLER,' 'A CATECHISM OF THE STEAM-ENGINE,' ETC.

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1873.

TO

GEORGE TURNBULL, ESQ., C.E., F.R.A.S., ETC.,

LATE ENGINEER-IN-CHIEF OF THE EAST INDIAN RAILWAY.

MY DEAR MR. TURNBULL,

In dedicating the present Work to you I am moved by two main considerations:—First, to testify in the best manner I can my regard and esteem for you personally; and Second, to mark my sense of the skill, tact, and abiding integrity which you brought to the onerous duty of constructing the first and greatest of the Indian railways, and of which, while in India, I had opportunities of forming a just appreciation.

The public in this country—traditionally so ignorant of India—has yet to learn the important fact, that the works carried out under your direction in that country, are greater and more difficult than most of those which are to be found at home; and that among other achievements, you constructed the largest bridge in the world—the great bridge over the St. Lawrence alone excepted. But these technical successes, important as they are, were not more eminent than those which you won over the discouragements and difficulties of the Indian official system—ending, too, in gaining the esteem and appro-

bation of the Indian Government, as well as of those for whom you zealously labored for so many years in India.

Whatever the benefits may be of the Indian railways, their *greatest* benefit is that they have taken to that country men who have impressed the people with their skill, and who have acquired an accurate perception of the physical wants of the country, together with all that practical knowledge of localities which will enable them to carry out with confidence, economy, and success, the numerous improvements still required by that great dependency, and upon which only a comparatively small beginning has yet been made.

I remain, my dear Mr. Turnbull,

Truly yours,

J. BOURNE.

PREFACE

THE present work, designed mainly as a Key to my 'Catechism of the Steam-Engine,' has, during its composition, been somewhat extended in its scope and objects, so as also to supply any points of information in which it appeared to me the Catechism was deficient, or whereby the utility of this Handbook as a companion volume would be increased.

The purpose of the Catechism being rather to enunciate sound principles than to exemplify the application of those principles to practice, it was always obvious to me that another work which would point out in the plainest possible manner the methods of procedure by which all computations connected with the steam-engine were to be performed—illustrated by practical examples of the application of the several rules—was indispensable to satisfy

the wants of the practical engineer in this department of enquiry. The present work was consequently begun, and part of it was printed, several years ago, but the pressure of other pursuits has heretofore hindered its completion; and in now sending it forth I do so with the conviction that I have spared no pains to render it as useful as possible to the large class of imperfectly educated engineers to whom it is chiefly addressed. It is with the view of enabling its expositions to be followed by those even of the most slender scientific attainments that I have introduced the first chapter, explaining those several processes of arithmetic by which engineering computations are worked out. For although there is no want of manuals imparting this information, there are none of them, that I know of, which have special reference to the wants of the engineer; and none of them deal with those associations, by way of illustration, with which the engineer is most familiar. Indeed, engineers, like sailors and other large classes of men, have an order of ideas, and, to some extent, even a species of phraseology of their own; and the avenues to their apprehension are most readily opened by illustrations based upon their existing knowledge and experience, such as an engineer can best supply. By this familiar method of exposition the idea of difficulty is dis-

pelled; and science loses half its terrors by losing all its mystery.

If I might infer the probable reception of the present work from the numerous anxious enquiries addressed to me from all quarters of the world during the last ten years, touching the prospects of its speedy appearance, I should augur for it a wider popularity than any work I have yet written. The questions propounded to me by engineers and others, in consequence of the offer I made in the preface to my 'Catechism of the Steam-Engine,' in 1856, to endeavour by my explanations to remove such difficulties as impeded their progress, have had the effect of showing more clearly than I could otherwise have perceived what the prevalent difficulties of learners have been; and I have consequently been enabled to give such explanations in the present work as appeared best calculated to meet those difficulties for the future.

To several of my correspondents I have to acknowledge myself indebted for the correction of typographical errors in my several works, and also for valuable suggestions of various kinds, which I have made use of in every case in which they were available.

I may here take occasion to notify that I have lately prepared an Introduction to my 'Catechism of the Steam-

Engine,' which reviews the most important improvements of the last ten years; and which, for the convenience of persons already possessing the Catechism, may be had separately. These three works taken together form a body of engineering information so elementary as to be intelligible by anybody, and yet so full that the attentive student of them will, I trust, be found not to fall far short of the most proficient engineers in all that relates to a knowledge of the steam-engine in its most important applications.

J. BOURNE.

Berkeley Villa, Regent's Park Road,
London: 1865.

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HANDBOOK

OF

THE STEAM-ENGINE.

CHAPTER I.

ARITHMETIC OF THE STEAM-ENGINE.

In this chapter I propose to explain as plainly and simply as I can those principles of arithmetic which it is necessary to know, that we may be able to perform all ordinary engineering calculations. In order that my remarks may be generally useful to working mechanics of little education, I shall proceed upon the supposition that the reader is not merely destitute of all arithmetical knowledge, but that he has no ideas of number or quantity that are not of the most vague and indefinite description. I have known many engineers—who were otherwise men of ability—to be in this condition; and the design of these observations is to enable such, with the aid of their own common sense and their familiar associations, to arrive at tangible ideas respecting the properties of numbers, and to perform with facility all the ordinary engineering calculations which occur in the requirements of engineering practice. These various topics are not beset with any serious difficulty. The processes of arithmetic are merely expedients for facilitating the discovery of results which every mechanic of ordinary ingenuity would find a means of discovering for himself, if really called upon to set about the task; and it is mainly be-

cause the rationale of these processes has not been much explained in school treatises, but the results presented as feats of legerdemain performed by the application of a certain rule—the reason of which is not made apparent—that the idea of difficulty has arisen in connection with such enquiries. The rudest and most savage nations have all some species or other of arithmetic suited to their requirements. The natives of Madagascar, when they wish to count the number of men in their army, cause the men to proceed through a narrow pass, where they deposit a stone for each man that goes through; and by subsequently arranging these stones in groups of ten each, and these again in groups of a hundred, and so on, they are enabled to arrive at a precise idea of the number of men the army contains. A labourer in counting bricks out of a cart or barge makes a chalk-mark on a board for every ten bricks he hands out; and these chalk-marks he arranges in groups of five or ten each, so that he may easily reckon up the total number of groups the board contains. These are expedients of numeration which the most moderate intelligence will suggest as conducive to the acquisition of the idea of quantity; and the rules of arithmetic are merely an extension and combination of such methods as experience has shown to be the most convenient in practice to accomplish the ends sought.

It will be obvious that the number of stones or chalk-marks collected into groups in the preceding examples may either be five, ten, twelve, or any other number; the only necessary condition being that the number in each group shall be the same. The concurrent practice of most nations, however, is to employ groups consisting of ten objects in each group; no doubt from the circumstances that mankind are furnished with ten fingers, and because the fingers are much used in most primitive systems of numeration. In some cases, however, objects are reckoned by the *dozen*, or *score*, or *gross*; or, in other words, a dozen, a score, or a gross of objects are collected in each group. But in the ordinary or *decimal* system of numeration, ten objects or units are supposed to be collected in each group, and ten of these primary groups are supposed to be collected in each higher or

larger group of the class immediately above, and so on indefinitely. The decimal system is so called from the Latin word *decem*, signifying *ten*, and the word *unit* is derived from the Latin word *unus*, signifying *one*. Ten units form a group of *ten*, and ten of these groups form a group of a *hundred*, and ten groups of a hundred form a group of a *thousand*, and so on for ever.

The Romans, whose numbers are still commonly used on clock faces, employed a mark or I to signify one; two marks or II to signify two; three marks or III to signify three; and four marks or IIII to signify four. But as it would have been difficult to count these marks if they became very numerous, they employed the letter V to signify five and the letter X or a cross to signify ten, and V is the same mark as one-half of X, which was no doubt the primary of the two characters. An I appended to the left-hand side of the V or X signified V or X diminished by one, whereas each additional I added to the right-hand side of the V or X, signified one added to V or X. Thus according to the Roman numeration IV signifies four; VI signifies six; IX signifies nine; XI signifies eleven; XII signifies twelve; and so on. A hundred is signified by the letter C, the initial letter of the Latin word *centum*, signifying a hundred; and a thousand is represented by the letter M, the initial letter of the Latin word *mille*, signifying a *thousand*.

It is clear that the Roman numeration, though adequate to the wants of a primitive people, was a very crude and imperfect system. It has therefore been long superseded for all arithmetical purposes by the system of notation at present in common use, and which has a distinct sign or figure for each number up to 9, and a cipher or 0, which has no individual value, but which affects the value of other figures. This system, which came originally from India, was brought into Europe by the Moors; and in common with most of the oriental languages, it is written from right to left instead of from left to right, like the languages of Europe, so that in performing a sum in arithmetic—as in writing a word in Sanscrit or Arabic—we have to begin at the right-hand side of the page. In this system the classes or orders of the objects or groups of objects is indicated by the place occu-

pied by the figures which express their value. Thus in the case of the groups of stones employed in Madagascar, the figure 3 may be employed to designate either three individual stones, or three groups of ten each, or three groups of a hundred each; but in using the figure it is quite indispensable that it should appear, by some distinctive mark, which order or class is intended to be designated. We might use the figure 3 to designate three single stones, and we might use the figure with a circle round it to denote groups of ten each, and with a square round it to denote groups of a hundred each. But on trial of such a system we should find it to be very cumbrous and perplexing, and the method found to be most convenient is to add a cipher after the three to show that groups of tens are intended to be signified, and two ciphers to show that groups of hundreds are intended to be signified. Three groups of tens, or thirty, are therefore expressed by 30, and three groups of hundreds are expressed by 300. Here the ciphers operate wholly in advancing the 3 into a higher and higher position, which, however, other figures will equally suffice to do if there are any such to be expressed. Three groups of one hundred stones in each, three groups of ten stones in each, and three individual stones, will therefore be represented by the number 333, in which the same figure recurs three times, but which is counted ten times greater at each successive place to which it is advanced, reckoning from the right to the left. Of course, the number three hundred and thirty-three might be represented in an infinite number of other ways, differing more or less from the one here indicated; and any of the properties belonging to the number would equally hold by whatever expedient of notation it was expressed. But the manner here described is that which the accumulated experience of mankind has shown to be the most convenient; and it is therefore generally adopted, though it is proper to understand that there is no more necessary relation between the number itself and the common mode of expressing it, than there is between the Latin word *equus*, a horse, and that most useful of quadrupeds. In each case the relations are wholly conventional, and might be altered without in any way affecting the object.

Arithmetic is the science of numbers. Numbers treat of magnitude or quantity; and whatever is capable of increase or diminution is a magnitude or quantity. A sum of money, a weight, or a surface, is a quantity, being capable of increase or diminution. But as we cannot measure or determine any quantity except by considering some other quantity of the same kind as known, and pointing out their mutual relation, the measurement of quantity or magnitude of all kinds is reduced to this: fix at pleasure upon any one known kind of magnitude of the same species as that which has to be determined, and consider it as the *measure* or *unit*, and determine the proportion of the proposed magnitude to this known measure. This proportion is always expressed by numbers; so that number is nothing more than the proportion of one magnitude to that of some other magnitude arbitrarily assumed as the unit. If, for example, we want to determine the magnitude of a sum of money, we must take some piece of known value—such as the pound or shilling—and show how many such pieces are contained in the given sum. If we wish to express the distance between two cities, we must use some such recognized measure of length as the foot or mile; and if we wish to ascertain the magnitude of an estate, we must employ some such measure of surface as the square mile or acre. The foot-rule is the measure of length most used for engineering purposes. The foot is divided into twelve inches, and the inch is subdivided into half inches, quarter inches, eighths, and sixteenths. It is clear that two half inches or four quarter inches make an inch, as also do eight eighths and sixteen sixteenths; and indeed it is obvious that into whatever number of parts the inch is divided, we shall equally have the whole inch if we take the whole of the parts of it. If the inch were to be divided into ten equal parts, then ten of these parts would make an inch. Fractional parts of an inch, or of any other quantity, are expressed as follows: a half, $\frac{1}{2}$; a quarter, $\frac{1}{4}$; an eighth, $\frac{1}{8}$; a sixteenth, $\frac{1}{16}$; and a tenth, $\frac{1}{10}$. The figure above the line is called the numerator, because it fixes the *number* of halves, quarters, or eighths, which is intended to be expressed; and the figure below the line is called the denominator, because it fixes the

order or *denomination* of the fraction, whether halves, quarters, eighths, or otherwise. Thus in the fractions $\frac{3}{4}$ths and $\frac{7}{8}$ths, the figures 3 and 7 are the numerators, and the figures 4 and 8 the denominators; and $\frac{4}{4}$ths, $\frac{8}{8}$ths, or $\frac{10}{10}$ths, are clearly equal to 1. So also $\frac{5}{4}$ths, $\frac{9}{8}$ths, and $\frac{11}{10}$ths are clearly greater than 1, the first being equal to $1\frac{1}{4}$th, the second to $1\frac{1}{8}$th, and the third to $1\frac{1}{10}$th.

The species of fractions here referred to is that which is called *vulgar fractions*, as being the kind of fractions in common use; and every engineer who speaks of $\frac{3}{4}$ths or $\frac{5}{8}$ths of an inch, and every housewife who speaks of $\frac{3}{4}$ of a pound of sugar, or $\frac{1}{2}$ a pound of tea, refers, perhaps unconsciously, to this species of numeration. There is another species of fractions, however, called *decimal fractions*, not usually employed for domestic purposes, but which is specially useful in arithmetical computations, and these fractions being dealt with in precisely the same manner as ordinary figures, are very easy in their application. In ordinary figures, the value of each succeeding figure, counting from the right to the left, is ten times greater than the preceding one, in consequence of its position; and in decimal fractions the value of each succeeding figure, counting from left to right, is ten times less. Thus the figures 1111 signify one thousand one hundred and eleven; and if after the last unit we place a period or full stop, and write a one after it thus, 1111·1, we have one thousand one hundred and eleven and one-tenth. The period, or *decimal point*, as it is termed, prefixed to any number, implies that it is—not a whole number—but a decimal fraction. Thus ·1 means one-tenth, ·2 two-tenths, ·3 three-tenths, ·4 four-tenths, and so on. So in like manner ·11 means one-tenth and one hundredth, or eleven hundredths; ·22 means two-tenths and two hundredths, or twenty-two hundredths; ·33, three-tenths and three hundredths, or thirty-three hundredths; and so on—each successive figure of the fraction counting from the left to the right, being from its position ten times less than that which went before it. The number ·1111 signifies one thousand one hundred and eleven ten thousandths, the first decimal place being tenths, the next hundredths, the next thousandths, the next ten thousandths, and so on. If we wish to express a hun-

dredth by this notation, we place a cipher before the unit thus, ·01; if a thousandth two ciphers, ·001; and so of all other quantities. The multiplication, division, and all the other arithmetical operations required to be performed with decimal fractions, are conducted in precisely the same manner as if they were ordinary numbers—the decimal progression being carried downwards in the one case precisely in the same manner as it is carried upwards in the other case; and it is easy to suppose that the stones used by the natives of Madagascar may not only be collected into groups of tens and hundreds, but that each stone may also be subdivided into tenths, hundredths, or thousandths, so that *parts* of a stone may be reckoned. Instead of dividing the stone into halves, and quarters, and eighths, and sixteenths, as would be done by the method of vulgar fractions, it is supposed by the decimal system of fractions to be at once divided into *tenths*, whereby the same system of grouping by tens, which is used *above* unity, is also rendered applicable to the fractional parts *below* unity—to the great simplification of arithmetical processes. In all cases a decimal fraction may be transformed into a vulgar fraction of equal value by retaining the significant figures as the numerator, and by using as the denominator 1, with as many ciphers as there are figures after the decimal point. Thus ·1 is equal to $\frac{1}{10}$; ·11 is equal to $\frac{11}{100}$; ·01 is equal to $\frac{1}{100}$; ·001 is equal to $\frac{1}{1000}$; 3·1459 is equal to 3 $\frac{1459}{10000}$; and ·7854 is equal to $\frac{7854}{10000}$.

In all countries there are certain recognised standards of magnitude for measuring other magnitudes by; such as the inch, foot, yard, or mile for measuring lengths; the square inch, square yard, or square mile, or square pole, rood, or acre, for measuring surfaces; the grain, ounce, pound, or ton for measuring weights; and the penny, shilling, and sovereign for measuring money. It is, of course, quite inadmissible in conducting any of the operations of arithmetic to confound these different kinds of magnitudes together, and there is as much difference between a linear foot and a square foot as there is between a ton weight and a pound sterling. A square surface measuring an inch long and an inch broad is a square inch. A strip of sur-

face 1 inch broad and 12 inches or 1 foot long will be equal to 12 square inches; and 12 such strips laid side by side, and therefore a foot long and a foot broad, will make 12 times 12 square inches, or 144 square inches. In each square foot, therefore, there are 144 square inches; and as there are 3 linear feet in a linear yard, there will be in a square yard 9 square feet, as we may suppose the square yard to be composed of three strips of surface, each 3 feet long and 1 foot wide, and therefore containing 3 square feet in each.

A cubic inch is a cube or dice measuring 1 inch long, 1 inch broad, and 1 inch deep. A square foot of board 1 inch thick will consequently make 144 cubic inches or dice if cut up. But as it will take twelve such boards placed upon one another to make a foot in depth, or, in other words, to make a cubic foot, it follows that there will be 12 times 144, or, in all, 1,628 cubic inches in the cubic foot. So, in like manner, as there are 3 linear feet in the linear yard, and 9 square feet in the square yard, there will be 3 times 9 or 27 cubic feet in the cubic yard—the cubic yard being composed of three strata 1 foot thick, containing 9 cubic feet in each.

Besides the square inch there is the circular inch by which surfaces are sometimes measured. The circular inch is a circle 1 inch in diameter, and as it is a fundamental rule in geometry that the area of different circles is proportional to the squares of their respective diameters, the area of any piston or safety-valve or other circular orifice will be at once found in circular inches by squaring its diameter, as it is called; or, in other words, by multiplying the diameter of such piston or orifice expressed in inches by itself. Thus as a square foot, or a square of 12 inches each way, contains 144 square inches, so a circular foot, or a circle of 12 inches diameter, contains 144 circular inches. There is a constant ratio subsisting between a circular inch or foot and the square circumscribed around it. The circular inch or foot is less than the square inch or foot by a certain uniform quantity; and this relation being invariable, it becomes easy when we know the area of any circle in circular inches to tell what the equivalent area will be in square inches,

as we have only to multiply by a certain number—which will be less than unity—in order to give the equivalent area. This number will be a little more than $\frac{3}{4}$, or it will be the decimal ·7854; and if circular inches be multiplied by this number, we shall have the same area expressed in square inches. Multiplying any quantity by a number *less* than unity, it may be here remarked, *diminishes* the quantity, just as multiplying by a number *greater* than unity *increases* it. To multiply by $\frac{1}{2}$ gives the same result as to divide by 2; and to multiply by the decimal ·7854 will have the effect of reducing the number by nearly a fourth, as it is necessary should be done in order to convert circular into square inches; for, seeing that the square inches are the larger of the two, there must be fewer of them in any given area.

Besides the cubic inch there are the spherical, the cylindrical, and the conical inch, all having definite relations to one another. The spherical inch is a ball an inch in diameter; the cylindrical inch is a cylinder an inch in diameter and an inch high; and the conical inch is a cone whose base is an inch in diameter, and which is an inch high. All these quantities are convertible into one another—just as the pound sterling is convertible into shillings or pence, and the ton weight is convertible into hundred-weights and pounds.

The foundation of all mathematical science must be laid in a complete treatise on the science of numbers, and in an accurate examination of the different methods of calculation which are possible by their means. Now *Arithmetic* treats of numbers *in particular*, but the science which treats of numbers *in general* is called *Algebra*. In algebra numbers are expressed by letters of the alphabet, and the advantage of the substitution is that we are enabled to pursue our investigations without being embarrassed by the necessity of performing arithmetical operations at every step. Thus if a given number be represented by the letter a, we know that $2\ a$ will represent twice that number, and $\frac{1}{2}\ a$ the half of that number, whatever the value of a may be. In like manner if a be taken from a, there will be nothing eft, and this result will equally hold whether a be 5, or 7, or

1,000, or any other number whatever. By the aid of algebra, therefore, we are enabled to analyse and determine the abstract properties of numbers without embarrassing ourselves with arithmetical details, and we are also enabled to resolve many questions that by simple arithmetic would either be difficult or impossible.

ADDITION.

The first process of arithmetic is Addition; and here the first steps are usually made by counting upon the fingers, as an aid to the perceptions of the total amount of the quantity that has to be expressed. For example, if we hold up 5 fingers of the one hand and 3 of the other, and are asked how much 5 and 3 amount to, we at once see that the number is 8, as we either actually or mentally count the other 3 fingers from 5, designating them as 6, 7, 8; when, the whole fingers being counted, we know that the total number to be reckoned is 8. Persons even of considerable arithmetical experience, will often find themselves either counting their fingers or pressing them down successively on the table, in order to assist their memory in performing addition. But the best course is to commit very thoroughly to memory an addition table, just as the multiplication table is now commonly committed to memory by arithmetical students—as such a table, if thoroughly mastered, will greatly facilitate all subsequent arithmetical processes. A table of this kind is here introduced, and it should be gone over again and again, until its indications are as familiar to the memory as the letters of the alphabet, and until the operation of addition can be performed without the necessity of mental effort. The sign + placed between the figures of the following table is the sign of addition termed *plus*, and signifies that the numbers are to be added together. The table is so plain as scarcely to require explanation. The figures in the first column are obtained by adding together the figures opposite to them in any of the other columns. Thus 4 and 9 make 13, as also do 5 and 8 or 6 and 7.

ADDITION TABLE.

2	1+1				
3	1+2				
4	1+3	2+2			
5	1+4	2+3			
6	1+5	2+4	3+3		
7	1+6	2+5	3+4		
8	1+7	2+6	3+5	4+4	
9	1+8	2+7	3+6	4+5	
10	1+9	2+8	3+7	4+6	5+5
11	2+9	3+8	4+7	5+6	
12	3+9	4+8	5+7	6+6	
13	4+9	5+8	6+7		
14	5+9	6+8	7+7		
15	6+9	7+8			
16	7+9	8+8			
17	8+9				
18	9+9				

GENERAL EXPLANATION OF THE METHOD OF PERFORMING ADDITION.

Write the numbers to be added under one another in such manner that the units of all the subsequent lines of figures shall stand vertically under the units of the first line—the tens under the tens, the hundreds under the hundreds, and so on. Then add together the figures found in the units column. If their sum be expressed by a single figure, write the figure under the units column, and commence the same process with the tens

column. But if the sum of the figures in the units column be greater than 9, it must in that case be expressed in more than one figure, and in such event write the last figure only under the units column, and carry to the column of tens as many units as are expressed by the remaining figure or figures. Proceed in the same manner with the column of tens, and so with all the other columns. When the column of the highest order, which is always the first on the left, has been added, including the number carried from the column last added up, then if the sum be expressed by a single figure, place that figure under the column. But if it be expressed in more figures than one, write those figures in their proper order, the last under the column and the others preceding it.

Examples.

Add together 1,904, 9,899, 5,467, and 2,708. The numbers are to be arranged as follows:

$$\begin{array}{r} 1904 \\ 9899 \\ 5467 \\ 2708 \\ \hline 19,978 \\ \hline \end{array}$$

Here, beginning at the right-hand column, we say 8 and 7 are 15, and 9 are 24, and 4 are 28. We write the 8 under the column of units, and carry the 2 tens to the next column of tens. Adding up this column, we have the 2 carried from the last column added to 6, which make 8, and 9 are 17. Here we write down the 7 and carry the 1 over to the next column. In the third column we have 1 carried from the last column added to 7, which makes 8, and 4 are 12, and 8 are 20, and 9 are 29. Here we write down the 9 and carry the 2 to the next column. In the fourth column we have the 2 carried from the last column, which added to 2 makes 4, and 5 are 9, and 9 are 18, and 1 are 19, which sum of 19 we write at the foot of the column, the 9 under the other figures and the 1 preceding it. The total sum of these several numbers therefore, when added together, is nineteen thousand nine hundred and seventy-eight.

Add together the following numbers:—

2808	1467	2708	5794
1407	5988	5467	9969
9969	2829	9899	1407
5794	9694	1904	2808
19,978	19,978	19,978	19,978

It is usual, for facility of reading the figures, to divide them, when they amount to any considerable number, into groups of three each, by means of a comma interposed. But the comma in no way affects the value of the quantity; but is merely used to save the trouble of counting the figures to make sure whether it is thousands, hundreds of thousands, or what other order of figures is intended to be expressed. Thus with the aid of the comma we see at once that the number 19,000 is nineteen thousand, or that the number 190,000 is one hundred and ninèty thousand, or that the number 1,900,000 is one million nine hundred thousand; whereas, without the aid of the commas, we should have to count the figures to make sure of the real value of the expression. The comma, therefore, has no such significance as the decimal point, and the number may be written with or without the comma at pleasure; but if written without it there will be more difficulty in reading the number, just as it would be more difficult to read a book if the stops were left out.

SUBTRACTION.

Subtraction is the reverse of addition. If we have a bag containing 20 shillings, and if we add thereto 5 shillings, 15 shillings, and 10 shillings, we can easily tell by the operation of addition that we must have 50 shillings in the bag. If, however, we now withdraw the 5 shillings, the 15 shillings, and the 10 shillings, or, in all, if we withdraw 30 shillings, we shall, of course, have the original 20 shillings left; and the operation of subtraction is intended to tell us, when we withdraw a less number from a greater, how much of the greater number we shall have left. As addition is signified by the sign + or *plus*,

so subtraction is signified by the sign — or *minus;* and two short parallel lines = are employed as a substitute for the words *equal to.* As the expression, therefore, $5+3$ means 5 increased by 3, or 8; so the expression $5-3$ means 5 diminished by 3, or 2. This in common arithmetical notation would be written $5+3=8$ and $5-3=2$.

When we have a number of quantities to subtract from a greater quantity, the usual course is to add together first all the quantities to be subtracted, in order that the subtraction may be performed at a single operation. Thus in the case of the bag containing 50 shillings, from which we successively withdraw 5 shillings, 15 shillings, and 10 shillings, we first add together the 5 shillings, the 15 shillings, and the 10 shillings, so as to have in one sum the whole quantity to be subtracted, and then we can suppose the operation to be performed at a single step, as, the subtraction having been performed at different times, will not affect the amount of the sum subtracted or the sum left. Thus $50-30=20$; or if we take the successive stages, we have $50-5=45$, and $45-15=30$, and $30-10=20$, which is the same result as before.

GENERAL EXPLANATION OF THE METHOD OF PERFORMING SUBTRACTION.

Write the less number under the greater in such manner that the units of the second line of figures shall stand vertically under the units of the first line—the tens under the tens, the hundreds under the hundreds, and so on, as in addition. Draw a straight line beneath the lower line of figures, and subtract the units of the lower line of figures from the units of the upper line, and place the remainder vertically under the units column and beneath the straight line which has been drawn. Subtract the tens from the tens in like manner, the hundreds from the hundreds, and so on until the whole is completed; and where there is no figure to be subtracted, the figure of the upper line will appear in the answer without diminution, as appears in following examples:

1864	Original number	1864	Original number
64	Number to be subtracted	32	Number to be subtracted
1800	Remainder	1832	Remainder

From	7854	From	89764384	From	785068473894
Subtract	6532	Subtract	41341073	Subtract	510054103784
Answer	1322	Answer	48423311	Answer	275014370110

In these examples all the figures of the second line are less than those of the first line, and we at once see what the remainder at each step will be by considering what sum we must add to the less number to make it equal to the greater. Thus in subtracting 6532 from 7854, we see that we must add 2 to the 2 of the lower line to make the 4 appearing in the upper; and we must add 2 to the 3 appearing in the lower line to make the 5 appearing in the upper. In cases, however, in which some of the figures of the lower line are larger than those existing in the upper, we must borrow a unit from the preceding column, which will count as ten in the column into which it is imported, and this unit so borrowed will be added to the sum to be subtracted when that preceding column comes to be dealt with. Thus in the groups of stones used by the natives of Madagascar—if we have 6 groups of 10 stones in each and 7 stones over, and if we want to withdraw 8 stones from the number, it is clear that, as the 7 stones not arranged in groups will not suffice to supply the 8 stones we have to furnish, we must break up one of the groups of 10 to enable the 8 stones to be surrendered. We shall then have only 5 groups, but with the 7 stones we had before we can supply the 8 by taking only one stone from one of the groups, leaving 9 stones in it, so that, after taking away the 8 stones, we shall have 5 groups of ten each and 9 stones left. This is expressed arithmetically as follows:

$$\begin{array}{r} 67 \\ 8 \\ \hline 59 \end{array}$$

Here we say we cannot subtract 8 from 7, so that we must borrow 1 from the previous column, which, when imported into the column of units, will be 10; and we therefore say 8 taken from 17 leaves 9, which 9 we place

in the remainder. But as we have taken one of the groups from the preceding column, we have to deduct that from the six groups remaining, and we therefore say 1 from 6 leaves 5. So, in like manner, if we had to take 29 shillings from 42 shillings, as we cannot take 9 from 2, we take 9 from 12, borrowing as before a unit from the preceding column. But as we have afterwards to return this unit, we do not say 2 from 4, but 3 from 4, which leaves 1; or, in other words, 29 taken from 42 leaves 13, as we can easily see must be the case, as 13 added to 29 make 42. To prove the accuracy of an answer in subtraction, it is only necessary to add together the two lower lines, which will produce the top one.

Examples.

From......	1864	From......	1864
Subtract....	14	Subtract...	97
Remainder	1850	Remainder	1767
From......	1864	From......	1864
Subtract....	975	Subtract....	1796
Remainder	889	Remainder	68

It will be seen that, by adding together the last two lines of figures in each of these examples, we obtain the first line.

MULTIPLICATION.

Multiplication is a process of arithmetic for obtaining the sum total of a quantity that is repeated any given number of times, and is virtually an abbreviated species of addition. If, for example, we have 6 heaps of stones, with 1,728 stones in each heap, we might ascertain the total number of stones in the six heaps by writing the 1,728 six times in successive lines, and adding up the sum by the method of procedure already explained under the head of Addition. But it is clear that this would be a very tedious process in cases in which the number

of heaps was great, and multiplication is an expedient for ascertaining the total quantity by a much less elaborate method of procedure.

All numbers whatever it is clear may be formed by the addition of units. The consecutive numbers 1, 2, 3, 4, 5, &c., may be derived as follows:

$$1=1$$
$$1+1=2$$
$$1+1+1=3$$
$$1+1+1+1=4$$
$$1+1+1+1+1=5$$

There are certain numbers which are formed by the continued addition of other numbers than 1; and the numbers which are formed by the continued addition of 2 may be shown as follows:

$$2=2$$
$$2+2=4$$
$$2+2+2=6$$
$$2+2+2+2=8$$
$$2+2+2+2+2=10.$$

In like manner, the numbers shown by the successive additions of 3 and 4 may be thus represented:—

$3=3$	$4=4$
$3+3=6$	$4+4=8$
$3+3+3=9$	$4+4+4=12$
$3+3+3+3=12$	$4+4+4+4=16$
$3+3+3+3+3=15$	$4+4+4+4+4=20$

Thus it will be seen that in the series of numbers proceeding upwards from 1, some can only be formed by the continued addition of 1, while others may be formed by the continued addition of 2, 3, or some higher number. The numbers 3, 5, and 7 cannot be produced by the continued addition of any other number than 1, while the intermediate numbers 4 and 6 may be formed, the first by the addition of 2, and the second by the continued addition of 2 or 3.

Those numbers which cannot be formed by the continued addition of any other number than 1 are termed *prime numbers.* The numbers 3, 5, 7, 11, 13, 17, &c., are prime numbers. All other numbers are termed *multiple numbers;* and they are said to be multiples of those lesser numbers by the continued addition of which they may be formed. Thus 6 is a multiple of 2, because it may be formed by the continued addition of 2. But it is also a multiple of 3, because it may be formed by the continued addition of 3. In like manner 12 is a multiple of 2, 3, 4, and 6.

In the ascending series of numbers, 1, 2, 3, 4, 5, &c., it will be obvious that each alternate number is a multiple of 2. Such numbers are called *even* numbers, and the intermediate numbers are called *odd* numbers. Thus 2, 4, 6, 8, 10, &c., are even numbers, and 1, 3, 5, 7, 9, &c., are odd numbers.

As every even number is a multiple of 2, it is clear that no even number except 2 itself can be a *prime number*, and every prime number except 2 itself must be an odd number. It by no means follows, however, that every odd number must be prime, and it is clear indeed that 9 is a multiple of 3, 15 of 3 and of 5, and so of other odd numbers, which cannot, therefore, be prime numbers.

If we take a strip of paper an inch broad and 12 inches long, like a strip of postage stamps, it is clear that this strip will contain 12 square inches; and if we take three such strips placed side by side, they will manifestly have a collective surface of 36 square inches. Nor will the result be different in whatever way we reckon the squares; and 12 multiplied by 3 will give just the same number as 3 multiplied by 12. In like manner, 7 multiplied by 5 is the same as 5 multiplied by 7, and so of all other numbers.

In order to be able to perform the operations of multiplication with ease and expedition, it is necessary to commit to memory the product of the multiplications of numbers from 1 to 9; and to enable this to be conveniently done, a table of these primary multiplications, called the *Multiplication Table*, forms part of the course of arithmetical instruction given at schools, where,

nowever, the tables used commonly carry the multiplications up to 12 times 12. A table containing all the multiplications necessary to be remembered is given below; and it is very material to the subsequent ease of all arithmetical processes, that this table should be thoroughly learned by heart, so as to obviate the hesitation and inaccuracy that must otherwise ensue.

MULTIPLICATION TABLE.

	2	3	4	5	6	7	8	9
9	18	27	36	45	54	63	72	81
8	16	24	32	40	48	56	64	
7	14	21	28	35	42	49		
6	12	18	24	30	36			
5	10	15	20	25				
4	8	12	16					
3	6	9						
2	4							

To find the product of two numbers by this table, we must look for the greater number in the first upright column on the left, and for the lesser number in the highest cross row. The product of the two numbers will be found in the same cross row with the greater number, and in the same upright column with the lesser number. Thus 6 times 3 are 18, 6 times 4 are 24, and 5 times 4 are 20. If we find the number 6 in the first column and pass our finger along the same line until we come vertically under the 3 in the top line, we find the number 18, which is the product required. By the same process we find the numbers 24 and 20.

Having committed the multiplication table to memory, we are in a condition for performing any multiplication of common

numbers without difficulty. If, for example, we wish to multiply 1,728 by 2, we write the 2 under the 8 and draw a line thus: —

$$\begin{array}{r} 1728 \\ 2 \\ \hline 3456 \\ \hline\hline \end{array}$$

We then say twice 8 are 16. We write down the 6 and carry the 1, which belongs to the order of tens next above, to that order. Twice 2 are 4, and the 1 carried from the 16 of the last multiplication make 5. The number 5 being less than 10, there is no figure to carry in this case. We therefore say twice 7 are 14, where again we write 4 and carry 1, and twice 1 are 2, and 1 carried over from the last multiplication make 3.

It is clear that the number 1,728 is made up of the numbers 1,000, 700, 20, and 8, and the result of the multiplication would not be altered if we were to multiply these quantities separately and add them together. A Saint Andrew's cross or $\times$ is the sign of multiplication; and

$$\begin{array}{r} 1000 \times 2 = 2000 \\ 700 \times 2 = 1400 \\ 20 \times 2 = \quad 40 \\ 8 \times 2 = \quad 16 \\ \hline 3456 \\ \hline\hline \end{array}$$

Here, then, we see we have precisely the same result as in the former case. But the first expedient is the simpler, and is therefore commonly used. We shall also obtain the same result by adding 1,728 to 1,728, thus:—

$$\begin{array}{r} 1728 \\ 1728 \\ \hline 3456 \\ \hline\hline \end{array}$$

In this particular case it is as easy to add the number to itself as to multiply by 2. But when the multiplication proceeds to 6, 8, or any greater number of times, it would be very inconvenient to have to add the number to itself 6 or 8 times, and it is much easier to proceed by the common method of multiplication here explained. The number we multiply with is called the *multiplier*, and the number we multiply is called the *multiplicand*, while the number resulting from the multiplication is called the *product*. In the above example 2 is the multiplier, 1,728 the multiplicand, and 3,456 the product.

If the multiplier consists of two figures. instead of one, the same mode of procedure is pursued, except that the whole of the figures resulting from the multiplication of the higher of the two figures is shifted one place to the left. Thus, if the number 1,728 has to be multiplied by 22, the mode of procedure is as follows:—

```
  1728
    22
 ------
  3456
 3456
 ------
 38,016
```

Here the arithmetical process of multiplication is precisely the same with each of the two figures, only that in the case of the second multiplication the resulting number is set one place more to the left; and the two lines of partial products are then added together for the answer. It is, therefore, a rule in all multiplications where the multiplier consists of more figures than one, that the first figure of the product shall be set under that particular figure of the multiplier with which that particular line of multiplication is performed. If instead of 22 the multiplier had been 222, then the operation would have been as follows:—

```
   1728
    222
 -------
   3456
  3456
 3456
 -------
 383,616
```

Here, it will be observed, the same partial product is repeated in every case, but set one place more to the left; and the several lines of partial products are then added up for the total product of the multiplication.

In cases where one of the figures of the multiplier is a cipher, the only effect is to shift the figures over to the left one place, and which may be done by adding a cipher to the product if the cipher forms the last figure of the multiplier. Thus, 1,728 multiplied by 20, is 34,560, multiplied by 200 is 345,600, and multiplied by 2,000 is 3,456,000. If the cipher comes in the middle of the multiplier, as in multiplying by 202, we proceed as follows:—

```
    1728
     202
 -------
    3456
  3456
 -------
 349,056
```

Here we pass over the cipher altogether, except that we begin the succeeding line of multiplication one place more to the left than we should have done if the cipher had not been present; or, in other words, we begin the line pertaining to the next figure of the multiplier under that figure, just as would be done if any other figure than a cipher intervened. Indeed we might

write a line of ciphers as resulting from multiplication by a cipher; but as this line could not affect the value of the sum total, it is left out altogether. In multiplying numbers terminating with ciphers, or in multiplying *with* numbers terminating with ciphers, the mode of procedure is to perform the multiplication as if there were no ciphers, and then to annex as many ciphers to the product as there are ciphers in the multiplier and multiplicand together. Thus 65,000 multiplied by 3,300 is treated as if 65 had to be multiplied by 33, and then five ciphers are added to the product to give the correct answer.

GENERAL EXPLANATION OF THE METHOD OF PERFORMING MULTIPLICATION.

The foregoing explanations of the method of performing the multiplication of numbers will probably suffice to enable all ordinary questions in multiplication to be readily performed. But for the sake of clearness, it may be useful to recapitulate the several steps of the process.

Place the multiplier under the multiplicand, as in addition. Multiply the multiplicand separately by each significant figure of the multiplier, by which we shall obtain as many partial products as there are significant figures in the multiplier. Write these products under one another, so that the last figure of each shall be under that figure of the multiplier by which it has been produced. Add the partial products thus obtained, and their sum will be the total product.

It will often facilitate arithmetical calculations if we have committed to memory the products of numbers larger than those found in the common multiplication tables, and it is very important that these elementary multiples should be accurately and promptly recollected. In the following table the products of numbers are given as high as 20 times 20:

MULTIPLICATION TABLE EXTENDING TO 20 TIMES 20.

	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
20	40	60	80	100	120	140	160	180	200	220	240	260	280	300	320	340	360	380	400
19	38	57	76	95	114	133	152	171	190	209	228	247	266	285	304	323	342	361	
18	36	54	72	90	108	126	144	162	180	198	216	234	252	270	288	306	324		
17	34	51	68	85	102	119	136	153	170	187	204	221	238	255	272	289			
16	32	48	64	80	96	112	128	144	160	176	192	208	224	240	256				
15	30	45	60	75	90	105	120	135	150	165	180	195	210	225					
14	28	42	56	70	84	98	112	126	140	154	168	182	196						
13	26	39	52	65	78	91	104	117	130	143	156	169							
12	24	36	48	60	72	84	96	108	120	132	144								
11	22	33	44	55	66	77	88	99	110	122									
10	20	30	40	50	60	70	80	90	100										
9	18	27	36	45	54	63	72	81											
8	16	24	32	40	48	56	64												
7	14	21	28	35	42	49													
6	12	18	24	30	36														
5	10	15	20	25															
4	8	12	16																
3	6	9																	
2	4																		

DIVISION.

When a number has to be separated into two, three, or any other number of equal parts, it is done by means of *Division*, which enables us to determine the magnitude of one of those parts. If, for example, we wish to divide 12 inches into four equal parts, the length of each of those parts will be 3 inches. If we wish to divide it into three equal parts, the length of each of the parts will be 4 inches; or if we wish to divide it into two equal parts, the length of each part will be 6 inches.

The number which is to be decomposed or divided is called the *dividend*, the number of equal parts into which the number sought to be divided is called the *divisor*, and the magnitude of one of those parts obtained from the division is called the *quo tient.* Thus in dividing 12 by 3,

12 is the dividend,
3 is the divisor,
4 is the quotient.

It follows from this explanation of the process of division, that if we divide a number into two equal parts, one of those parts taken twice will reproduce the original number; or if we divide it into three equal parts, one of those parts taken three times will reproduce the original number. In all cases, indeed, the quotient multiplied by the divisor will produce the dividend. Hence division is said to be a rule which teaches us to find a number which, multiplied by the divisor, will reproduce the dividend. For example, if 35 has to be divided by 5, we seek for a number which, multiplied by 5, will produce 35. This number is 7, since 5 times 7 is 35. The manner of expression employed in this division is 5 in 35 goes 7 times, and 5 times 7 makes 35. The dividend, therefore, may be considered as a product, of which one of the factors is the divisor and the other the quotient. Thus, supposing we have 63 to divide by 7, we endeavour to find such a product that, taking 7 for one of its factors, the other factor multiplied by this shall produce exactly 63. Now 7×9 is such a product, and, consequently, 9 is the quotient obtained when we divide 63 by 7.

In the same sense in which multiplication above unity may be looked upon as a continued addition, so may division be looked upon as a continued subtraction. Thus as $7 \times 9 = 7+7+7+7+7+7+7+7+7$, so also $63 \div 9 = 63-7-7-7-7-7-7-7-7-7$. This may easily be seen by performing the operation of addition or subtraction. Thus 7 and 7 are 14, and 14 and 7 are 21, and 21 and 7 are 28, and 28 and 7 are 35, and 35 and 7 are 42, and 42 and 7 are 49, and 49 and 7 are 56, and 56 and 7 are 63. So in like manner 63 less 7 are 56, and 56 less 7 are 49, and 49 less 7 are 42, and 42 less 7 are 35, and 35 less 7 are 28, and 28 less 7 are 21, and 21 less 7 are 14, and 14 less 7 are 7, and 7 less 7 is 0.

We have seen that when we divide 12 inches by 4, we obtain 3 inches as the quotient. But if we divide 13 inches by 4 we shall have 4 parts of 3 inches each and 1 inch over, and if this inch be also divided into 4 equal parts, each of these parts will be one quarter of an inch. Hence 13 inches divided by 4 gives $3\frac{1}{4}$ inches. So if we divide 63 feet into lengths of 7 feet each we shall have exactly 9 of such lengths. But if we divide 64 feet into lengths of 7 feet each, we shall, after having performed the division, have 1 foot over. This foot is obviously just one sixty-third of the total length; and if we wish to distribute this residual foot equally among the whole of the other divisions, we must either divide it into 9 equal parts, and add 1 of these parts to each division, or we must divide it into 63 equal parts, and add 1 of these parts to each foot, or 7 of them to each division. It follows that 64 divided by 9 is equal to $7\frac{1}{9}$, or to $7\frac{7}{63}$, which is the same thing. So in dividing a plank 50 feet long into lengths of 4 feet each, we shall have 12 such lengths in the length of the plank, and we shall have 2 feet over. If we wish to distribute these 2 feet equally among the 12 divisions, so that no part of the plank may be cut to waste, then we must increase the length of each foot one forty-eighth part of 2 feet, or we must increase the length of each division one-twelfth part of 2 feet, or two-twelfth parts of 1 foot. Now, as the foot consists of 12 inches, two-twelfth parts are equal to 2 inches. Moreover, as a twenty-fourth part of a foot is equal to half an inch,

and a forty-eighth part of a foot is equal to a quarter of an inch, it follows that 8 forty-eighth parts are equivalent to 8 quarters of an inch, or to 2 inches, as before. Each division of the plank of 50 feet, therefore, must be 4 feet 2 inches long, in order that it may be cut without waste into 12 equal lengths.

If we have a number 50 which we wish to divide by another number 12, then we write the number as follows:—

$$\begin{array}{r} 12)50 \\ \hline 4\frac{2}{12} \end{array}$$

We say the twelves in 50, 4 times and 2 over, which two-twelfths is written as a vulgar fraction, and forms part of the quotient. But if we wish the answer to be in decimal fractions, we place a decimal point after the 50, and add thereto any number of ciphers, continuing the division in precisely the same manner as if the number were not a fraction at all. Thus—

$$\begin{array}{r} 12)50{\cdot}00000 \\ \hline 4{\cdot}16666, \text{ \&c.} \end{array}$$

Here we say, as before, the twelves in 50, 4 times and 2 over, which 2 we carry to the next succeeding place of figures, and say the twelves in 20 once and 8 over, the twelves in 80 6 times and 8 over, the twelves in 80 6 times and 8 over, and so on to infinity. We thus see not merely that the fraction $\frac{2}{12}$ths or $\frac{1}{6}$th, called the remainder, is left over when we divide 50 by 12, but that this fraction may be expressed decimally under the form of the infinite series of numbers ·16666, &c., which numbers, if carried on for ever, will be continually coming nearer to the quantity $\frac{1}{6}$th, but will never be absolutely equal to it, though sufficiently near thereto to answer all the purposes of practical computation.

A very little consideration will suffice to show us the reason of the process in division in which we carry the residual number to the next place of figures immediately succeeding. Thus, if we have to divide the number 963 by 3, we may, if we please, perform the operation by dividing the whole number into 900, 60, and 3, and dividing them separately. Now the third of 900 is obviously 300, the third of 60 is 20, and the third of 3 is 1, so that the third of the total number of 963 is 321. If, however, the number which we had to divide by 3 was 954, then in dividing the constituent numbers as before, we should have the third of 900 which is 300, the third of 50 which is 16, leaving 2 over,

which 2 has to be added to the 4 not yet divided, making it up to 6; and the third of 6 is 2. These numbers may be written as follows:—

900 divided by 3=300	900 divided by 3=300
60 divided by 3= 20	50 divided by 3= 16
3 divided by 3= 1	6 divided by 3= 2
321	318

By the ordinary method of division, the quantity would be written thus:—

Divisor 3)963 Dividend	Divisor 3)954 Dividend
321 Quotient.	318 Quotient.

Here, in the first example, we say the threes in 9, three times, which 3 we write under the 9; the threes in 6 twice, which 2 we write under the 6; and the threes in 3 once, which 1 we write under the 3. In the second example we say, as before, the threes in 9 three times; but the threes in 5 will only go once, leaving 2 as a remainder, which 2 when imported into the next inferior place of figures, will count ten times greater, or as 20; and we then say the threes in 24 eight times, which 8 we write under the 4. It will be recollected that as the second place of figures from the right is groups of tens, two of these groups when resolved into units must necessarily be 20.

The method of division here described is that used when any number has to be divided by another number consisting of only one figure. It is called *Short Division*. In the case of quantities which have to be divided by numbers consisting of two or more figures, this method would not be convenient, and another method called *Long Division* is commonly employed. If, for example, we had to divide 4967398 by 37, we may, no doubt, perform the question by the method of short division. But the remainders, when there are several figures in the di-

visor, become so large and perplexing, that it is much better to employ the method of long division, which is as follows:

```
               Dividend
  Divisor 37)4967398(134254 Quotient
             37
             ---
             126
             111
             ---
              157
              148
              ---
                93
                74
               ---
               199
               185
               ---
                148
                148
                ===
```

Here we first find how many times 37 are contained in 49, and it is clear it is contained only once. We write therefore 1 in the quotient, and multiply the divisor by it, placing the product under the 49, and we subtract the 37 from the 49, which shows that there is a remainder of 12. To this remainder we next bring down the figure of the original number which immediately succeeds the 49, and which in this case is 6. We then consider how many times 37 are contained in 126, and we find that it is three times. We write the 3 in the quotient, and multiply the divisor by it, when we find that the product is 111, which sum we subtract from the 126, and find that we have a remainder of 15. To this 15 we next bring down the figure of the original sum succeeding to that which we brought down before, and which in this case is 7, and we consider how many times 37 will go in 157. We find that it will go four times, and we write the 4 in the quotient as before, and proceed to multiply the divisor by it, and to subtract the product 148 from the 159, which will leave a remainder of 9. Carrying on this process until we have successively brought down all the figures of

the original sum that had to be divided, we find that the quotient is 134254, which number, if multiplied by 37, will reproduce the 4967398 with which we set out. When after performing the division there is found to be a remainder, it may either be written as the numerator of a vulgar fraction in the answer, the divisor being the denominator, or a decimal point may be introduced after the last figure, and any desired number of ciphers may be added thereto, when, by continuing the division, the remainder will be obtained in decimal fractions.

The operation of division is indicated by the sign $\div$ and as $12 \times 12 = 144$, so $144 \div 12 = 12$.

In cases in which the divisor is composed of two factors, it is a common practice, instead of employing the method of long division to divide successively by the two factors by the method of short division, which is more rapidly done. Thus if a number has to be divided by, say 36, the same result will be obtained if it is divided by 6 and the quotient be then again divided by 6. Or, if we have to divide by 42, we may divide by 6 and then by 7; if we have to divide by 63, we may divide by 9 and then by 7; and so of all other numbers possessing similar factors.

As, by annexing a cipher at the end of any number, we multiply its amount by 10, so by abstracting a cipher from the end of any number we divide its amount by 10. Thus $2 \times 10 = 20$ and $20 \times 10 = 200$. So also $200 \div 10 = 20$ and $20 \div 10 = 2$. If, therefore, we have a divisor containing a number of ciphers, we may leave them out of the account in performing division: but in such case we must count off as decimals an equal number of figures as we have excluded of ciphers. Thus $1728 \div 10 = 172{\cdot}8$ or $1728 \div 100 = 17{\cdot}28$ or $1728 \div 1000 = 1{\cdot}728$. So $444 \div 20 = 22{\cdot}2$ and $999 \div 30 = 33{\cdot}3$ or $999 \div 300 = 3{\cdot}33$.

GENERAL EXPLANATION OF THE METHOD OF PERFORMING DIVISION.

Short Division.—Divide the first figure of the dividend by the divisor, and place the quotient under the same figure of the dividend. Prefix the remainder to the next figure of the divi-

dend and divide the number thus obtained by the divisor. Place the quotient under the second figure of the dividend, and prefix the remainder to the third figure of the dividend. Divide the number thus obtained by the divisor, and proceed as before, continuing this process until you arrive at the units place of the dividend, when the division will be complete.

Long Division.—Write the divisor on the left of the dividend, separated from it by a line. Place another line to the right of the dividend after the units place to separate the quotient from the dividend—the quotient being afterwards written on the right of that line.

Count off from the left of the dividend or from its highest place as many figures as there are places in the divisor. If the number formed by these be less than the divisor, then count off one more. Consider these figures as forming one number, and find how often the divisor is contained in that number. It will always be contained in it less than ten times, and therefore the quotient will always consist of a single figure. Place this single figure as the first figure of the quotient.

Multiply the divisor by this single figure, and place the product under those figures of the dividend which were taken off on the left, and subtract such product from the number above it, by which we obtain the first remainder. This remainder must always be less than the divisor.

On the right of the first remainder place that figure of the dividend which next succeeds those which were cut off to the left. Find how often the divisor is contained in the number thus formed, and place the resulting figure of the quotient next to the figure of the quotient already found. Multiply the divisor by this figure, and proceed as before, until all the succeeding figures of the dividend have been brought down, when the division will be complete.

NATURE AND PROPERTIES OF FRACTIONS.

It has already been stated that a fraction which has the same numerator and denominator is exactly equal to 1, and therefore

such a fraction is of the same value as an *integer* or whole number. For example, the fractions

$$\tfrac{2}{2},\ \tfrac{3}{3},\ \tfrac{4}{4},\ \tfrac{5}{5},\ \tfrac{6}{6},\ \tfrac{7}{7},\ \tfrac{8}{8},\ \tfrac{9}{9},\ \&c.,$$

are all equal to 1, and are therefore equal to one another.

All fractions of which the numerator is less than the denominator have a less value than unity; for if a number be divided by another number greater than itself, the result must be less than 1. If we cut a lath 2 feet long into three equal lengths, one of those lengths will certainly be shorter than a foot. Hence it is evident that $\frac{2}{3}$ is less than 1, and for the reason that the numerator 2 is less than the denominator 3. If, on the contrary, the numerator be greater than the denominator, then it will follow that the fraction will be greater than 1. Thus $\frac{3}{2}$ is greater than 1, for $\frac{3}{2}$ is equal to $\frac{2}{2}$ and $\frac{1}{2}$, and as $\frac{2}{2}$ is equal to 1, then $\frac{3}{2}$ will be equal to $1\frac{1}{2}$. In the same manner $\frac{4}{3}$ is equal to $1\frac{1}{3}$, $\frac{5}{3}$ to $1\frac{2}{3}$, $\frac{7}{3}$ to $2\frac{1}{3}$, and so on. In all such cases it is sufficient to divide the upper number by the lower, and if there is a remainder, to write it as the numerator of the residual fraction, and the divisor as the denominator. If, for example, the fraction were $\frac{43}{12}$, we should divide the 43 by 12, when we should get 3 as the integer and $\frac{7}{12}$ as a remainder; or, in other words, we should obtain the number $3\frac{7}{12}$. Fractions like $\frac{43}{12}$, which have the numerator greater than the denominator, are termed *improper fractions*, to distinguish them from fractions properly so called, which, having the numerator less than the denominator, are less than unity, or an integer.

As we can only understand what the fraction $\frac{7}{12}$ is when we know the meaning of $\frac{1}{12}$, we may consider the fractions whose numerator is unity as the foundation of all others. Such are the fractions

$$\tfrac{1}{2},\ \tfrac{1}{3},\ \tfrac{1}{4},\ \tfrac{1}{5},\ \tfrac{1}{6},\ \tfrac{1}{7},\ \tfrac{1}{8},\ \tfrac{1}{9},\ \tfrac{1}{10},\ \tfrac{1}{11},\ \tfrac{1}{12},\ \tfrac{1}{13},\ \&c.,$$

and it is observable that these fractions go on continually diminishing, for the more we divide an integer, or the greater the number of parts into which we distribute it, the less does each part become. Thus $\frac{1}{100}$ is less than $\frac{1}{10}$; $\frac{1}{1000}$ is less than $\frac{1}{100}$; $\frac{1}{10000}$ is less than $\frac{1}{1000}$; and as we increase the denominator of

the fraction, the less does the value of the fraction become. If, therefore, we suppose the denominator to be made infinitely large, the fraction will become equal to nothing. To express the idea of infinity, we make use of the symbol ∞, and we may, therefore, say that the fraction $\frac{1}{\infty}=0$. Now, we know that if we divide the dividend 1 by the quotient $\frac{1}{\infty}$, which is equal to nothing, we obtain again the divisor ∞. Hence we learn that infinity is the quotient arising from the division of 1 by 0. Thus 1 divided by 0 expresses a number infinitely great. But $\frac{1}{0}$ is certainly only the half of $\frac{2}{0}$, or the third of $\frac{3}{0}$; so that it would appear as if one infinity may be twice or three times greater than another. It will be obvious that as the fractions

$$\tfrac{2}{2},\ \tfrac{3}{3},\ \tfrac{4}{4},\ \tfrac{5}{5},\ \tfrac{6}{6},\ \tfrac{7}{7},\ \tfrac{8}{8},\ \tfrac{9}{9},\ \&c.,$$

are all equal to one another, each of them being in fact equal to 1, so also the fractions

$$\tfrac{2}{1},\ \tfrac{4}{2},\ \tfrac{6}{3},\ \tfrac{8}{4},\ \tfrac{10}{5},\ \tfrac{12}{6},\ \tfrac{14}{7},\ \&c.,$$

are all equal to one another, each of them being in fact equal to 2; for the numerator of each divided by the denominator gives 2. So likewise the fractions

$$\tfrac{3}{1},\ \tfrac{6}{2},\ \tfrac{9}{3},\ \tfrac{12}{4},\ \tfrac{15}{5},\ \tfrac{18}{6},\ \tfrac{21}{7},\ \&c.,$$

are equal to one another, since in fact each of them is equal to 3.

Now it is clear that as $\frac{6}{2}$ is the same as $\frac{12}{4}$, and as $\frac{9}{3}$ is the same as $\frac{18}{6}$, both being equal to 3, the value of a fraction will not be changed if we multiply numerator and denominator by the same number. Thus in the case of the fraction $\frac{1}{2}$, if we multiply numerator and denominator by 4, we shall have $\frac{4}{8}$ which is clearly equal to $\frac{1}{2}$. So also the fractions

$$\tfrac{1}{2},\ \tfrac{2}{4},\ \tfrac{3}{6},\ \tfrac{4}{8},\ \tfrac{5}{10},\ \tfrac{6}{12},\ \tfrac{7}{14},\ \tfrac{8}{16},\ \tfrac{10}{20},\ \&c.,$$

are equal, each of them being equal to $\frac{1}{2}$. The fractions

$$\tfrac{1}{3},\ \tfrac{2}{6},\ \tfrac{3}{9},\ \tfrac{4}{12},\ \tfrac{5}{15},\ \tfrac{6}{18},\ \tfrac{7}{21},\ \tfrac{8}{24},\ \tfrac{9}{27},\ \tfrac{10}{30},\ \&c.,$$

are also equal, each being equal to $\frac{1}{3}$; and the fractions

$$\tfrac{2}{3},\ \tfrac{4}{6},\ \tfrac{6}{9},\ \tfrac{8}{12},\ \tfrac{10}{15},\ \tfrac{12}{18},\ \tfrac{14}{21},\ \tfrac{16}{24},\ \tfrac{18}{27},\ \&c.,$$

are also equal, each of them being equal to $\frac{2}{3}$.

Now of all the equivalent fractions

$$\tfrac{2}{3},\ \tfrac{4}{6},\ \tfrac{6}{9},\ \tfrac{8}{12},\ \tfrac{12}{18},\ \tfrac{14}{21},\ \tfrac{16}{24},\ \tfrac{18}{27},\ \text{\&c.},$$

the quantity $\frac{2}{3}$ is that of which it is easiest to form a definite idea. It is usual, therefore, when we have any such fraction as $\frac{16}{24}$ or $\frac{18}{27}$, to *reduce it to its lowest terms*, by dividing numerator and denominator by some number that will divide each without a remainder. This division, it is clear, will not affect the numerical value of the fraction; for if we can multiply both numerator and denominator by the same number without affecting the value, so we may divide both without affecting the value; as by such division we bring back the fraction of which both portions had been multiplied to the original expression.

The number by which the numerator and denominator of a fraction may be divided without leaving a remainder is called a *common divisor;* and so long as we can find for the numerator and denominator a common divisor, it is certain that the fraction may be reduced to a lower form. But if we cannot find such common divisor, the fraction is in its lowest form already. Thus in the fraction $\frac{48}{120}$, we at once see that both terms are divisible by 2, and, performing this division, the fraction becomes $\frac{24}{60}$; which, if again divided by 2, becomes $\frac{12}{30}$, and which in like manner, by another division by 2, becomes $\frac{6}{15}$. This, it will be obvious, cannot any longer be divided by 2, but it may be by 3, when the expression becomes $\frac{2}{5}$; and as this cannot be divided by any other number than 1, it follows that the fraction is now in its lowest terms. Now $2 \times 2 \times 2 \times 3 = 24$, and instead of the successive divisions by 2, by 2, by 2, and by 3, we may divide at once by the product of these quantities, or 24; and dividing numerator and denominator of $\frac{48}{120}$ by 24, we have $\frac{2}{5}$ as before.

The property of fractions retaining the same value, whether we multiply or divide their numerator and denominator by the same number, carries this important consequence—that it enables fractions to be easily added or subtracted, after having first brought them to the same denomination. If, for example, we had to add together $\frac{3}{8}$, $\frac{1}{4}$, $\frac{1}{16}$, and $\frac{2}{32}$ of an inch, we could not do so easily unless we brought the whole of these quantities to

2*

thirty-seconds. When so reduced the quantities will be $\frac{12}{32}$, $\frac{8}{32}$, $\frac{2}{32}$, $\frac{2}{32}$, the sum of which is clearly $\frac{24}{32}$, or, dividing numerator and denominator by 8, the expression becomes $\frac{3}{4}$.

All whole numbers, it is clear, may be expressed by fractions, since any whole number may be divided into any number of parts. For example, 6 is the same as $\frac{6}{1}$. It is also the same as $\frac{12}{2}$, $\frac{18}{3}$, $\frac{24}{4}$, $\frac{36}{6}$, and an infinite number of other expressions which all have the same value.

ADDITION AND SUBTRACTION OF FRACTIONS.

When fractions have the same denomination there is no more difficulty in adding or subtracting them than there is in adding or subtracting whole numbers. Thus $\frac{1}{8}+\frac{5}{8}$ is manifestly $\frac{6}{8}$, and $\frac{5}{8}-\frac{1}{8}$ is obviously $\frac{4}{8}$. So also

$$\frac{7}{100}+\frac{9}{100}-\frac{12}{100}-\frac{15}{100}+\frac{20}{100}=\frac{9}{100}.$$
$$\frac{24}{50}-\frac{7}{50}-\frac{12}{50}+\frac{31}{50}=\frac{36}{50} \text{ or } \frac{18}{25}.$$
$$\frac{16}{20}-\frac{3}{20}-\frac{11}{20}+\frac{14}{20}=\frac{16}{20} \text{ or } \frac{4}{5}.$$
$$\text{Also } \frac{1}{3}+\frac{2}{3}=\frac{3}{3}=1 \text{ and } \frac{2}{4}-\frac{3}{4}+\frac{1}{4}=\frac{0}{4}=0.$$

But when fractions have not the same denominators, then, before we can add or subtract them, we must change them for others of equal value which have the same denominators. For example, if we wish to add the fractions $\frac{1}{2}$ and $\frac{1}{3}$, we must consider that $\frac{1}{2}$ is the same as $\frac{3}{6}$, and that $\frac{1}{3}$ is equivalent to $\frac{2}{6}$. We have, therefore, instead of the fractions first proposed, the equivalent fractions $\frac{3}{6}$ and $\frac{2}{6}$, the sum of which is $\frac{5}{6}$. If the two fractions were united by the sign $-$, we should have $\frac{1}{2}-\frac{1}{3}$ or $\frac{3}{6}-\frac{2}{6}=\frac{1}{6}$. Again, if the fractions proposed be $\frac{3}{4}+\frac{5}{8}$, then as $\frac{3}{4}$ is the same as $\frac{6}{8}$, the sum will be $\frac{6}{8}+\frac{5}{8}=\frac{11}{8}=1\frac{3}{8}$. If the sum of $\frac{1}{3}$ and $\frac{1}{4}$ were required, then as $\frac{1}{3}=\frac{4}{12}$ and $\frac{1}{4}=\frac{3}{12}$, the sum is $\frac{4}{12}+\frac{3}{12}=\frac{7}{12}$.

These cases are simple and easily reduced. But we may have a great number of fractions to reduce to a common denomination, which require a more elaborate process. For example, we may have $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{4}$, $\frac{4}{5}$, $\frac{5}{6}$, to reduce to a common denomination, in order that we may add them together. The solution of such a case depends upon finding a number that shall be divisible by

all the denominators of these fractions. Here we proceed according to the following rule:

TO REDUCE FRACTIONS TO A COMMON DENOMINATION.

RULE.—*Multiply each numerator into every denominator except its own for a new numerator, and multiply all the denominators together for a common denominator.*

When this operation has been performed, it will be found that the numerator and denominator of each fraction have been multiplied by the same quantity, and consequently that the fractions retain the same value, while they are at the same time brought to a common denomination.

Example. Reduce $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{4}$, $\frac{4}{5}$, and $\frac{5}{6}$, to a common denomination.

$$1\times3\times4\times5\times6=360 \text{ and } 360\div6=60 \text{ and } 60\div2=30$$
$$2\times2\times4\times5\times6=480 \text{ and } 480\div6=80 \text{ and } 80\div2=40$$
$$3\times2\times3\times5\times6=540 \text{ and } 540\div6=90 \text{ and } 90\div2=45$$
$$4\times2\times3\times4\times6=576 \text{ and } 576\div6=96 \text{ and } 96\div2=48$$
$$5\times2\times3\times4\times5=600 \text{ and } 600\div6=100 \text{ and } 100\div2=50$$
$$2\times3\times4\times5\times6=720 \text{ and } 720\div6=120 \text{ and } 120\div2=60$$

Here, then, we first multiply 1, which is the numerator of the fraction $\frac{1}{2}$, by the denominators of all the other fractions in succession. We next multiply the number 2, which is the numerator of the fraction $\frac{2}{3}$, by the denominators of all the other fractions—excepting always its own denominator—and we proceed in this manner through all the fractions whatever their number may be. We next multiply all the denominators together for the common denominator. Proceeding in this way we find the first numerator to be 360, the second 480, the third 540, the fourth 576, and the fifth 600; while the new denomiinator we find to be 720. It is clear, however, that these fractions are not in their lowest terms, and that the numerator and denominator of each may be divided by some common number without leaving a remainder. We may try 6 as such a divisor, and we shall find that the numerators will then become 60, 80, 90, 96, and 100, and the denominator 120. These numbers,

however, are still divisible by 2, and performing the division the numerators become 30, 40, 45, 48, and 50, and the denominator becomes 60. The same result would have been attained if we had divided at once by 12. And as we cannot effect any further division upon all of the numbers by one common number, without leaving a remainder in the case of some of them, the fractions, we must conclude, are now in their lowest common terms. To add together these fractions we have only to add together the numerators, and place the common denominator under the sum. Performing this addition we find that in this case we have $\frac{213}{60}$, and as $\frac{60}{60}$ are equal to 1, it follows that $\frac{213}{60}$ are equal to 3 and $\frac{33}{60}$, or $3\frac{11}{20}$.

It is easy to prove that the fractions

$$\tfrac{1}{2},\ \tfrac{2}{3},\ \tfrac{3}{4},\ \tfrac{4}{5},\ \text{and}\ \tfrac{5}{6}$$

are of precisely the same value as the fractions

$$\tfrac{30}{60},\ \tfrac{40}{60},\ \tfrac{45}{60},\ \tfrac{48}{60},\ \tfrac{50}{60}$$

which have been substituted for them. Dividing numerator and denominator of the first term by 30 we obtain $\frac{1}{2}$; dividing numerator and denominator of the second term by 20 we obtain $\frac{2}{3}$; 15 is the divisor in the case of the third term when we obtain $\frac{3}{4}$; 12 is the divisor in the case of the fourth term when we obtain the fraction $\frac{4}{5}$; and 10 is the divisor in the last case when we obtain the fraction $\frac{5}{6}$. Dividing the numerator and denominator of each of the transformed fractions, therefore, by the greatest number that will divide both without a remainder, we get the fractions

$$\tfrac{1}{2},\ \tfrac{2}{3},\ \tfrac{3}{4},\ \tfrac{4}{5},\ \text{and}\ \tfrac{5}{6}$$

which, it will be seen, are the fractions with which we set out, and they are now in their lowest terms, but are no longer of one common denomination. The lowest terms with a common denominator are

$$\tfrac{30}{60},\ \tfrac{40}{60},\ \tfrac{45}{60},\ \tfrac{48}{60},\ \text{and}\ \tfrac{50}{60}$$

as determined above.

The subtraction of fractions from one another is accomplished by reducing them to a common denomination as for ad-

dition, and then by subtracting the less numerator from the greater. Thus if we have to subtract $\frac{2}{3}$ from $\frac{5}{7}$, we must reduce them to a common denomination by the process already explained, when the first becomes $\frac{14}{21}$, and the second $\frac{15}{21}$, so that $\frac{5}{7}$ exceeds $\frac{2}{3}$ in magnitude by $\frac{1}{21}$. So also if we have to subtract $\frac{3}{5}$ from $\frac{5}{8}$, the first fraction becomes by the process of reduction $\frac{24}{40}$, and the second $\frac{25}{40}$, so that $\frac{3}{5}$ taken from $\frac{5}{8}$ leaves $\frac{1}{40}$.

As whenever the numerator of a fraction is a larger number than the denominator, the value of the fraction is greater than unity, and is equal to unity when numerator and denominator is the same, we have only to divide the numerator by the denominator to find the number of integers which the fraction contains. So in subtracting a fraction from a whole number, we must break one or more integers up into fractions of the same denomination as that which has to be subtracted. Thus if we have to take $\frac{20}{60}$ from 1, we must instead of the 1 write $\frac{60}{60}$, and $\frac{20}{60}$ taken therefrom obviously leaves $\frac{40}{60}$. If we have to add together such sums as $3\frac{1}{2}$ and $2\frac{2}{3}$, we see at once that the whole numbers when added will be 5, and the equivalent fractions under a common denominator will be $\frac{3}{6}$ and $\frac{4}{6}$ or $\frac{7}{6}$, which is $1\frac{1}{6}$, so that the total quantity will be $6\frac{1}{6}$.

The addition and subtraction of decimal fractions are performed in precisely the same way as the addition and subtraction of whole numbers—the only precaution necessary being to place the decimal point in the proper place. Thus 78963·874 + 83952·2 + 364·003 + 10000·997 are added together as follows:

$$
\begin{array}{r}
78963{\cdot}874 \\
83952{\cdot}2 \\
364{\cdot}003 \\
10000{\cdot}997 \\
\hline
173281{\cdot}074
\end{array}
$$

Here, beginning as in the addition of whole numbers with the first column to the right, we find that 7 and 3 are 10 and 4 are 14. We set down the 4 beneath the column and carry 1 to the next column. Adding up the next column, we find only two significant figures in it, and we say 1 added to 9 makes 10, which added to 7 makes 17. We set down the 7 and carry the 1 as before to the next column, which when added up we find to be 20. This means 20 tenths, and we set down the 0 and carry the 2 to the next column just as in simple addition. So likewise in subtraction, if we take 2·25 from 4·75, the result

will be 2·50; or if we take 1·79 from 3, the result is 1 21. In such a case we write the 3 thus:

3·00
1·79
1·21

Here we write the 3 with a decimal point after it, and we add as many ciphers after the decimal point as there are decimal figures to be subtracted, or we suppose those ciphers to be added. This does not alter the value of the 3, as 3 with no fractions added to it is just 3. Performing the subtraction we say 9 from 10 leaves 1, and 8 taken from 10 leaves 2, and 2 from 3 leaves 1, just as in simple subtraction.

MULTIPLICATION AND DIVISION OF FRACTIONS.

If we wish to multiply a fraction any number of times, it is clear that it is only the numerator we must multiply. Thus if we multiply $\frac{1}{8}$ of an inch by 3, it is obvious that we shall get $\frac{3}{8}$ of an inch as the product of the multiplication, or $\frac{1}{8}$ repeated 3 times. We have already seen that to multiply both terms of a fraction by any number does not alter the value of the fraction, and if we were to multiply the numerator and denominator of the fraction $\frac{1}{8}$ by 3 we should get $\frac{3}{24}$, which is just the same as $\frac{1}{8}$. Thus also—

3 times $\frac{1}{2}$ makes $\frac{3}{2}$ or $1\frac{1}{2}$.
3 times $\frac{1}{3}$ makes $\frac{3}{3}$ or 1.
3 times $\frac{1}{6}$ makes $\frac{3}{6}$ or $\frac{1}{2}$.
4 times $\frac{5}{12}$ makes $\frac{20}{12}$ or $1\frac{8}{12}$ or $1\frac{2}{3}$.

Instead, however, of multiplying the numerator, we may attain the same end by dividing the denominator, and this is a preferable practice when it can be carried out, as it shortens the arithmetical operation. Thus $\frac{1}{4}$ multiplied by 2 is $\frac{2}{4}$ or $\frac{1}{2}$. But by dividing the denominator of $\frac{1}{4}$ by 2, we obtain the same quantity of $\frac{1}{2}$ at one operation. So also if we have to multiply $\frac{8}{9}$ by 3 we obtain $\frac{24}{9}$, or $\frac{8}{3}$. But if, instead of multiplying the numerator, we divide the denominator, we obtain the $\frac{8}{3}$ at one operation. In the same way $\frac{13}{24}$ multiplied by 6 is equal $\frac{13}{4}$, or $3\frac{1}{4}$.

Where the integer with which the multiplication is performed is exactly equal to the denominator of the fraction, the product will be equal to the numerator. Thus—

$$\tfrac{1}{2}\times 2=1$$
$$\tfrac{2}{3}\times 3=2$$
$$\tfrac{3}{4}\times 4=3$$

Having now shown how a fraction may be multiplied by an integer, the next step is to show how a fraction may be divided by an integer; and just as a fraction may be multiplied by dividing the denominator, so may a fraction be divided by multiplying the denominator. It is clear that if we divide half an inch into two parts, each of these parts will be $\frac{1}{4}$ of an inch, and we divide quarter of an inch into two parts, each of those parts will be $\frac{1}{8}$ of an inch, so that $\frac{1}{2}\div 2=\frac{1}{4}$ and $\frac{1}{4}\div 2=\frac{1}{8}$, which quantities we obtain by successively multiplying the denominators. We may accomplish the same object by dividing the numerator where it is divisible without a remainder. Thus $\frac{2}{3}$ divided by 2 is clearly $\frac{1}{3}$, and $\frac{6}{7}$ divided by 3 is $\frac{2}{7}$. Thus also

$\frac{12}{25}$ divided by 2 gives $\frac{6}{25}$,
$\frac{12}{25}$ divided by 3 gives $\frac{4}{25}$,
$\frac{12}{25}$ divided by 4 gives $\frac{3}{25}$.

When the numerator is not divisible by the divisor without a remainder, the fraction may be put into some equivalent form, when the division may be effected. Thus if we had to divide $\frac{3}{4}$ by 2, we might turn it into the equivalent fraction $\frac{6}{8}$, which, divided by 2, gives $\frac{3}{8}$. But the same number is more conveniently found by multiplying the denominator instead of by dividing the numerator.

We have next to consider the case where one fraction has to be multiplied by another. Thus if the fraction $\frac{2}{3}$ has to be multiplied by the fraction $\frac{4}{5}$, we have first to remember that the expression $\frac{2}{3}$ means 2 divided by 3, and we may first multiply by 4, which produces $\frac{8}{3}$, and then divide by 5, which produces $\frac{8}{15}$. Hence, in multiplying a fraction by a fraction, we multiply the numerators together for the new numerator, and the denominators together for the new denominator. Thus,

$\frac{1}{2}\times\frac{2}{3}$ gives the product $\frac{2}{6}$ or $\frac{1}{3}$,
$\frac{2}{3}\times\frac{4}{5}$ gives $\frac{8}{15}$,
$\frac{3}{4}\times\frac{5}{12}$ gives $\frac{15}{48}$ or $\frac{5}{16}$.

Finally, we have to show how one fraction may be divided by another. If the two fractions have the same number for a denominator, the division takes place only with respect to the numerators. An inch being $\frac{1}{12}$ of a foot, it is clear that $\frac{3}{12}$ is contained in $\frac{9}{12}$ just as often as 3 inches is contained in 9 inches or 3 times; and in the same manner, in order to divide $\frac{8}{12}$ by $\frac{9}{12}$, we have only to divide 8 by 9, which gives $\frac{8}{9}$. So also $\frac{6}{20}$ is contained 3 times in $\frac{18}{20}$, and $\frac{7}{100}$ 9 times in $\frac{49}{100}$. But when the fractions have not the same denominator, then we must reduce them to a common denominator by the method of reduction already explained. This result, expressed in words, will be as follows:—Multiply the numerator of the dividend by the denominator of the divisor for the new numerator, and the denominator of the dividend by the numerator of the divisor for the new denominator. Thus $\frac{5}{8}$ divided by $\frac{2}{3}=\frac{15}{16}$, and $\frac{3}{4}$ divided by $\frac{1}{2}=\frac{6}{4}$ or $\frac{3}{2}$, or $1\frac{1}{2}$, and $\frac{25}{48}$ divided by $\frac{5}{6}=\frac{150}{240}$ or $\frac{5}{8}$. This rule is commonly expressed in the following form:—Invert the terms of the divisor so that the denominator may be in the place of the numerator. Multiply the fraction which is the dividend by this inverted fraction, and the product will be the quotient sought.

Thus $\frac{3}{4}$ divided by $\frac{1}{2}=\frac{3}{4}\times\frac{2}{1}=\frac{6}{4}=1\frac{1}{2}$. Also, $\frac{5}{8}$ divided by $\frac{2}{3}=\frac{5}{8}\times\frac{3}{2}=\frac{15}{16}$, and $\frac{25}{48}$ divided by $\frac{5}{6}=\frac{25}{48}\times\frac{6}{5}=\frac{150}{240}$ or $\frac{5}{8}$.

If we have a line 100 feet long, and if we divide it in half, we shall manifestly have two lines each 50 feet long. So if we divide it into lengths of 25 feet, we shall have 4 such lengths; if we divide it into lengths of 2 feet each, we shall have 50 such lengths; and if into 1 foot lengths, we shall have 100 of them; if into lengths of half a foot, we shall have 200 lengths; and if into lengths of $\frac{1}{4}$ of a foot, we shall have 400 such lengths. Hence,

100 divided by 100 = 1
100 divided by 50 = 2
100 divided by 25 = 4
100 divided by 1 100
100 divided by $\frac{1}{2}$ = 200
100 divided by $\frac{1}{4}$ = 400

We see, therefore, that to divide a number by the fraction $\frac{1}{4}$

is equivalent to multiplying it by 2; to divide by the fraction $\frac{1}{4}$ is the same as to multiply by 4. So, further, if we divide 1 by the fraction $\frac{1}{1000}$, the quotient is 1,000, and 1 divided by $\frac{1}{10000}$ is 10,000. As, then, the fraction gets smaller and smaller, the quotient gets greater and greater, so that we are enabled to conceive that a number divided by 0 will be indefinitely great, since in fact there will be an indefinitely great number of nothings in it.

As every number whatever, divided by itself, produces unity, so a fraction, divided by itself, produces unity. Thus $\frac{3}{4} \div \frac{3}{4} = \frac{3}{4} \times \frac{4}{3} = 1$.

The multiplication of decimal fractions is performed in precisely the same way as the multiplication of whole numbers, and we must mark off in the product as many decimal places as there are in the multiplier and multiplicand together. Thus 1·0025 multiplied by 2·5 = 2·50625; also, ·0048 multiplied by ·000012 = 0000000576.

The division of decimals is performed in the same way as the division of common numbers; and if the number of decimal places in the divisor be the same as in the dividend, the quotient thus obtained will be the quotient required, and will be a whole number. But if the number of decimals in the dividend exceed that in the divisor, mark off in the quotient obtained by this division as many decimal places as make up the difference. But if the number of decimals in the divisor exceed that in the dividend, annex as many ciphers to the quotient as make up the difference. Thus ·805 divided by 2·3 = ·35, and 2·5 divided by ·32 = 7·8125.

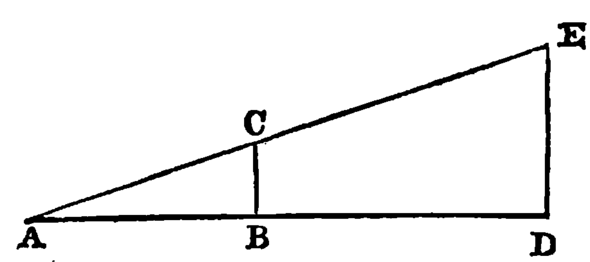
The number 3·045 denotes 3 *units*, 0 *tenths*, 4 *hundredths*, and 5 *thousandths*, and it might be written $3+\frac{0}{10}+\frac{4}{100}+\frac{5}{1000}$, and the number 3·47 might be written $3+\frac{4}{10}+\frac{7}{100}$, or it might be written $\frac{300+40+7}{100}=\frac{347}{100}$. So also $13·75 = 13\frac{75}{100} = 13\frac{3}{4}$, and $23·0625 = 23\frac{625}{100000} = 23\frac{1}{16}$. Also, $4·35 = 4+\frac{3}{10}+\frac{5}{100}$, or to $\frac{4}{1}+\frac{3}{10}+\frac{5}{100}$; or by reducing the fractions to the same denomination it is $\frac{400}{100}+ {}^{30} +\frac{5}{100}=\frac{435}{100}$. So $\frac{562}{100}$, put in the form of a decimal, will be 5·62, for $\frac{562}{100}=\frac{500}{100}+\frac{60}{100}+\frac{2}{100}$. But $\frac{100}{100}=1$, and therefore $\frac{500}{100}=5$, and $\frac{60}{100}=\frac{6}{10}$, and $5+\frac{6}{10}+\frac{2}{100}=5·62$.

PROPORTION.

The *Proportion* or *Ratio* of one quantity to another is the number which expresses what fraction the former is of the latter, and is therefore obtained by dividing the former by the latter.

The most distinct idea of proportion is obtained by reference to a triangle such as that here figured, where AB has the same proportion to BC that AD has to DE. It is clear that if the quantities AB, AD, and BC are fixed, the quantity DE will also be determined, as we have only to draw the line AE through C until

Fig. 1.



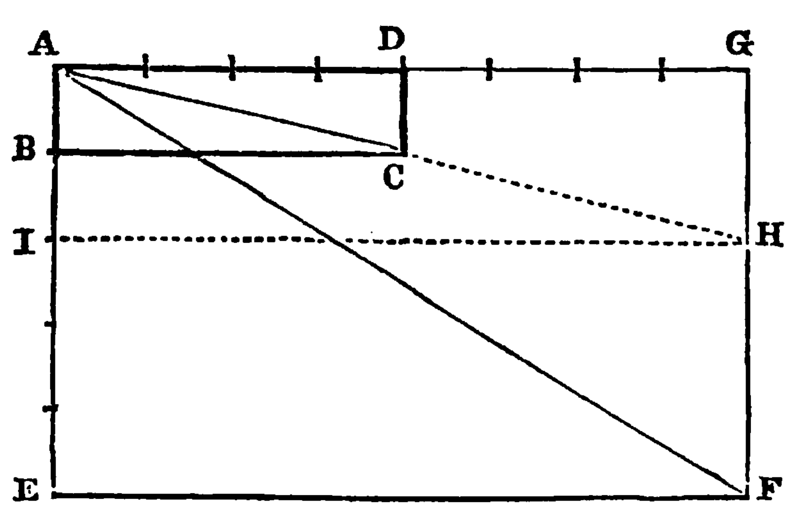
it intersects the vertical line DE, which it will thereby cut off to the proper length. Thus also the ratio 108 to 144, or as it is written 108 : 144, is $\frac{108}{144} = \frac{3}{4}$. A proportion is usually stated as follows: 2 is to 4 as 4 is to 8, or 2 : 4 :: 4 : 8; and in all cases of proportion the product of the first and fourth terms are equal to the product of the second and third terms. This is expressed by saying that the product of the extremes is equal to the product of the means. So $2 \times 8 = 4 \times 4$. Conversely, if the product of any two numbers equal the product of other two, then the four numbers are proportionals. The method by which we find a fourth proportional to three given quantities, by multiplying together the second and third and dividing by the first, is what is termed the RULE OF THREE. If a yard of calico costs 1 shilling, it is clear that 20 yards will cost 20 shillings; and we say, therefore, 1 yard is to 20 yards as 1 shilling is to 20 shillings; or we say, 3 inches : 12 inches :: 12 inches : 48 inches. Here we obtain the 48 by multiplying together 12 and 12, which makes 144, and which divided by 3 gives 48.

Proportion is in fact a mere question of scale. If we make

a model or drawing of a house or a machine, we may make it on the scale of $\frac{1}{4}$ of an inch to the foot, or $\frac{1}{2}$ an inch to the foot, or 1 inch to the foot, or $1\frac{1}{2}$ inches to the foot, or on any scale whatever. But the object, when constructed of the full size, will be precisely the same on whatever scale the model or drawing has been formed. If the scale be $\frac{1}{4}$ of an inch to the foot, then it is clear the object when formed of full size will be 48 times larger than the model or drawing—that is, it will be 48 times longer, 48 times broader, and 48 times higher. So in like manner if the $\frac{1}{2}$ inch scale be employed, the object will be 24 times larger; if the scale be 1 inch, it will be 12 times larger; and if the scale be $1\frac{1}{2}$ inches to the foot, it will be 8 times larger. So in like manner £20 bears the same proportion to £1 that 20 shillings bears to 1 shilling. But £20 are 400 shillings, and £1 are 20 shillings. Hence, by transforming the pounds into shillings, we see that 400 shillings bear the same relation to 20 shillings that 20 shillings bear to 1 shilling; or, in other words, 400 : 20 :: 20 : 1.

If we take a rectangular figure such as ABCD, say 4 inches long and 1 inch wide, and if we enlarge this figure by making it 4 inches longer and 4 inches broader, we see at a glance that the resulting rectangle AEFG is not of the same shape, and in fact is not the same kind of figure as the original rectangle ABCD. This

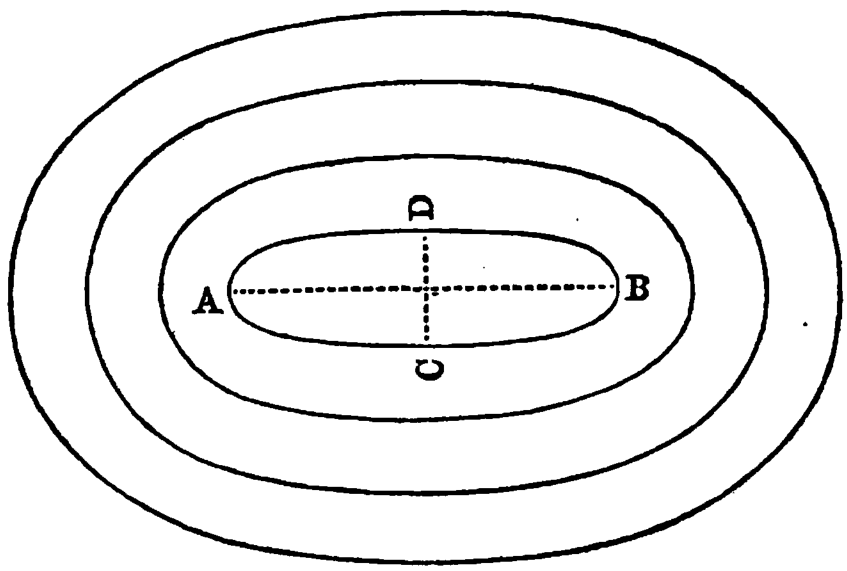
Fig. 2.



is because the enlargement was not made proportionally, and the diagonal AF consequently does not lie in the same line as the

diagonal AC. To make the enlargement proportional, we should only have extended AB 1 inch, when we extended AD 4 inches

Fig. 8.



Such an extension is shown by the rectangle AIHG; and the diagonal of that rectangle lies in the same line as that of the original rectangle ABCD. In like manner, if the elliptical figure AB be enlarged by equal quantities in the line AB and in the line CD, each successive ellipse becomes more circular, and to maintain the original figure the enlargements should be in the proportion of the length and breadth.

ON THE SQUARES AND SQUARE ROOTS OF NUMBERS.

The product of a number multiplied by itself is called a square, and the quotient obtained by dividing this product by the number is the square root of the product. Thus 12 times 12 is 144, which is the square of 12; and 144 divided by 12 is 12, which is the square root of 144. In like manner, the square root of 12 is the particular number which, multiplied by itself, produces 12. Such number is neither 3 nor 4, as 3 times 3 is 9 and 4 times 4 is 16, of which the one is less than 12 and the other greater. The square root of 12 will be some number between 3 and 4, and what the particular number is it is the object of the process for determining square roots to discover. The origin of the term is traceable to the language of geometry, where a rectangular surface is produced by the multiplication of one linear dimension

with another, or a square is produced by the multiplication of one linear dimension by itself. Thus a piece of board a foot long and a foot broad has a surface of one square foot, or, if we count the dimensions in inches, as the length is 12 inches and the breadth 12 inches, the superficies will be 12 times 12, or 144 square inches. The square of 1 is 1, since $1 \times 1=1$. The square of 2 is 4, since $2 \times 2=4$. The square of 3 is 9, since $3 \times 3=9$. Contrariwise 1, 2, and 3 are the square roots of 1, 4, and 9.

If we write the numbers

1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13,

and their squares

1, 4, 9, 16, 25, 36, 49, 64, 81, 100, 122, 144, 169,

it will be seen that if each square number is subtracted from that which immediately follows, we obtain the series of odd numbers

3, 5, 7, 9, 11, 13, 15, 17, 19, 21, &c.,

in which the numbers go on increasing by 2.

The square of a fraction is obtained by multiplying the fraction by itself, in the same manner as a whole number. Thus $\frac{1}{2}\times\frac{1}{2}=\frac{1}{4}$; $\frac{1}{3}\times\frac{1}{3}=\frac{1}{9}$; $\frac{2}{3}\times\frac{2}{3}=\frac{4}{9}$; $\frac{1}{4}\times\frac{1}{4}=\frac{1}{16}$; and $\frac{3}{4}\times\frac{3}{4}=\frac{9}{16}$. So also $\frac{1}{2}$ is the square root of $\frac{1}{4}$; $\frac{1}{3}$ is the square root of $\frac{1}{9}$, and $\frac{1}{4}$ is the square root of $\frac{1}{16}$.

When the square of a mixed number, consisting of an integer and a fraction, has to be determined, we may reduce the mixed number to a fraction by multiplying the integer by the denominator, and adding the numerator to form a new numerator with the same denominator for the denominator of the new fraction. Thus $3\frac{7}{8}=\frac{24}{8}+\frac{7}{8}=\frac{31}{8}$ and the square of $\frac{31}{8}=\frac{961}{64}$ or $15\frac{1}{64}$. Thus also, as the square of $\frac{5}{7}$ is $\frac{25}{49}$, the square root of $\frac{25}{49}$ is $\frac{5}{7}$, and the square root of $12\frac{1}{4}$ or $\frac{49}{4}=\frac{7}{2}=3\frac{1}{2}$. But when the number is not a square, it is impossible to extract its square root precisely, though the root may be approximated to with any required degree of nearness. We have already seen that the square root of 12 must be more than 3 and less than 4. We have also seen that this root is less than $3\frac{1}{2}$, as the square of $3\frac{1}{2}$ is $12\frac{1}{4}$. Neither is the root $3\frac{7}{15}$ or $\frac{52}{15}$ the square of which is $\frac{2704}{225}$ or $12\frac{4}{225}$, which

is still greater than 12. So if we try the number $3\frac{6}{13}$ or $\frac{2025}{169}$, we shall find the number to be too small, for 12 reduced to the same denomination is $\frac{2028}{169}$, so that $3\frac{6}{13}$ is $\frac{3}{169}$ too small, while $3\frac{7}{13}$ is too great. The fact is, whatever fraction we annex to 3, the square of that sum will always contain a fraction, and will never be exactly 12; and although we know that $3\frac{7}{13}$ is too great, and $3\frac{6}{13}$ is too small, we cannot fix upon any intermediate number which multiplied by itself shall produce 12; whence it follows that the square root of 12, though a determinate magnitude, cannot be expressed by fractions. There is therefore a kind of numbers which cannot be specified by fractions, but which still are determinate quantities, and of these numbers the square root of 12 is an example. These numbers are called *irrational numbers*, and they occur whenever we attempt to find the square root of a number that is not a square. These numbers are also called *surds* or *incommensurables*. The square roots of all numbers which are not perfect squares, are indicated by the sign $\sqrt{}$, which is read *square root.* Hence $\sqrt{12}$ means the square root of 12; $\sqrt{2}$ the square root of 2; $\sqrt{3}$ the square root of 3; $\sqrt{\frac{2}{3}}$ the square root of $\frac{2}{3}$, and $\sqrt{a}$ the square root of a. As, moreover, the square root of a number multiplied by itself will produce the number, $\sqrt{2}$ multiplied by $\sqrt{2}$ will produce 2; $\sqrt{3} \times \sqrt{3}=3$; $\sqrt{5} \times \sqrt{5}=5$; $\sqrt{\frac{2}{3}} \times \sqrt{\frac{2}{3}}=\frac{2}{3}$; and $\sqrt{a} \times \sqrt{a}$ produces a.

Although these irrational quantities cannot be expressed in fractions, it will not therefore be supposed that they are visionary or impossible. On the contrary, they are real quantities, which may be dealt with in the same way as common numbers; and however difficult of appreciation such a number as the square root of 12 may be, we at least know this much of it, that it is such a number as multiplied by itself will produce 12.

It is easy to approximate to the square root of a number by taking a trial number and squaring it, when it will be at once seen whether such supposititious number is too great or too small. It is also easy to find the square root by means of logarithms. But the ordinary arithmetical process for finding the square root is not difficult, and will be readily understood by one or two examples.

Thus, in extracting the square roots of 15,625 and 998,001, the mode of procedure is as follows:—

```
     15625(125              998001(999
     1                      81
     --                     ---
   22)56                 189)1880
      44                     1701
     ----                    -----
  245)1225              1989)17901
      1225                   17901
     ----                    -----
```

Here, in the first place, we separate the numbers into groups of two figures each, beginning at the right, by making a short line over each pair of figures, or by pointing them off into groups by such point or mark as shall not be confounded with the decimal point. We then find the next lowest square of the first group, which we set under that group, and subtract as in long division, setting the quotient in the usual place according to the mode of procedure in that process. We next double the quotient for the next trial divisor, and the quotient which we think we shall obtain we also place in the divisor, of which it forms a constituent part; and dividing by the divisor thus increased, we perform the division, setting the quotient in the usual place as in long division. We then subtract, and for the next trial divisor we use the first term of the last divisor, and double the last term of the quotient. In the first example, consisting of five figures, we have only one figure in the first group, and that figure is 1. Now the square root of 1 is 1, which number we set in the quotient, and double it for the next trial divisor, which therefore becomes 2; and as 2 will go twice in 5, we set 2 in the quotient, and also add it to the trial divisor to make the true divisor; and so on. In the second example, the first group consists of the figures 99, the nearest square to which is 81, and we therefore set 9 in the quotient, and put twice 9, or 18, for the next trial divisor, and we see that the number to be added thereto to exhaust the dividend must be large, as 18 is contained 10 times in 188. The number to be added to the trial divisor we find to

be 9, and we set it in the quotient, and double it to add to the first trial divisor to form the second trial divisor; and so on through all the terms, bringing down at each stage a group of two figures, instead of a single figure, as in long division. When there is a remainder after all the figures have been brought down, the number is not a complete square, and its exact root cannot be found, but it may be approximated to by using decimals to carry on the division with sufficient nearness for all useful purposes.

ON THE CUBES AND CUBE ROOTS OF NUMBERS.

When any number is multiplied twice by itself, or, what is the same thing, when the square of a number is multiplied by the number, the product is the cube of the number. Thus $2\times 2\times 2=8$, and 8 therefore is the cube of 2. Also 4 is the square of 2, and $4\times 2=8$. In like manner, $3\times 3\times 3=27$, and 27 is the cube of 3; $4\times 4\times 4=64$, and 64 is the cube of 4; $a\times a\times a=a^3$, and a^3 is the cube of a; or $a^2\times a=a^3$. The cubes of the first nine numerals are 1, 8, 27, 64, 125, 216, 343, 512, and 729, and the respective differences of these numbers are 7, 19, 37, 61, 127, 169, 217, 271, where we do not discern any law of increase. But if we take the respective differences of these last numbers, we obtain the numbers 12, 18, 24, 30, 36, 42, 48, 54, 60, where it is evident that the addition of the number 6 to each successive term produces the next one.

In the cubes of fractions the same law holds as in the case of the squares of fractions. Thus as the square of $\frac{1}{2}$ is $\frac{1}{4}$, so the cube of $\frac{1}{2}$ is $\frac{1}{8}$. So also $\frac{1}{27}$ is the cube of $\frac{1}{3}$; $\frac{8}{27}$ is the cube of $\frac{2}{3}$, and $\frac{27}{64}$ is the cube of $\frac{3}{4}$.

In the case of the cubes of mixed numbers, we first reduce those mixed numbers to an improper fraction, and then cube them as above. Thus the cube of $1\frac{1}{2}$ is the same as the cube of $\frac{3}{2}$, which is $\frac{27}{8}$ or $3\frac{3}{8}$, and the cube of $3\frac{1}{4}$ or $\frac{13}{4}$ is $\frac{2197}{64}$, or $34\frac{21}{64}$.

The cube of $a\,b$ is $a^3\,b^3$, whence we see that if a number has factors, we may find its cube by multiplying together the cubes of the factors. Thus the cube of 12 is 1728. But 12 is composed of the factors 3 and 4; and the cube of 3 is 27, and the

cube of 4 is 64. Hence $27 \times 64 = 1728$ will be the cube of 12, as by multiplying 12 by itself twice it is found to be. The cube of a positive number will always be positive, and of a negative number, negative. This is obvious on considering that $+a \times +a \times +a = +a^3$, and that $-a \times -a = +a^2$, and this multiplied again by $-a$ produces $-a^3$. So the cube of -1 is -1, the cube of -2 is -8, the cube of -3 is -27, and so of all negative numbers.

The cube root of a number is expressed by the sign $\sqrt[3]{}$, and it is easy to determine the cube root of a number when the number is really a cube. Thus we see at once that the cube root of 1 is 1, that the cube root of 8 is 2, that the cube root of 27 is 3, that the cube root of 64 is 4, and that the cube root of 125 is 5. We further see that the cube root of $\frac{8}{27}$ will be $\frac{2}{3}$, of $\frac{27}{64}$ will be $\frac{3}{4}$, and of $2\frac{10}{27}$, or $\frac{64}{27}$, is $\frac{4}{3}$ or $1\frac{1}{3}$. But if the proposed number be not a cube, it cannot any more than in the case of the square root be expressed accurately, either by whole or fractional numbers, though an approximate expression may be obtained that will be sufficiently near the truth for all useful purposes. For instance, 43 is not a perfect cube, and it is impossible to specify any number, whether whole or fractional, which, multiplied by itself twice, will produce 43. If we take a number as nearly as we can to that which we suppose the cube root should be, and multiply it twice by itself, we shall at once see whether such trial number is too great or too small. Thus if we fix upon $3\frac{1}{2}$ or $\frac{7}{2}$ as the trial number, then we find that the cube of $\frac{7}{2}$ being $\frac{343}{8}$, or $42\frac{7}{8}$, the number will err in defect, $42\frac{7}{8}$ being $\frac{1}{8}$ less than 43. By taking other numbers, we may approximate still more nearly to the true root, but we shall never be able to express it in figures precisely, and—as in the similar case in the doctrine of square roots—such quantities are termed irrational quantities.

ON POWERS AND ROOTS IN GENERAL.

The product arising from multiplying a number once or many times by itself is termed a *power*. The square of a number is sometimes called its second power; the cube is sometimes called its third power, and we may have its fourth power, its fifth power, or any power depending on the number of the multipli-

cations, or we may say that the number has been raised to the second, third, fourth, or fifth degree. The fourth power of a number is sometimes called its *biquadrate*, but after this degree powers cease to have any other than numerical appellations.

It is difficult to make the reason or processes of the ordinary arithmetical rule for the extraction of the cube root very intelligible without the aid of Algebra, of the processes of which the rule is only a translation. But an example will show the mode of procedure.

Let us suppose that we had to extract the cube root of the number 80,677,568,161.

		80677568161(4321
		64
123	4800	16677
	369	
	5169	15507
1292	554700	1170568
	2584	
	557284	1114568
12961	55987200	56000161
	12961	
	56000161	56000161

Here we first divide the number, beginning at the right hand, into groups of three figures in each—just as in extracting the square root we divide the number into groups of two figures in each. In the last of the groups we thus form there happens, in this example, to be only two figures, and sometimes there will be only one.

We now consider what is the next lower cube to the number 80, and we find that it is 64, which is the cube of 4. We set the figure 4 in the quotient, and subtract its cube 64 from 80, which leaves a remainder of 16. We next bring down the following period 677.

The next step is to set the *triple* of the first figure of the root (12) at some distance to the left of the remainder. (There is 123 in the sum, but the 3 will be accounted for presently.) We then multiply this triple by the first figure of the root, and place the product 48 between the 12 and the remainder, annexing two ciphers to it.

We now divide the remainder by this 4800, as a trial divisor, and set the quotient 3 as the second figure in the root, and also after the 12, making 123. We next multiply this 123 by 3, the second figure of the root, set the product 369 under the 4800, and add them together. The resulting sum, 5169, is the first real divisor. We next multiply the divisor by the second figure of the root, and subtract the product 15507, as in long division, bringing down the next period 568.

To obtain the next real divisor we proceed as follows:—We first triple the last figure 3, of 123, which gives 129. (There is 1292 put down, but the last figure, 2, will be accounted for presently.) The other quantity, 5547, is found by adding 9, the square of the second figure of the root, to the two preceding middle lines, 369 and 5169. We now add two ciphers and repeat the whole process, and we find the next figure of the root to be 2, which is the 2 added to the 129.

In the case of decimals occurring in any number of which we have to extract the cube root, the distribution of the figures into groups of three each will begin at the decimal point, and will proceed to the left for integers, and to the right for fractions—adding ciphers where necessary to make up the required number of figures. Thus if we had to extract the cube root of ·01, we might write the number $\cdot\overline{010}$, and in like manner 24·1 might be written $\overline{24\cdot100}$

It will now be shown that to add the exponents of numbers is equivalent to multiplying the numbers.

ON ROOTS AS REPRESENTED BY FRACTIONAL EXPONENTS.

The multiplication or division of numbers is indicated by adding or subtracting their exponents, and as 2 may be written

as 2^1, then $2^{\frac{1}{2}} \times 2^{\frac{1}{2}} = 2^1$, since $\frac{1}{2}+\frac{1}{2}=1$. As, too, the third, fourth, fifth, &c., powers of a number are represented by the expressions 2^3, 2^4, 2^5, &c., so the third, fourth, fifth, &c., roots are represented by the expressions $\sqrt[3]{2}$, $\sqrt[4]{2}$, $\sqrt[5]{2}$, &c. The square root may be written $\sqrt[2]{}$, or more simply $\sqrt{}$. Now as we have seen that $2^{\frac{1}{2}} \times 2^{\frac{1}{2}} = 2$, and as $\sqrt{2} \times \sqrt{2}$ also $= 2$, it follows that $2^{\frac{1}{2}}$ is another form of expression for $\sqrt{2}$. So also $2^{\frac{1}{3}} = \sqrt[3]{2}$, $2^{\frac{1}{4}} = \sqrt[4]{2}$, $2^{\frac{1}{5}} = \sqrt[5]{2}$; and so of all other roots whatever. Since also $2^1 \times 2^{\frac{1}{2}} = 2^1 = 2^{\frac{2}{2}}$, it follows that $2^{\frac{2}{2}}$ is the same as $\sqrt{2^2}$. In like manner, $2^{\frac{2}{3}} = \sqrt[3]{2^2}$ and $2^{\frac{3}{4}} = \sqrt[4]{2^3}$.

When the fraction which represents the exponent exceeds unity, it may either be expressed in the form of an improper fraction, or in that of a mixed number. For example, the fraction $2^{\frac{5}{2}}$ may be expressed in the form $2^{2\frac{1}{2}}$. But $2^{2\frac{1}{2}}$ is the product of 2^2 by $2^{\frac{1}{2}}$, and it may be written in the form $2\sqrt[2]{2^5}$.

ON THE CLASS OF FRACTIONAL EXPONENTS TERMED LOGARITHMS.

Since the square root of a given number is a number whose square is equal to that given number, and since the cube root of a given number is a number whose cube is equal to that given number, and so of all roots whatever, it follows that any number whatever being given, we may always suppose such roots of it that, raised to their respective powers, they shall always be equal to the given number. Since, also, powers with negative exponents are fractions, and powers with positive exponents are whole numbers, and as all numbers whatever may be expressed by whole numbers and fractions, it is clear that if we take any given number, such as 10, we may raise it to such a power either positive or negative as will make it equal to any number whatever that we may think proper to assign. Thus if we fix upon the number 4, it is certain that there is a certain power of the number 10, which is equal to 4. Or if we fix upon the number 40, or 47, or 57, or 381, or any other number whatever, then

there will be some power or other of 10 that will be equal to those several numbers. Putting b for this unknown exponent, then $10^b = 381$, or any other number depending on the value of b. If instead of 10 we write the letter a, and instead of 381, or a raised to the power b, we write the letter c, then we obtain the expression $a^b = c$. Here c is the given number, a is the root or *radix*, and b is the exponent or *logarithm* of the number c with the radix a. The radix of the common system of logarithms is the number 10, and the logarithm of a given number is the power to which 10 must be raised to be equal to that given number. Every number whatever has its corresponding logarithm; and when we know its logarithm, we may, instead of the number, use the logarithm, with this conspicuous advantage, that when we have to multiply two numbers together we shall accomplish that end by adding their logarithms to obtain a new logarithm, the number corresponding to which will be the correct product of the two numbers; or if we have to divide one number by another, we shall accomplish the object by subtracting the logarithm of the one from that of the other—the difference constituting a new logarithm, which will be the logarithm of the quotient. This quality of logarithms is apparent when we recollect that they are all exponents of a given number a, and that $a^2 \times a^3 = a^5$, or that $a^5 \times a^8 = a^{13}$, where the multiplication is signified by adding the exponents. So also as $a^{2\times3} = a^6$, $a^{3\times3} = a^9$, $a^{3\times4} = a^{12}$, $a^{4\times5} = a^{20}$, it follows that to multiply a logarithm by 3, 4, 5, or any other number, is equivalent to raising the number to the third, fourth, fifth, or other corresponding power; and contrariwise, to divide the logarithm by 3, 4, 5, or any other number, is equivalent to the extraction of the third, fourth, fifth, or any other root of the number. From these considerations it will be at once apparent that by the use of logarithms an enormous amount of labour may be saved in performing arithmetical computations, and to facilitate such computations the logarithms of all the numbers usually occurring in calculations have been ascertained and arranged in tables, so as to facilitate their employment. All positive numbers, such as 1, 2, 3, 4, 5, &c., are logarithms of the root or radix a, and of its positive powers, and

are consequently logarithms of numbers greater than unity. On the contrary, the negative numbers —1, —2, —3, —4, —5, &c., are the logarithms of the fractions $\frac{1}{a}, \frac{1}{a^2}, \frac{1}{a^3}, \frac{1}{a^4}$, &c., which are less than unity and greater than nothing. Now as every significant number can only be positive or negative, and as the logarithms of numbers greater than unity are positive, and the logarithms of numbers less than unity but greater than nothing are negative, there is no sign left to express numbers less than nothing, or negative numbers, and we must therefore conclude that the logarithms of negative numbers are impossible.

It has already been stated that in the logarithmic tables at present in common use, the radix, of which the logarithmic number is the exponent, is 10. If we denote this radix by a, then the logarithm of any number c is the exponent to which we must raise the radix a or 10, in order that the power resulting from it may be equal to the number c. If we denote the logarithm of c by log. c, then $10^{\log. c}=c$. Now as $a^0=1$ and $a^1=a$, so $10^0=1$ and $10^1=10$. But as the exponents are the logarithms of the numbers, it follows that the logarithm of 1 is 0, and the logarithm of 10 is 1. So also log. 100 or $10^2=2$; log. 1000 or $10^3=3$; log. 10000 or $10^4=4$; log. 100000 or $10^5=5$, and log. 1000000 or $10^6=6$. In like manner log. $\frac{1}{10}= -1$; log. $\frac{1}{100}= -2$; log. $\frac{1}{1000}= -3$; log. $\frac{1}{10000}= -4$; log. $\frac{1}{100000}= -5$; log. $\frac{1}{1000000}= -6$; and so on indefinitely.

Since log. $1=0$ and log. $10=1$, it is plain that the logarithms of all numbers between 1 and 10 must be less than unity and greater than nothing. Let us suppose that it was required to determine the logarithm of the number 2. If we represent this logarithm by the letter x, then we shall have this expression $10^x=2$. In order to determine the value of x, we may make a few approximate suppositions. If we suppose x to be $\frac{1}{2}$, we shall have $10^{\frac{1}{2}}=2$, which is manifestly too great, since $9^{\frac{1}{2}}=3$ and $10^{\frac{1}{2}}$ must therefore be more than 3. If we suppose x to be $\frac{1}{3}$, the quantity will still be too great. For if $10^{\frac{1}{3}}=2$, then $10^{\frac{3}{3}}=2^3$, or 10^1 or $10=8$, which shows that $\frac{1}{3}$ is too much. If we take $\frac{1}{4}$ as

the exponent, then we have $10^{\frac{1}{4}}=2$, or $10^{\frac{4}{4}}=2^4$, or $10=16$, which shows that $\frac{1}{4}$ is too small, while $\frac{1}{3}$ is too great.

By pursuing the investigation in this manner, we should find with any required degree of accuracy what the exponent would be that, if 10 were raised to that power, would be equal to 2. This exponent or logarithm, as it is termed, would in point of fact be 0·3010300, or a little less than $\frac{1}{3}$, and in the logarithmic tables in common use the logarithms are always expressed in decimal fractions, as being the most convenient form for purposes of computation. The value of this decimal expressed in vulgar fractions is $\frac{3}{10}+\frac{0}{100}+\frac{1}{1000}+\frac{0}{10000}+\frac{3}{100000}+\frac{0}{1000000}+\frac{0}{10000000}$. Logarithmic tables are commonly computed to seven places of decimals, as decimals carried to 7 places, though not expressing the result with absolute exactness, will, it is considered, give results that are sufficiently accurate for all ordinary purposes. According to this method of expressing logarithms, the logarithm of 1 will be 0·0000000, since it is really=0. The logarithm of 10 will be 1·0000000, since it is=1. The logarithm of 100 will be written 2·0000000,=2, and so on. The logarithms of all numbers intervening between 10 and 100, and consequently composed of 2 figures, will be greater than 1 and less than 2, and are expressed by 1+ a decimal fraction. Thus log. 50=1·6989700. The logarithms of numbers between 100 and 1000 are expressed by 2+ a decimal fraction; the logarithms of numbers between 1000 and 10,000 are expressed by 3+ a decimal fraction. The logarithms of numbers between 10,000 and 100,000 are expressed by 4 and a decimal fraction, and the number prefixed to the decimal will always be 1 less than the number of figures in the given number. Thus the logarithm of 2290 is 3·3598355, for as there are four figures in 2290, the number prefixed to the decimal will be 3. The number prefixed to the decimal, or the integral part of the logarithm, is termed the *characteristic;* and when a number consists of four figures, such as the number 2290, its characteristic is invariably 3. If the number be reduced to 229, its characteristic will be 2; if reduced to 22 its characteristic will be 1, and if reduced to 2 its characteristic will be 0. There are therefore two parts to be con-

sidered in a logarithm: first the characteristic, which we can at once determine when we know the number of figures of which the given number consists; and second the decimal fraction, which is determined by the nature of those figures. So also we know, at the first sight of the characteristic of a logarithm, what is the number of figures composing the number of which it is the logarithm. If for example the logarithm 6·4771213 be presented, we know at once that the number of which it is the logarithm must consist of 7 figures, and must be over 1,000,000. The integral part of a logarithm therefore being so easily found, the main part requiring consideration is the decimal part, and it is that part alone which is given in the logarithmic tables in common use. To show the manner of using these tables, we may multiply together the numbers 343 and 2401 by the aid of logarithms. Here—

$$\begin{array}{rl}
\text{Log. } 343 = 2{\cdot}5352941 & \rule{0pt}{0pt}\\
\text{Log. } 2401 = 3{\cdot}3803922 & \text{added.}\\
\hline
5{\cdot}9156863 & \text{their sum.}\\
\text{Log. } 823540 = 5{\cdot}9156847 & \text{nearest tabular log.}\\
\hline
16 & \text{difference.}
\end{array}$$

We look in the table of logarithms opposite the figures 343, and we find the number 5352941, which we know constitutes the fractional part of the logarithm, while the integral part will be 1 less than the number of figures in 343, or in other words the integral part will be 2. In like manner we find the logarithm of 2401, and adding these logarithms together, we find their sum to be 5·9156863. We then look in the table to find the next less logarithm to this, which we find to be 5·9156847. We see at once by the magnitude of the characteristic that the number of which this is the logarithm must consist of six figures, and we find the number answering to this logarithm to be 823540. The difference between the logarithm formed by the addition of the two original logarithms and its next lower tabular logarithm is 16, and in the tables there is a column of differences intended to fix the numerical value of such differences, and which in this

case would amount to the number 3. With this correction the product of 343 and 2401 will become 823543.

It is in the extraction of roots, however, that logarithms become of the most eminent service. If, for instance, we had to extract the square root of 10, we should only have to divide the logarithm of 10 which is 1·0000000 by 2, which gives 0·5000000 as the logarithm of the root required; and by referring to the table of logarithms, we should find that the number answering to this logarithm was 3·16228, which consequently is the square root of 10. So also if we had to extract the fifth root of 2 we should divide the logarithm of 2, which is 0·3010300, by 5, which gives a quotient of 0·0602060, the number answering to which in the tables is 1·1497, which consequently is the fifth root of 2.

ON THE COMPUTATION OF COMPOUND QUANTITIES.

Hitherto our investigations have been restricted to the modes of calculation suited to the measurement of simple quantities; but many of the quantities with which we have to deal in engineering practice are compound quantities made up of simple quantities in different forms of combination, and it is now necessary to consider the mode of computing the values of these compound quantities. One of the most familiar forms of a compound quantity is a sum of money expressed in pounds, shillings, and pence, or in other coins of different values. Another variety is a given weight expressed in tons, hundredweights, quarters, and pounds, or in other different kinds of weights. If we wish to know what number of pence there is in a sum of money, or what number of pounds or ounces there is in a given weight, the operation is termed *reduction*, and is performed by multiplying the given quantity by the number which shows how many of the *next* lower denomination makes one of the higher. Thus if we wish to know how many pence there are in 37*l.*, we first multiply the 37*l.* by 20, which will show the number of shillings there are in 37*l.*, for as there are 20 shillings in 1*l.*, there will be 20 times 37 in 37*l.* Now $37 \times 20 = 740$

shillings, and as there are 12 pence in 1 shilling, there will be 12 times 740=8880 pence in 37*l.* If the sum were 37*l.* 16*s.* 8*d.* in which we wished to find the number of pence, it is clear that the number of pence in 16*s.* 8*d.* must be added to the number already found. Now as there are 12 pence in 1 shilling, 12 times 16=192, the number of pence in 16 shillings, to which, if we add the 8 pence remaining, we shall have 200 pence to add to the 8880, or in other words we shall have 9080 pence as the answer. So if we wish to ascertain the number of pounds weight in 3 tons, we have first to ascertain by a reference to a table of weights how many pounds there are in the ton, and which we shall find to be 2240. This number multiplied by 3 will obviously be the number of pounds weight contained in 3 tons. But if the weight which we were required to find the number of pounds in were 3 tons 7cwt. 2 quarters and 8 pounds, we should first have to multiply the 3 tons by 20 to reduce them to cwts., and as there are 20 cwt. in the ton, 3 tons would be 60 cwt. But besides these we have 7 cwt. more, so that we have in all 67 cwt. Now as there are 4 quarters in the cwt., there will in 67 cwt. be 4 times 67=268 quarters, to which we have to add the two quarters of the original sum, making in all 270 quarters in the weight. But as there are 28 lbs. in 1 quarter, there will be 28 times 270=7560 lbs.; and as there are 8 pounds besides to be added, the sum total of the weight will be 7568 lbs. So if we wished to know how many square inches there were in 2¼ square feet, it is plain that as there are 144 square inches in the square foot, there will be 288 square inches in 2 square feet, and 36 square inches in ¼ of a square foot, and 288+36=324 square inches. In performing these and similar operations it is of course necessary to have access to proper tables of weights and measures, or, in other words, to certain standard magnitudes, as it is impossible to form an idea of any magnitude except by comparing it with some other magnitude, such as a pound, a foot, or a gallon, of which we have a definite conception.

On the addition of compound quantities.—The first step in performing this addition is to set the quantities to be added under one another, so that terms of the same kind may be in the

same column. When the relation between the different quantities is known—as it is in all cases of arithmetical addition—we add up the numbers in the right-hand column, and divide by the number in this column which makes 1 in the next column. We then set the remainder, if any, under the first column, and carry the quotient to be added to the next, and so on through all the columns. Thus in adding up the pounds, shillings, and pence here set down we proceed as follows:

£	s.	d.
13	0	8
2	5	6
23	4	7
37	8	10
12	9	7
0	13	4
89	2	6

We first arrange the pounds, shillings, and pence in three columns, with the units under the units, the tens under the tens, and so on, as in simple addition. We then add up the column of pence, and find how many pence it contains. But as every group of 12 pence makes 1 shilling, we divide the total number of pence by 12 to find how many of such groups there are, or, in other words, how many shillings there are in the total number of pence. These shillings we transfer to the shillings column, and as after we have done this there are 6 pence left, we write the 6 beneath the pence column, and then proceed to add up the shillings, beginning with the number of shillings we have brought from the pence column. Having thus ascertained the total number of shillings, we find how many pounds there are in that number of shillings by dividing by 20, there being 20 shillings in the pound sterling; and after having found this number of pounds, we carry it to the pounds column, and the 2 shillings which we find remaining we write under the shillings column. We then proceed to add the pounds column, beginning with the number of pounds in shillings which we have carried from the shillings column.

In adding up cwts., quarters, and pounds, the mode of procedure is precisely the same, only as there are 28 lbs. in 1 quarter, 4 quarters in 1 cwt., and 20 cwt. in 1 ton, the divisors we use at each step must vary correspondingly. This will be plain from the following example:

Here we find the sum to be 20 cwt. 3 qrs. and 17 lbs., or 1 ton 0 cwt. 3 qrs. and 17 lbs.; for, after adding the first column, and dividing the sum by 28, we have 17 left, and after add-

	cwt.	qr.	lbs.
	3	3	12
	5	1	24
	2	3	18
	6	2	19
	2	0	0
1 ton	0	3	17

ing the second or quarters column with the addition of the number of quarters in lbs. that we have carried over from the lbs. column, we divide the number so obtained by four to obtain the number of cwts. there are in all these quarters. We carry the cwt. so obtained to the cwts. column, and write beneath the quarters column the 3 quarters which we find are left. Proceeding in the same way with the cwts. column, we find its sum to be 20 cwts. or 1 ton; and the total quantity to be 1 ton 0 cwt. 3 qrs. 17 lbs., as stated above.

Subtraction of compound quantities.—When we wish to subtract one compound quantity from another, we write the less under the greater, so that the terms of the same kind may be in the same column, as in the case of addition. We then subtract the right-hand term of the lower line from that of the upper, if possible. But if this cannot be done, we must transform a unit of the next higher term into its equivalent number of units of the first term, and then performing the subtraction, we write the difference under the first column, and we increase by 1 the next term to be subtracted to compensate for the unit previously borrowed. In algebra, the usual process of subtraction is to change the signs of the lower line, and then to proceed as in addition.

If we had to take 27*l.* 8*s.* 4½*d.* from 34*l.* 17*s.* 9¾*d.*, we should write down the greater sum first and the less under it, so that

£34	17	9¾
£27	8	4½
£7	9	5¼

pounds should fall under pounds, shillings under shillings, and pence under pence. Taking ½*d.* from ¾*d.* we have ¼*d.* over, which we write down, and then taking 4*d.* from 9*d.* we have 5*d.* over, which we also write below the column of pence. Next taking 8*s.* from 17*s.* we have 9*s.* left, and taking 7*l.* from 14*l.* we have 7*l.*, and carrying 1 to the 2 appearing in the next place we have 3 from 3, which leaves nothing. The difference, therefore, between 34*l.* 17*s.* 9¾*d.* and 27*l.* 8*s.* 4½*d.* is 7*l.* 9*s.* 5¼*d.* If we had to subtract 22*l.* 18*s.* 11¾*d.* from 23*l.* 6. 0½*d.*, we should proceed thus:—

£23	6	$0\frac{1}{2}$
£22	18	$11\frac{3}{4}$
£0	7	$0\frac{3}{4}$

Here taking $\frac{3}{4}d.$ from $\frac{1}{2}d.$ we have to borrow $1d.$ or 4 farthings from the next term, and we have then 6 farthings to be subtracted from, and $\frac{3}{4}d.$ subtracted from $\frac{6}{4}d.$ leaves $\frac{3}{4}d.$ In the next term we have $11d.$, which must be increased to $12d.$ on account of the penny before borrowed; and as we have no pence to subtract from we must borrow $1s.$ from the next term, and change it into 12 pence, and 12 pence taken from 12 pence leaves nothing. In the next term of shillings we have 18, which must be increased to 19 in consequence of the previous borrowing of $1s.$ to carry to the column of pence, and $19s.$ taken from $1l.$ $6s.$ or $26s.$ leaves $7s.$ In the next term the 2 has to be increased to 3 to make up for the $1l.$ imported into the column of shillings, and 23 taken from 23 leaves nothing. The difference between these two sums is consequently $7s.$ $0\frac{3}{4}d.$

If we have to take 5 tons 12 cwt. 3 qrs. $27\frac{1}{2}$ lbs. from 93 tons 8 cwt. 1 qr. 6 lbs., we proceed as follows:

tons	cwt.	qr.	lbs.
93	8	1	6
5	12	3	$27\frac{1}{2}$
87	15	1	$6\frac{1}{2}$

Here $\frac{1}{2}$ lb. taken from 1 lb. leaves $\frac{1}{2}$ lb., and 28 lbs. taken from 1 qr. and 6 lbs. or 34 lbs., leaves 6 lbs. Then 4 qrs. taken from 1 cwt. and 1 qr. or 5 qrs. leaves 1 qr.; and 13 cwt. taken from 1 ton and 8 cwt. or 28 cwt. leaves 15 cwt. Lastly, 6 tons taken from 93 tons leaves 87 tons.

If we wish to subtract $6-2+4$ from $9-3+2$, we may either perform the subtraction by first adding the quantities together, and then subtracting the sum of the one from that of the other, or we may change the signs of the quantity to be subtracted, and then add all together, which will give the same result. Thus $6-2=4$, and $4+4=8$. So also $9-3=6$, and $6+2=8$. Subtracting now one sum from the other, we get $8-8=0$. But if we change the signs of $6-2+4$, and add it to $9-3+2$, we have $9-3+2-6+2-4=0$.

Multiplication of compound quantities.—When we wish to perform the multiplication of any compound number, such as pounds, shillings, or pence, or hundredweights, quarters, and pounds, we set the multiplier under the right-hand term of the multiplicand, multiply that term by it, and find what number

of times one of the next higher term is contained in the product, which number is to be carried to the next term, while the remainder, if any, is to be written under the right hand or lowest term. We must then multiply the next term in like manner, and so until the whole have been multiplied. Thus if we had to multiply 23*l.* 13*s.* 5*d.* by 4, we should proceed as follows:

£23	13	5
		4
£94	13	8

Here we first multiply the pence, and 4 times 5 pence is 20 pence, which is 1*s.* 8*d.*; and so we put down 8 and carry 1. In the shillings term we say 4 times 3 are 12, and with the addition of the 1 shilling brought over from the pence term, the 12 becomes 13. Then 4 times 10 is 40 shillings, which make just 2 pounds, so we carry the 2 pounds to the pounds place, leaving the 13 previously obtained in the shillings place. Proceeding to the pounds, we say 4 times 3 are 12 and 2 are 14, and 4 times 2 are 8 and 1 are 9. Hence the product is 94*l.* 13*s.* 8*d.*, which sum would also be obtained by writing down 23*l.* 13*s.* 5*d.* four times under one another, and ascertaining their sum by addition.

When the multiplier is large, but is composed of two or more factors, we may, instead of multiplying by the number, multiply successively by its factors. Thus if we have such a sum as £23 11*s.* 4¾*d.* to multiply by 36, then as 36 is a number represented by the factors 6 × 6, 4 × 9 or 3 × 12, we shall obtain the same result by multiplying by any set of these factors as by multiplying by the 36 direct. Thus—

£23	11	4¾		£23	11	4¾		£23	11	4¾
		6				4				3
141	8	4½		94	5	7		70	14	2¼
		6				9				12
£848	10	3		£848	10	3		£848	10	3

In like manner if we had to multiply the sum £17 3*s.* 0½*d.* by 140, then as 140 is made up of the factors 7 × 20, or 4 × 5 × 7, we may multiply by these numbers instead of the

£17	3	$0\frac{1}{2}$
		4
68	12	2
		5
343	0	10
		7
£2401	5	10

140. In cases however in which the multiplier cannot be broken up into factors, we must multiply each term by it consecutively. Thus if £23 11*s.* $4\frac{3}{4}$*d.* be multiplied by 37, we have first 3 farthings multiplied by 37, which gives 111 farthings or 27 pence and 3 farthings. Writing down the 3 farthings and carrying the 27 pence, we have 37 times 4 pence or 148 pence, and adding the 27 pence we have 175 pence, which as there are 12 pence in the shilling we divide by 12 and get 14 shillings and 7 pence. We set down the 7 in the pence place and carry the 14 to the shillings place, and we thus proceed through all the terms until the multiplication is completed. The same mode of procedure is adopted if, instead of pounds, shillings, and pence, we have hundredweights, quarters, and pounds or any other quantities whatever.

Division of compound quantities.—In the arithmetical division of compound quantities, we set the divisor in a loop to the left of the dividend and divide the left-hand term by it, setting the quotient under that term. If there is any remainder we reduce it to the next lower denomination, adding to it that term, if any, of the dividend which is of this lower denomination. We then divide the result by the divisor and so on, until all the terms have been divided. Thus if we had to divide £38 6*s.* $8\frac{1}{4}$*d.* by 3, we should proceed as follows:—

£	*s.*	*d.*
3)38	6	$8\frac{1}{4}$
12	15	$6\frac{3}{4}$

Here we find that 3 is contained in 3 once, and in 8, 2 times and 2 over. But 2 pounds are 40 shillings, and 6 are 46 shillings, and 46 divided by 3 gives 15 and 1 over, which 1 shilling is equal to 12 pence, and adding to this the 8 pence in the dividend, we have 20 pence to be divided by 3. Now 20 divided by 3 gives 6 and 2 over, which 2 pence are 8 farthings, and adding thereto the 1 farthing in the dividend, we have 9 farthings to divide by 3, or 3 farthings. It is clear that £12 15*s.* $6\frac{3}{4}$*d.* multiplied by 3 will again give the £38 6*s.* $8\frac{1}{4}$*d.* of the dividend.

If we have to divide a number by 10, we may accomplish the division by pointing off one figure as a decimal, if by 100 we point

off two figures, if by 1000 three figures, and so on. Thus if we have to divide £2315 14*s.* 7*d.* by 100, we may proceed as follows:

£	*s.*	*d.*
23·15	14	7
·20		
3·14		
12		
1·75		
4		
3·00		

Here we point off two figures of the highest term as decimals, which leaves £23. We next multiply the residual decimal by 20 to reduce it to shillings, bringing down the 14 shillings in the dividend, and we obtain 3 shillings and ·14 of a shilling, which fraction we multiply by 12 to bring it to pence, and we bring down thereto the 7 pence in the dividend. We obtain as a product 1·75 pence, and multiplying in like manner ·74 by 4 to bring it to farthings, we obtain 3 farthings, making the total quotient £23 3*s.* 1¾*d.* This sum multiplied by 100 will make £2315 14*s.* 7*d.* When the divisor is large but may be broken up into factors, we may divide separately by those factors. Thus if we wish to divide £3762 3*s.* 6*d.* by 24, then as 24 = 4 × 6 or 3 × 8 or 2 × 12, we may divide the sum by any pair of factors instead of by the 24.

£	*s.*	*d.*		£	*s.*	*d.*		£	*s.*	*d.*
4)3762	3	6		3)3762	3	6		2)3762	3	6
6)940	10	10½		8)1254	1	2		12)1881	1	9
£156	15	1¾		£156	15	1¾		£156	15	1¾

When the number cannot be broken up into factors we must proceed by the method of long division. Thus if we had to divide £3715 18*s.* 9*d.* by 47 we should proceed as follows:—

£	*s.*	*d.*			
47)3715	18	9(79	1	3	
329					
425					
423					
2					
20					
58(1					
47					
11					
12					
141(3					
141					

Here we find first how often 47 will go in 371, and we find it will be 7 times, when we write the 7 in the quotient and multiply the divisor by it, setting the product under the first three figures of the dividend. Subtracting now the 329 from the 371, we find that the remainder is 42, and we bring down the next figure of the dividend and find how often 47 is contained in 425. We find that it is 9 times, which completes the division of the pounds. The 2 pounds remaining we next multiply by 20 to bring them to

shillings, adding the 18 shillings of the dividend, which together make 58 shillings, the 47th part of which is 1 shilling and $\frac{11}{47}$ths over. Multiplying this by 12 to bring it to pence, and dividing by 47, we get 3 pence, which completes the operation.

In cases where we have to divide a compound quantity by another of the same kind, such as money by money or weights by weights, the requirement is equivalent to that of finding what number of times the one amount is comprehended in the other. We cannot of course divide a quantity by another of a different kind, as money by weight, nor can we multiply money by money or weight by weight. If we are required to divide such a sum as £3 7*s.* 6*d.* by 16*s.* 10½*d.*, we reduce both the numbers to the lowest denomination appearing in either, which in this case is half pence, and we then divide the greater number by the less. Now £3 7*s.* 6*d.* = 1620 half pence and 16*s.* 10½*d.* = 405 half pence, and 1620 ÷ 405 = 4. So if we had to divide 3 tons 2 cwt. 2 qrs. 21 lbs. by 2 qrs. 7 lbs., then as the first amount is equal to 6993 lbs. and the second to 63, the question becomes one of dividing 6993 by 63, which we find gives 111. It follows consequently that 2 qrs. 7 lbs. multiplied by 111 = 3 tons 2 cwt. 1 qr. 21 lbs.

As a square foot contains 144 square inches, we must, in ascertaining the number of square feet in any given number of square inches, divide by the number 144, and as a cubic foot contains 1728 cubic inches, we must, in ascertaining what number of cubic feet there are in any number of cubic inches, divide by the number 1728. So also there are nine square feet in a square yard, and 27 cubic feet in a cubic yard. A cubic foot contains *very nearly* 2200 cylindric inches or solid cylinders 1 inch in diameter and 1 inch high; 3300 spherical inches or balls 1 inch diameter; and 6600 conical inches or cones 1 inch diameter and 1 inch high.

ON THE RESOLUTION OF FRACTIONS INTO INFINITE SERIES.

We have already explained that in decimal fractions the decrease at every successive figure is ten times, just as in common

numbers the increase at every successive number is ten times. Thus the number 666 means 600 + 60 + 6, so that the first figure by virtue of its position alone is ten times greater than the second, and the second by virtue of its position alone is ten times greater than the third. Precisely the same law holds when we descend below unity, as we do in every case in which the decimal point is introduced, as the meaning of the decimal point is, that all the numbers to the right of it are less than unity, and that they diminish ten times at each successive figure, just as ordinary numbers do. The expression 666·666 therefore means six hundred and sixty-six with the addition of 6 *tenths*, six hundredths, and six thousandths, or, what is the same thing, of 666 thousandths. The expression might therefore be written $666+\frac{6}{10}+\frac{6}{100}+\frac{6}{1000}$ or $666\frac{666}{1000}$. Every decimal fraction may consequently be considered as a vulgar fraction, with a denominator of 10 or 100 or 1000 understood, according to the position of the decimal. Thus ·1 is equivalent to $\frac{1}{10}$, ·01 is equivalent to $\frac{1}{100}$, and ·001 is equivalent to $\frac{1}{1000}$. Now the fraction $\frac{1}{3}$ is 1 divided by 3, and if we perform the division we shall have

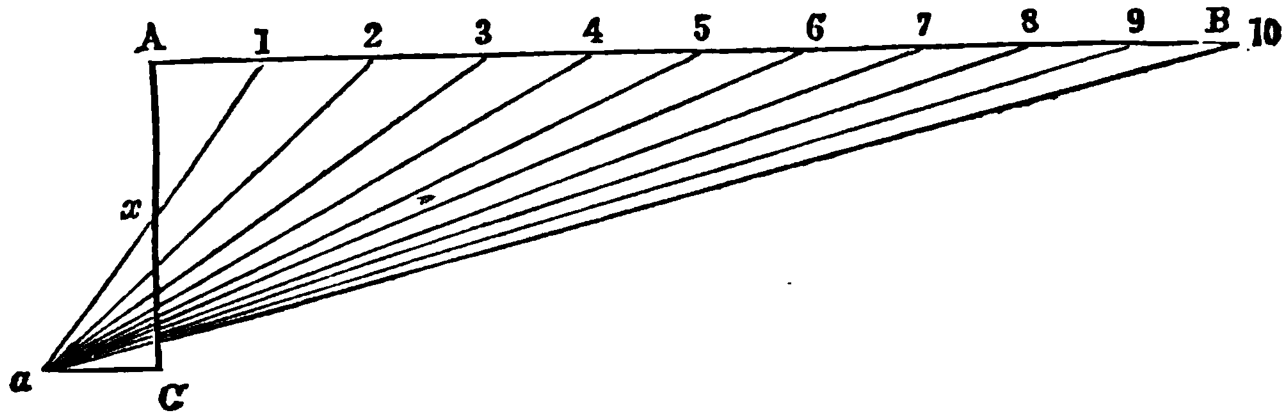
$$\begin{array}{r} 3)1{\cdot}00000 \\ \hline {\cdot}33333, \text{ \&c.,} \end{array}$$

and so on to infinity. The vulgar fraction $\frac{1}{3}$ is consequently equal to the infinite series ·33333, &c., which, at each successive term to which it is carried, becomes more nearly equal to the fraction of $\frac{1}{3}$, but never becomes exactly equal thereto. Any vulgar fraction may be at once converted into its equivalent decimal by dividing the numerator by the denominator, adding as many ciphers to the numerator as may be necessary to enable the division to be carried on. But some of the divisions thus performed, it will be found, may be carried on for ever, and such a series of numbers is termed an infinite series. As a visible exemplification of the continual approach of two quantities to one another without ever becoming equal, we may take the following example:

Here we have a line A B which we may divide into any num-

ber of equal parts, and we draw the line A C at right angles with A B: at C we draw another short line ac parallel to A B, and we set off the distance ca equal to A1. If now we draw the diagonal line $1a$ we shall cut off the half of A C, or shall bisect it in the point x, and by drawing the lines $2a$, $3a$, $4a$, $5a$, &c.,

Fig. 4.



we cut off successive portions of xc, and therefore continually diminish it. But we never can cut it all off, however extended we may make the line A B, and however numerous the additional portions cut off may be. The quantity xc becomes more and more nearly equal to xA, the greater the length of the line A B, and the more numerous the fractional quantities successively cut off. But no extension of the operation short of infinity could make the portions cut off from xc equal to xA.

ARITHMETICAL EXAMPLES.

Having now illustrated with adequate fulness of detail the elementary principles of engineering arithmetic, it is only necessary that we should add some examples of the method of performing such computations as are most likely to be required in practice.

Reduction.—This is the name given to the process of converting a quantity expressed in one denomination into an equivalent quantity expressed in another denomination, such as tons expressed in ounces, or miles in yards.

Example 1.—Reduce 15*l.* 7*s.* 0¾*d.* to farthings.

15	7	0¾
20		
307*s.*		
12		
3684*d.*		
4		
14739 *f.* *Ans.*		

Here we first multiply the pounds by 20, there being 20 shillings in the pound, and we bring down the 7 shillings, making 307 shillings. We then multiply the shillings by 12, there being 12 pence in the shilling, and here we have no pence to bring down. Finally, we multiply by 4, there being 4 farthings in each penny, and we bring down the 3 farthings, making 14,739 farthings in all.

Example 2.—Reduce 23 tons to pounds avoirdupois.

By a reference to a table of weights and measures, we find that there are 2,240 pounds in the ton; 23 times 2,240, there fore, or 51,520 lbs., is the answer required.

Example 3.—Reduce 100 square yards to square inches. Here, as each square yard contains 9 square feet, and each square foot 144 square inches, there will be 9 times 144 or 1,296 square inches in each square yard, and 100 times this, or 129,600 square inches, in 100 square yards. It may be well here to remark that 100 square yards is a very different quantity from 100 *yards square*, which would, in fact, contain an area of 10,000 square yards.

Example 4.—Reduce 7 cubic yards, 20 cubic feet, to cubic inches. As there are 27 cubic feet in a cubic yard, there will be 27 times 7, or 189 cubic feet in 7 cubic yards, to which adding 20, we have 209 cubic feet in all; and as there are 1,728 cubic inches in a cubic foot, we have 1,728 times 209, or 361,152 cubic inches as the answer required.

Quantities are brought to a higher denomination by the reverse of the process indicated above, that is, by dividing, instead of multiplying. Thus, by dividing by 4, 12, and 20, it will be found that 14,739 farthings are equal to 15*l.* 7*s.* 0¾*d.*; by dividing 51,520 lbs. by 2,240, that the quotient is equal to 23 tons; and by dividing 129,600 square inches by 144, and then by 9, that the result is 100 square yards. So also by dividing by 1,728, it will be found that 361,152 cubic inches are equal to 209 cubic feet, and dividing again by 27, we find the answer tc be 7 cubic yards and 20 cubic feet.

Mensuration of Surfaces and Solids.—The area of a rectangular surface is obtained by multiplying the length by the breadth. The area of a circle in circular inches is obtained by multiplying the diameter by itself; and the area of a circle in square inches is obtained by multiplying the diameter by itself, and by the decimal ·7854. The circumference of a circle is 3·1416 times its diameter. The capacity of a rectangular solid is obtained by multiplying together its length, depth, and thickness; and the capacity of a cylinder in cubic feet or inches is obtained by multiplying the area of its cross section or mouth, expressed in square feet or inches, by its depth in feet or inches.

Example 1.—What is the quantity of felt required to cover the side of a marine boiler that is 17 feet 8 inches long, and 3 yards high?

Here we first reduce the measurements to inches, and as 17 ft. 8 in. is equal to 212 inches, and as 3 yards or 9 feet is equal to 108 inches, we have an area represented by 212 multiplied by 108 inches, or 22,896 square inches. Now, as there are 144 square inches in each square foot, we shall, by dividing 22,896 by 144, find that the area is 159 square feet, and dividing this by 9 to bring the quantity into square yards, we find that the area is 17 square yards and 6 square feet over.

Since the area is obtained by multiplying the length by the breadth, it will follow that if we divide the area by the length we shall get the breadth, and if we divide the area by the breadth we shall get the length.

Example 2.—What is the weight required to be placed on top of a safety-valve 4 inches diameter, to keep it down until the steam attains a pressure of 20 lbs. on each square inch?

Here $4 \times 4 = 16$ circular inches, and $16 \times {\cdot}7854 = 12{\cdot}566$ square inches, which $\times 20$ the pressure on each square inch $=$ 251·32 lbs.

Example 2.—The engine of the steamer 'Arrogant' is a trunk engine, in which the piston rod is widened into a hollow trunk or pipe 24 inches diameter, which correspondingly reduces the effective area of the piston. As the cylinder is 60 inches diameter, reduced by a circle 24 inches diameter, what

will be the diameter of a common cylinder to have an equal area?

Here $60^2 \times \cdot 7854 = 2827 \cdot 44$ square inches, and $24^2 \times \cdot 7854 = 452 \cdot 39$ square inches, and 2827·44 diminished by 452·39 = 2375·05 square inches. This is as nearly as possible the area of a cylinder 55 inches in diameter, which is 2375·83 square inches.

Example 3.—The steamer 'Black Prince' has two direct-acting trunk engines, with cylinders equal to 104¼ inches diameter, and the length of the stroke is 4 feet. The engines make 55 revolutions per minute. What will be the number of cubic feet of steam required per hour to fill the cylinder?

Here the diameter being 104¼ inches, the area of each cylinder will be ·7854 times 104¼ squared, or it will be 8835·7 square inches, or 61·3 square feet. As the piston travels backwards and forwards at each revolution, it will pass through 8 feet during each revolution; and the volume of steam required by each cylinder in each revolution will be 8 times 61·3, or 490·4 cubic feet. As there are two engines, the total volume of steam required in each revolution will be twice 490·4, or it will be 980·8 cubic feet; and as there are 55 strokes in each minute, the expenditure per minute will be 55 times 980·8, or 53,944 cubic feet. The expenditure per hour will, of course, be 60 times this, or 3,236,640 cubic feet. In all modern engines the steam is not allowed to enter the cylinder from the boiler during the whole stroke; and the expenditure of steam will be less the sooner it is cut off or prevented from entering the cylinder. But the cylinder, nevertheless, will still be filled with steam, though of a less tension, than if the supply from the boiler had not been interrupted; and the space traversed by the piston will always be a correct measure of the steam consumed, taking that steam at the pressure it has at the end of the stroke.

Example 4.—The 'Black Prince' has an area of immersed midship section of 1,270 square feet; or, in other words, if the vessel were cut across in the middle, the area of that part below the water would be 1,270 square feet. The diameter of the screw is 24 feet 6 inches, the nominal power is 1,250, and the indicated power 5,772 horses. What is the ratio, or proportion,

of the area of midship section to the area of the circle in which the screw revolves? and what is the ratio of the immersed mid ship section to the indicated power?

Here the diameter of the screw being $24\frac{1}{2}$ feet, the area of the circle in which it revolves will be 471·436 square feet, and 1,270 divided by 471·436 being 2·69, it follows that the ratio of immersed midship section to screw's disc is 2·69 to 1. So, in like manner, the indicated power 5,772, divided by 1,250, gives a ratio of indicated power to immersed midship section of 4·54 to 1. With these proportions the speed was at the rate of nearly 15 knots per hour, so that to ensure such a speed in a vessel like the 'Black Prince,' it is necessary that there should be $4\frac{1}{2}$ or 5 indicated horse-power for each square foot of immersed midship section of the hull.

Example 5.—If it were desired to encircle the screw of the 'Black Prince' with a sheet-iron hoop, what length of hoop would be required for the purpose?

The diameter of the screw being $24\frac{1}{2}$ feet, the circumference of the circle in which it revolves will be 3·1416 times $24\frac{1}{2}$, or it will be 76·969 feet.

Example 6.—A single acting feed pump has a ram of $2\frac{1}{2}$ inches diameter and 18 inches stroke, and makes 50 strokes per minute. How much water ought it to send into the boiler every hour?

Here the area of the ram will be 4·9 square inches, and the stroke being 18 inches, 18 times 4·9 or 88·2 cubic inches will be expelled at every stroke, supposing that there is no loss by leakage or otherwise. As there are 50 strokes made in the minute, the discharge per minute will be 50 times 88·2, or 4,410 cubic inches; and there will be 60 times this, or 264,600 cubic inches discharged in the hour. As there are 1,728 cubic inches in the cubic foot, we get the hourly discharge in cubic feet by dividing 264,600 by 1,728, and we shall find the discharge to be 153·125 cubic feet. A cubic inch of water will make about a cubic foot of steam, of the same pressure as the atmosphere.

Example 7.—A cubic foot of water weighs 1,000 ounces. What will be the weight of water in a vessel which is filled to the brim, and which measures a yard each way?

As there are 27 cubic feet in a cubic yard, the weight required will be 27,000 ounces, which, divided by 16, the number of ounces in a pound, gives 1,687 lbs. and 8 oz., and dividing again by 112, the number of lbs. in each cwt., we get 15 cwt. 7 lbs. 8 oz.

Example 8.—Two steamers being started together on a race, it was found that the faster went 5 feet ahead of the other in each 55 yards: how much will she have gained in half a mile?

As a mile is 1,760 yards, half a mile is 880 yards, and there are 16 times 55 yards, therefore, in half a mile. As in each 55 yards 5 feet are gained, there will be 16 times 5 feet, or 80 feet gained in the half mile, or 26 yards 2 feet.

Example 9.—The 'Warrior,' a steamer of 6,039 tons burden, and 1,250 nominal horse-power, attained a speed on trial of 14·356 knots per hour, the engines exerting an actual power of 5,469 horses. The screw was 24½ feet diameter, and 30 feet pitch, or, in other words, the twist of the blades was such that it would advance 30 feet at each revolution, if the advance were made without any resistance. The engines made 54·25 revolutions per minute, and if the screw advanced 30 feet in each revolution, it would advance 1627·5 per minute, or 16·061 knots per hour. In reality, however, the screw only advanced through the same distance as the ship, namely, 14·356 knots per hour. The actual advance, therefore, was less than the theoretical advance by 1·705 knots per hour, which difference is called the *slip* of the screw; for 1·705 added to 14·356 makes 16·061, which would be the speed of the vessel at this speed of the screw if there was no slip.

Example 10.—What is the diameter of a piston of which the area is 2827·44 square inches?

Here 2827·44, divided by ·7854, = 3600, the square root of which is 60. This is the diameter required.

Example 11.—A cubical vessel of water weighs 5 tons, excluding the weight of the vessel. What is the length of the side?

As there are 1,000 ounces in a cubic foot of water, we know that there will be the same number of cubic feet in the vessel as

the number of times 1,000 ounces is contained in 5 tons. Now as there are 2,240 lbs. in the ton, there will be 5 times this, or 11,200 lbs. in 5 tons, or 179,200 ounces. Dividing this by 1,000, we have 179·2 cubic feet as the content of the vessel.

To find the length of the side we must extract the cube root of 179·2. We soon see that the root must lie between 5 and 6, for the cube of 5 is 125, and the cube of 6 is 216. Taking 5 as the next lowest root, we set this number as the first figure of the quotient and subtract its cube as in long division, bringing down three more figures at each stage, and here two of these must be ciphers.

		179·2(5·63
		125
156	7500	54·200
	936	
	8436	50·616
1683	940800	3584000
	5049	
	945849	2837547
		746453

We now triple the root 5, and set down the 15 to the left, and we multiply this triple number by the first figure of the root making 75, which number we set down between the 15 and the remainder, adding two ciphers to it, which make it 7500. We now consider how often the trial divisor 7500 will go into the remainder 54200, after making some allowance for additions to the divisor, and we find it will be 6 times. We place the 6 as the second figure of the root, and we also place it after the 15. We multiply the 156 by the 6, and place the product under the 7500. The resulting number, 8436, is the first true divisor.

We now bring down the next period of three figures, and as there are no figures remaining to be brought down, we introduce three ciphers. We triple the last figure of 156, which gives 168, and we add the square of 6, which is 36, to the sum of the two last lines, 936 and 8436, making in all 9408, to which we add two ciphers, making 940800, and we then see how often this sum is contained in 3584000. We find that it will be 3 times, and we set down the 3 as the next figure of the root, and also after the 168, making 1683, and we add three times this to the 940,800, making 945849, which is the second real divisor. We

now multiply this divisor by the last figure of the quotient, and subtract the product as in long division, leaving as a remainder 746453, to which, if we wished to carry the answer to another place of decimals, we should annex three ciphers, and proceed as before. For all ordinary purposes, however, an extraction to the second place of decimals is sufficient, and if we cube 5·63, we shall find the resulting number to be 178·453547, being a little less than 179·2.

Example 12.—The density or specific gravity of mercury is 13·59 times greater than that of water, and the specific gravity of water is 773·29 times greater than that of air of the usual atmospheric pressure. What will be the height of a column of water that will balance the usual barometric pressure of 30 inches of mercury, and what also will be the height of a column of air of uniform density that will be required to balance that pressure?

Here the mercury being 13·59 times more dense than the water, or in other words, the water being 13·59 times more light than the mercury, it will be necessary that the height of the column of water should be 13·59 times greater than that of the column of mercury, in order to balance the pressure. If, therefore, the column of mercury be 30 inches high, the height of the balancing column of water must be 13·59 times 30 inches, or 33·975 feet, and the height of the balancing column of air must be 773·29 times this, or 26271·52775 feet. In point of fact, the height will be a little more than this, as mercury is 13·59593 times heavier than water, whereas, for simplicity, it has been taken here at only 13·59 times heavier.

EQUATIONS.

When one quantity is set down as equal to another quantity with the sign of equality (=) between the two, the whole expression is termed an equation. Thus 1 lb. = 16 oz. is an equation; and if we represent lbs. by the letter A, and oz. by the letter B, then we shall have the equation in the form 1A or A = 16B. It is clear that the equality subsisting in such an ex-

pression will not be extinguished by any amount of addition, subtraction, multiplication, division, or other arithmetical process to which it may be subjected, provided it be simultaneously applied to both sides of the equation—just as the equality of weight shown by a pair of scales between 1 lb. and 16 oz. will not be altered if we add an ounce, or pound, or any other weight to each scale, or subtract an ounce, or pound, or any other weight from each scale. If we add an ounce to each scale, then we shall have the equation $A + B = 16B + B$, or if we subtract an ounce from each scale, the equation becomes $A - B = 16B - B$, both of which expressions are obviously just as correct as the first one. We may, consequently, add any quantity to each side of an equation, or subtract any quantity from it without altering the value of the expression.

If we have such an expression as $A - B = 16B - B$, and wish thereby to know the value of A, we shall ascertain it by adding the quantity B to each side of the equation, which will then become $A - B + B = 16B - B + B$. Now $A - B + B$ is obviously equal to A, for the value of any quantity is not changed by first subtracting and then adding any given quantity to it. So likewise $16B - B + B$ is obviously equal to $16B$, as the $-B$ and $+B$ destroy one another. The equation thus cleared of redundant figures becomes $A = 16B$. as at first.

If now we divide both sides of the equation by any number, or mutiply both sides by any number, we shall find the value of the expression to remain without change. For example, if we divide by 16 we shall get $\frac{A}{16} = B$, or if we multiply by 2 we shall get $2A = 32B$. Both of these expressions are obviously as true as the first one, as they amount to saying that $\frac{1}{16}$th of a pound is equal to an ounce, and that 2 lbs. are equal to 32 oz.

If we have such an expression as $a + b = c$, and wish to know the value of a, we subtract b from both sides of the equation, which we have seen we can do without error, whatever quantity b may be supposed to represent. Performing this subtraction we get $a + b - b$, or $a = c - b$; and if we know the values of c and b, we at once get the value of a. If we know the

values of a and c, and wish to find the value of b, we shall ascertain it by substracting a from each side of the equation, which will then become $b = c - a$. In both of these subtractions we may see that we have merely shifted a letter from one side of the equation to the other, at the same time changing its sign; and we hence deduce this general law applicable to all equations, that we may without error transfer any quantity from the one side to the other, if we at the same time change its sign.

If we have the equation $a = \frac{x}{b}$, and if we know the values of a and b, but not of x, then, to find the value of x, we multiply both sides of the equation by b, which reduces the equation to the form $ab = x$. If, then, $a = 2$ and $b = 4$, it is clear that $x = 8$. It may be here remarked that ab is the same as $a \times b$, and which is quite a different expression from $a + b$, the one meaning a multiplied by b, and the other a added to b. So likewise $\frac{ab}{b} = a$ and $\frac{ab}{a} = b$.

The utility of such equations in engineering computations is very great, not merely as simplifying arithmetical processes, but as presenting compendious expressions of important laws, both easily remembered and easily recorded. Thus it is found that in steam-vessels the power necessary to be put into them, to achieve any given speed with any given form of vessel, and any given area of immersed midship section, varies as the cube of the speed required. If we represent the indicated power by P, the speed in knots per hour by S, the area in square feet, and the cross section below the water line by A, and if by C we denote a certain multiplier or coefficient, the value of which varies with the form of the vessel, but is constant in the same species of vessel, then $P = \frac{S^3A}{C}$ is an equation which expresses these relations, and we can find the value of P from this equation if we know the value of the other quantities, or we can find the value of S, or of A, or of C, if we know the values of the other quantities in the equation. Thus if we multiply both sides of the equation by C, we get $PC = S^3A$, and if we now divide by P we

get $C = \frac{S^3A}{P}$. So also if we divide the equation $PC = S^3A$ by S^3, we get the value of A, as we shall then have $\frac{PC}{S^3} = A$; or if we divide by A we get $\frac{PC}{A} = S^3$, and taking out the cube root of both sides we get $\sqrt[3]{\frac{PC}{A}} = S$. If, therefore, we know the indicator power of a steamer, the immersed area of midship cross section, and the coefficient proper for the order of vessel to which the particular vessel under examination belongs, we can easily tell what the speed will be, as we have only to multiply the indicator power in horses by the coefficient, and divide by the sectional area in square feet, and finally to extract the cube root of the quotient, which will give the speed in knots per hour. The coefficients of different vessels have been ascertained by experiment. The following are the coefficients of some of the screw-vessels of the navy:—

'Shannon,' 550; 'Simoom,' 500; 'Windsor Castle,' 493; 'Penguin,' 648; 'Plover,' 670; 'Curaçoa,' 677; 'Himalaya,' 695; 'Warrior,' 824; 'Black Prince,' 674. The coefficient of the Royal Yacht 'Fairy' is 464, and the original coefficient of the 'Rattler' was 676; but the performance has latterly fallen off, and is not now above 500, or thereabout. The original coefficient of the 'Frankfort,' a merchant screw steamer, was 792, which was about the best performance at that time attained. The larger the coefficient the better is the performance.

CHAPTER II.

MECHANICAL PRINCIPLES OF THE STEAM-ENGINE.

LAW OF CONSERVATION OF FORCE.

THE fundamental principle of Mechanics, as of Chemistry, Physiology, and every department of physical science, is that a force once in being can never cease to exist, except by its transformation into some other equivalent force, which, however, does not involve the annihilation of the force, as it continues to exist in another form. This principle, usually termed the *conservation of force*, and sometimes the *conservation of energy*, is only now beginning to receive that wide and distinct recognition which its importance demands; and it will be found that the clear apprehension of this pervading principle will greatly simplify and aid all our investigations in natural science. One very obvious inference from the principle is that we cannot manufacture force out of nothing, any more than we can manufacture time, or space, or matter; and in the various machines for the production of power—such as the steam-engine, the wind or water mill, or the electro-motive machine—we merely develop or liberate the power pent up in the material which we consume to generate the power; just as in setting a clock in motion, we liberate the power pent up in the spring. Coal is virtually a spring that has been wound up by the hand of nature; and in using it in an engine we are only permitting it to uncoil—im-

parting thereby to some other agent an amount of power equal to that which the coal itself loses. The natural agent employed in winding up the springs which our artificial machines uncoil is the sun, which by its action on vegetation decomposes the carbonic acid which combustion produces, and uses the carbon to build up again the structure of trees and plants, that, by their subsequent combustion, will generate power; and as coal is only the fossil vegetation of an early epoch, we are now using in our engines the power which the sun gave out ages ago. So in windmills and waterwheels, it is the sun that, by rarefying some parts of the atmosphere more than others, causes the wind to blow that impels windmills, and the vapours to exhale, which, being afterwards precipitated as rain, form the rivers that impel waterwheels. In performing these operations the sun must lose as much power, in the shape of heat or otherwise, as it imparts; and one of two consequences must ensue—either that the sun is gradually burning out, or that it is receiving back in some other shape the equivalent of the power that it parts with.

LAW OF VIRTUAL VELOCITIES.

One branch of the principle of conservation of force is well known in mechanics as the principle of *virtual velocities*. This principle teaches that, as the power exerted in a given time by a machine, such as a steam-engine or waterwheel, is a definite quantity, and as power is not mere pressure or mere motion, but the product of pressure and motion together, so in any part of the machine that is moving slowly, the pressure will be great, and in any part of the machine moving rapidly, the pressure must be small, seeing that under no other circumstances could the product of the pressure and velocity—which represents or constitutes the power—be a constant quantity. A horse power is a dynamical unit, or a unit of force, which is represented by 33,000 lbs. raised one foot high in a minute of time; and this unit is usually called an *actual* horse power to distinguish it from the *nominal* or commercial horse power, which is merely an expression for the diameter of cylinder and length of stroke,

or a measure of the dimensions of an engine without any reference to the amount of power actually exerted by it. If we suppose that an engine makes one double stroke of 5 feet in the minute—which is equal to a space of 10 feet in the minute that the piston must pass through, since it has to travel both upward and downward—and that this engine when at work exerts one horse power, it is easy to tell what pressure must be exerted on the piston in order that this power may be exactly attained; for it must be the 10th of 33,000 or 3,300 lbs.; since 3,300 lbs. multiplied by 10 feet is equivalent to 33,000 lbs. multiplied by 1 foot. Such an engine, if making 10 strokes in the minute, would exert 10 horses' power; if making 20 strokes in the minute would exert 20 horses' power; if making 30 strokes in the minute would exert 30 horses' power; and in general the pressure on the piston in lbs. multipled by the space passed through by the piston in feet per minute, and divided by 33,000, will give the number of horses' power exerted by the engine.

It will be clear from these considerations that the circumstance which determines the power exerted by any engine during each stroke is—with any uniform pressure of steam—the *capacity* of the cylinder. A tall and narrow cylinder will generate as much power each stroke, and will consume as much steam, as a short and broad one, if the capacities of the two are the same. But the strain to which the piston-rod, the working-beam, and the other parts are subjected, will be greatest in the case of the short cylinder, since the weight or pressure on the piston must be greatest in that case in order to develop the same amount of power. Since, too, in the case of an engine exerting a given power, the quantity of power is a constant quantity, which may be represented by a small pressure acting through a great space, or a great pressure acting through a small space, so long as the product of the space and pressure remain invariable, it follows that in any part of an engine through which the strain is transmitted, and of which the motion is very slow, the pressure and strength must be great in the proportion of the slowness, since the pressure multiplied by the motion, at any other part of the engine, must always be equal to the pressure multiplied by the

motion of the piston. In the case of any part of an engine, therefore, or in the case of any part of any machine whatever, it is easy to tell what the strain exerted will be when we know the relative motions of the piston, or other source of power, and of the part the strain on which we wish to ascertain, since, if the motion of such part be only $\frac{1}{2}$ of that of the moving force, the strain will be twice greater upon that part than upon the part where the force is first applied. If the motion of the part be $\frac{1}{3}$ of that of the moving force, the strain upon it will be 3 times greater than that due to the direct application of the moving force; if the motion be $\frac{1}{4}$, the strain will be 4 times greater; if $\frac{1}{5}$, it will be 5 times greater; if $\frac{1}{10}$, it will be 10 times greater; if $\frac{1}{100}$, it will be 100 times greater: and if any motion of the prime mover imparts no appreciable motion to some other part of the machine, the strain becomes infinite, or would become so only for the yielding and springing of the parts of the machine. We have an example of a strain of this kind in the Stanhope printing press, or in the elbow-jointed lever, which consists of two bars jointed to one another like the halves of a two-foot rule. If we suppose these two portions to be opened until they are nearly but not quite in the same straight line, and if they are then interposed between two planes, and are forced sideways so as to bring them into the same straight line, the force with which the planes will be pressed apart will be proportional to the relative motions of the hand which presses the elbow-joint straight, and the distance through which the planes are thereby separated. As it will be found that this distance is very small indeed, relatively with the motion of the hand, when the two portions of the lever come nearly into the same straight line, and ceases altogether when they are in the same straight line, so the pressure acting in separating the planes will be very great indeed when the parts of the lever come into nearly a straight line, and is infinite when they come really into a straight line; or it would be so but for the compressibility of the metal and the yielding of the parts of the apparatus.

It is perfectly easy, with the aid of the law of virtual velocities, to determine the strains existing at any part of a machine,

and also the weight which the exertion of any given force at the handle of a crane, winch, screw, hydraulic press, differential screw, blocks and tackle, or any other machine will lift; for we have only to determine the first and last velocities, and in the proportion in which the last velocity is slow, the weight lifted will be great. Thus, suppose we have a crane, moved by a handle which has a radius of 2 feet, which turns a pinion of 6 inches diameter gearing into a wheel of 4 feet diameter, on which there is a barrel of 1 foot diameter for winding the chain upon, it is easy to tell what weight—excluding friction—will be balanced or lifted by, say a force of 30 lbs. applied at the handle. The handle, it is clear, will describe a circle of 4 feet diameter, while the pinion describes only a circle of 6 inches diameter, which gives us a relative velocity of 8 to 1; or, in other words, the strain exerted at the circumference of the pinion will be 8 times greater than the strain of 30 lbs. applied at the end of the handle; so that it will be 240 lbs. Now the strain of the pinion is imparted to the circumference of the wheel with which it gears; and the strain of 240 lbs. at the circumference of a wheel of 4 feet diameter will be 4 times greater at the circumference of a barrel of 1 foot diameter, placed on the same shaft as the wheel, and revolving with it. The weight on the barrel, therefore, which will balance 30 lbs. on the handle, will be 4 times 240 lbs., or 960 lbs., but for every foot through which the weight of 960 lbs. is raised, the handle must move through 32 feet, since 30 lbs. moved through 32 feet is equivalent to 960 lbs. moved through 1 foot. So also in the case of a screw press, the screw of which has a pitch of say half an inch, and which is turned round by a lever say 3 feet long, pressed with a weight of 30 lbs. on the end of it, we have here a moving force acting in a circle of 6 feet diameter; and as at each revolution of the screw it is moved downward through a distance equal to the pitch, which is ½ inch, we have the relative velocities of ½ inch, and the circumference of a circle 6 feet in diameter. Now the proportion of the diameter of a circle to its circumference being 1 to 3·1416, the circumference of a circle 6 feet diameter will be 18·8496 feet, or say 18·85 feet, which, multipled by 12 to reduce

it to inches, since the pitch is expressed in inches, gives us 226·2 inches, and the relative velocities, therefore, are 226·2 to $\frac{1}{2}$, or 452·4 to 1. It follows, consequently, that a pressure of 30 lbs. applied at the end of the lever employed to turn such a screw as has been here supposed, will produce at the point of the screw a pressure of 452·4 times 30, or 13,572 lbs., which is a little over 6 tons. Whatever the species of mechanism may be—whether a hydraulic press, a lever, ropes and pulleys, differential wheels, screws, or pulleys, or any other machine or apparatus, this invariable law holds, that with any given pressure or strain at the point where the motion begins, the pressure or strain exerted at any part of the machine will be in the inverse proportion of its velocity—the stress or pressure on any part being great, just in the proportion in which its motion is slow.

In the case of a lever like the beam of a pair of scales, which has its fulcrum in the middle of its length, the application of any force or pressure at one end of the beam will produce an equal force or pressure at the other end; and both of the ends will also move through the same distance if motion be given to either. But if the fulcrum, instead of being placed in the middle of the beam, be placed intermediately between the middle and one end, we shall then have a lever of which the long end is 3 times the length of the short one, and a pound weight placed at the extremity of the long end, will balance 3 lbs. weight placed at the extremity of the short end. If, however, the short end be moved through 1 foot, the long end will be simultaneously moved through 3 feet; and 3 lbs. gravitating through 1 foot expresses just the same amount of mechanical power as 1 lb. gravitating through 3 feet. In a safety-valve, pressed down by a lever 5 feet long, while the point which presses on the spindle of the safety-valve is 6 inches distant from the fulcrum, we have a lever, the ends of which have a proportion of $\frac{1}{2}$ to 5, or 1 to 10; so that every pound weight hung at the extremity of the long end of such a lever, will be equivalent to a weight of 10 lbs. placed on the top of the valve itself. In the case of a set of blocks and tackle, say with 3 sheaves in each block, and, therefore, with 6 ropes passing from

one block to the other, it is clear that if the weight to be lifted be raised a foot, each of the ropes will have been shortened a foot, to do which—as there are 6 ropes—the rope to which the motive power is applied must have been pulled out 6 feet. We have, here, therefore, a proportion of 6 to 1; or, in other words, a weight of 1 cwt. applied to the rope which is pulled, would balance 6 cwt. suspended from the blocks.

It is a common practice among sailors in tightening ropes—after having first drawn the rope as far as they can by pulling it towards them—to pass the end of the rope over some pin or other object, and then to pull it sideways in the manner a harp string is pulled, taking in the slack as they again release it. This action is that of the elbow-jointed lever reversed; and inasmuch as the tightened rope may be pulled to a considerable distance sideways, without any appreciable change in its total length, the strain imparted by this side pulling is great in the proportion of the smallness of the distance through which any given amount of side deflection will draw the rope on end.

A hydraulic press is a machine consisting of a cylinder fitted with a piston, beneath which piston water is forced by a small pump; and at each stroke of the pump the piston or ram of the hydraulic cylinder is raised through a small space, which will be equal to the capacity of the pump spread over the area of the hydraulic piston. If, for example, the pump has an area of 1 square inch, and a stroke of 12 inches, its capacity or content will be 12 cubic inches; and if the piston has an area of 144 square inches, it is clear that the pump must empty itself 12 times to project 144 cubic inches of water into the cylinder, and which would raise the piston or ram 1 inch. In other words, the plunger of the pump must pass through 12 times 12 inches, or 144 inches, to raise the piston of the hydraulic cylinder 1 inch, so that the motion of the piston or ram of the hydraulic cylinder being 144 times slower than that of the plunger of the pump, it will exert 144 times the pressure that is exerted on the piston of the pump to move it. When, therefore, we know the amount of pressure that is applied to move the plunger of the pump, we can easily tell the weight that the hydraulic piston

will lift, or the pressure that it will exert; and, indeed, this pressure will be greater than that on the pump in the proportion of the greater area of the hydraulic piston, relatively with that of the pump plunger, and which in the case supposed is 144 to 1.

There are various forms of differential apparatus for raising weights, or imparting pressure, in which the terminal motion is rendered very slow, and therefore the terminal pressure very great, by providing that it shall be the difference of two motions, very nearly equal, but acting in opposite directions. Thus, if the bight of a rope be made to hang between two drums or barrels on which the different ends of the rope are wound, and one of which barrels pays the rope out, while the other winds it up at a slightly greater velocity than that with which it is unwound by the other, the bight of the rope will be very slowly tightened; and any weight hung upon the bight will be lifted up with a correspondingly great force. Then there are forms of the screw press in which the screw winds itself up a certain distance at one end, and unwinds itself nearly the same distance at the other end; so that, at each revolution, it advances the object it presses upon through a distance equal to the difference of the winding and unwinding pitches; and as this difference may be made as small as we please, so the pressure may be made as great as we please. The effect of using these differential screws is the same as would be obtained if we were to use a single common screw having a pitch equal to the differences of the pitches. But in practice such a pitch would be too fine to have the necessary strength to resist the pressure; and consequently differential screws are in every respect preferable.

It is easy to tell what the pressure exerted by a differential screw will be, when we know the actual advance it makes at each revolution. Thus, suppose the pitch of the unwinding or screwing-out part of the screw to be half an inch, or $\frac{500}{1000}$ of an inch, and the pitch of the winding or screwing-in part of the screw to be $\frac{499}{1000}$ of an inch, then the distance between the winding and unwinding nuts will be increased $\frac{500}{1000}-\frac{499}{1000}$ or $\frac{1}{1000}$th part of an inch at each revolution. The pressure exerted

by such a screw will consequently be the same as if the pitch were $\frac{1}{1000}$th part of an inch; and such pressure may be easily computed in the manner already explained.

There are various forms of differential gearing employed in special cases—not generally for the purpose of generating a great pressure, but for the purpose of generating a slow motion with few wheels; though a great pressure is an incident of the arrangement, if the terminal motion be resisted. Thus, if we place two bevel wheels on the same shaft, with the teeth facing one another, and cause the two wheels to make the same number of revolutions in opposite directions, and, further, if we place between the two wheels, and on the end of a crank or arm capable of revolving between them, a bevel pinion, gearing with the two wheels, then it will follow—if the two wheels have the same number of teeth—that the bevel pinion will merely revolve on its axis, but that this axis or crank will be itself stationary. If, however, one wheel is made with a tooth more than the other wheel, then it will follow that the crank or arm carrying the bevel pinion will be advanced through the distance of one tooth by each revolution of the wheels, and the arm will consequently have a very slow motion round the shaft, and will impart a correspondingly great pressure to any object by which that motion is resisted. Differential gearing is principally employed for drawing along, very slowly, the cutter block in boring mills; and many of its forms are very elegant. It is also employed in various kinds of apparatus for recording the number of strokes made by an engine in a given time. But the same conditions which render the motion slow, also render it forcible; without any reference to the forms of apparatus by which the transformation is produced.

These expositions are probably sufficient to show how the pressure exerted by any machine may be computed; and as the pressure is only another name for the strain, we may thence discover how to apportion the material to give the necessary strength. The very same considerations will enable us to determine the strains existing at any part of an engine, or at any part of any structure whatever; and when we know the

amount of the strain, it becomes easy to tell how much material, of any determinate strength, we must apply in order to resist it. Let us suppose, for example, that we wished to know the strain which exists at any part of the main beam of a land engine, in order that we may determine what quantity of metal we should introduce into it to give it the necessary strength. Now if we suppose the fly wheel to be jammed fast when the steam is put on the engine, it is clear that the connecting-rod end of the beam will be thereby fixed, and will become a fulcrum round which the piston-rod will endeavour to force up the beam, lifting the main centre with twice the pressure that the piston exerts; since if we suppose the main centre to be a weight, and the fulcrum to be at the end of the beam, this weight would only be moved through one inch, when the piston moved through 2 inches, so that the lifting pressure upon this point would be twice greater than that upon the piston, and the main centre must consequently be made strong enough to withstand this strain. If, however, we suppose the main centre to be sufficiently strong, we may dismiss all consideration respecting it, and may consider the beam, which will be thus fixed at two points, as a beam projecting from a wall, which an upward or downward pressure is applied to break.

Now in any well-formed engine beam, and indeed in all metal beams of proper construction, the strength is collected at the edges; and the web of the beam acts merely in binding into one composite mass the areas of metal which are to be compressed and extended. The edges of the beam may be in fact regarded as pillars, which it is the tendency of the strain applied to the beam to crumple up on the one edge, and tear asunder on the other edge; and the whole strength of the beam may be supposed to reside in these pillars, since if they were to break the rest of the beam would at once give way. The strength of any given material to resist compression is not necessarily, nor always the same as the strength to resist compression. In the case of wrought-iron the stretching strength is about twice greater than the crumpling strength; whereas, in the case of cast-iron the crushing strength is between 5 and 6 times

greater than the tensile strength. In the case of an engine beam, which has the strain applied alternately in each direction, the weakest strength must necessarily be that on which our computations are based; and in machinery it is not advisable to load cast-iron with a greater weight than 2,000 lbs. per square inch of section. Now if we suppose, for the sake of simplifying the computation, that the depth of the beam at the centre is equal to its length, then it is clear that if the end of the beam moves through any given distance, a point on the edge of the beam over or below the main centre will move through the same distance, having the same radius; and if we suppose that the depth of the beam is equal to half its length, then a point on the edge of the beam, over or below the main centre, will move through half the space that the end of the beam moves through, and at such point there will consequently be twice the amount of strain existing than is exerted upon the piston. For every 2,000 lbs., therefore, of pressure on the piston, there ought to be strength enough at the edge of the beam to withstand a strain of 4,000 lbs.; but as this strength has to be divided between the two edges of the beam, there should be strength enough at each end to bear 2,000 lbs. without straining the metal more than 2,000 lbs. per square inch of section. In other words, with such a proportion of beam there ought to be a square inch of section in the top and bottom flanges or mouldings of the beam, for each 2,000 lbs. pressure or load upon the piston. In land engines a common proportion for the depth of the beam is the diameter of the cylinder; and a common proportion for the length of stroke is twice the diameter of the cylinder, while the length of the beam is commonly made equal to three times the length of the stroke. With these proportions the length of the beam will be equal to six times its depth; and as the edge of the beam, above or below the main centre, will in such a beam have only one-sixth of the motion that the end of the beam has, the strain at that part divided between the two edges of the beam will be six times as great as the stress exerted on the piston. For every 2,000 lbs. pressure, therefore, on the piston, there must be about three square inches of sec-

tional area in the upper and lower flanges or mouldings of the beam, or six square inches between the two; while the web of the beam is made merely strong enough to keep the upper and lower flanges in their proper relative positions.

It will be obvious from these considerations, that the principle of *virtual volocities* enables us to compute the amount of strain existing at any part of any machine or engine, as we have only to suppose the part to be broken, and to see what amount of motion the broken part will have relatively with the motion of the prime mover, to determine the amount of the strain. We can also easily discern, by keeping this principle in view, how it comes that, in the case of marine or other engines arranged in pairs, with the cranks at right angles with one another, one of the engines is so often broken by water getting into the cylinder; and how necessary, therefore, it is that such engines should be provided with safety-valves, so enable the water shut within the cylinder to escape. For if water gets into one cylinder, and if at or near the end of the stroke the slide-valve shuts off the communication both with the boiler and with the condenser, as is a common state of things, it will follow that the water shut within the cylinder, being unable to escape, will resist the descent of the piston. As, moreover, the crank of one engine is vertical, while that of the other is horizontal, and as when vertical the crank is virtually an elbow-jointed lever, it will follow that one engine, with its greatest leverage of crank, is moving into the vertical position the crank of the other engine, in which position it will act like an elbow-jointed lever, or the lever of a Stanhope press, in forcing down the piston on the water, with a pressure that is infinite; and as the water is nearly incompressible, and as in the absence of escape-valves it cannot get away, some part of the engine must necessarily break. The smaller the quantity of water shut within the cylinder, so long as it resists the piston, the greater the breaking pressure will be; as the crank will, in such case, come more nearly into the vertical position where the downward thrust that it exerts is greatest; whereas, if there be any large volume of water shut within the cylinder, the piston will encounter it before the crank comes

near the vertical position, and also before the crank of the other engine comes into the horizontal position in which it exerts the greatest leverage in turning round the shaft, as it does when the engine is at half stroke. In these as in all other cases in which we wish to investigate the strain produced in any machine, or in any *part* of any machine, by any given pressure applied in any direction, whether oblique or otherwise, we have only to consider the amount of motion—in the direction in which the strain acts—of that particular part which endures the strain or communicates the pressure, relatively with the amount of simultaneous motion in the prime mover. And if the ultimate motion be a tenth, a hundredth, or a thousandth part of the original motion, so will the strain or pressure exerted by the prime mover at the part where the motion is first communicated be multiplied ten, a hundred, or a thousand fold.

NATURE OF MECHANICAL POWER.

Mechanical power, or, as it is sometimes defined, *work*, or *vis viva*, is pressure acting through space; and the law of the conservation of force teaches that power once produced cannot be annihilated, though it may be transformed into other forces of equivalent value. In all machines a certain proportion of the power resident in the prime mover is lost, while the rest is utilised and is rendered available for the performance of those labours for which power is required. Thus, in a waterwheel, the theoretical value of the fall is that due to a certain weight of water gravitating through a certain number of feet in the minute; and if we know the height of the fall, and the discharge of water in a given time, the theoretical value of such a fall can be easily computed. But by no species of hydraulic instrument, whether a waterwheel, a turbine, a water-pressure engine, a Barker's mill, or any other machine, can the whole of the power be abstracted from the fall, and be made available for useful purposes. About 80 per cent. of the theoretical power of a waterfall is considered to be a very satisfactory result to obtain in practice; and the rest is lost by impact and eddies, and by

the friction of the water and of the machine. In the steam-engine the motive force is not gravity, but heat; and just in the same way as power is imparted by water in descending from a higher to a lower level, so is power imparted by heat in descending from a higher to a lower temperature. These two temperatures are the the temperature of the boiler, and the temperature of the condenser; and it is clear that if the condenser were to be made as hot as the boiler, the motion of the engine would cease. And just as in a waterfall there is a certain theoretical power due to the quantity of gravitating matter and the difference of level, so in a steam-engine there is also a certain theoretical power due to the quantity of heated matter, and the difference of temperature; but in utilising the power of steam-engines, this theoretical limit is not approached so nearly as in hydraulic machines. The great fault of the steam-engine is that the larger part of the attainable fall is lost. Thus, if we suppose the temperature of the furnace to be 2,500° Fahrenheit, and the temperature of the boiler to be 250°, while that of the condenser is 100°, we utilise pretty effectually the power represented by the difference in temperature between 100° and 250°; but the difference between 250° and 2,500° is not utilised at all. The consequence of this state of things is that not above *one-tenth* of the power theoretically due to the fuel consumed, is utilised in the best modern steam-engines—the rest being thrown away.

MECHANICAL EQUIVALENT OF HEAT.

If the law of the conservation of force be an invariable law of nature, we shall naturally expect to find that the power which is consumed when a steam-engine or other machine is set to execute useful work, reappears as an equivalent force in some other form. This consequently is the case. When an engine is employed to pump water, we have obviously the equivalent of the force in the water pumped to a higher level; and if this water were suffered to flow back again, so as in its descent to generate power, we should again have the power we before

spent, with the deductions due to the imperfections of the apparatus employed. In the case of an engine, however, which expends its power in friction, or in such work as the propulsion of a vessel through the water, the reproduction of the equivalent of the power expended is not so easily perceived. But in these cases, also, it has been proved by careful experiment, that the law of the conservation of force equally obtains. Friction, whether of solids or liquids, produces heat; and in the case of an engine which expends its power on a friction brake, or on any other analogous object, an amount of heat will be produced, such as, if it could be used without loss in a perfect engine, would exactly reproduce the amount of power expended. In the case of a vessel propelled through the water, the power is mainly consumed in overcoming the friction of the water on the bottom of the vessel, and a part is also expended in moving the water to a greater or less extent; and whatever motion the water acquires, implies a corresponding loss of power by the engine, which power is ultimately expended in moving the particles of water upon one another. In such operation heat is produced; which heat, if it could be utilised without loss in an engine, would exactly reproduce the power expended. It has been found by careful experiment, that if the power developed by the descent of a pound weight through 772 feet be expended in agitating a pound of water, it will raise the temperature of that water 1° Fahrenheit. The fall of any given quantity of water through 772 feet is consequently called the *Mechanical Equivalent* of the heat required to raise the same quantity of water one degree in temperature; since theoretically the two values are equivalent, and practically the power will produce the heat. But we have not yet any form of apparatus by which the heat would produce the power; and before we can possess such, we must have an engine ten times better than the best form of steam-engine at present in use. There is every reason to believe that there is a definite quantity of mechanical power or energy in the universe, the amount of which can neither be increased nor diminished, though it may be transformed from one shape into another; and heat, light, electricity, and all

chemical and vital phenomena are merely phases, more or less complex and disguised, of the same elementary force.

LAWS OF FALLING BODIES.

Bodies falling to the earth by gravity are drawn thither by a species of attraction—constant in amount—which acts in a manner similar to that which reveals itself when two bodies in opposite electrical states are brought into proximity. We do not know with any certainty the *cause* of gravity. But we know that it would be quite impossible for one body to act upon another without some link to connect the two together; and the most probable supposition is, that as sound is a pulsation of the air, caused by pulsations of the sounding body, and as light is a pulsation in the ether which fills all space, caused by pulsations of the illuminating body, so gravity is a similar pulsation in the ether, or a pulsation in another kind of ether, caused by the pulsations of the attracting body. We know by experience that similar pulsations may be generated in a piece of iron by sending an electric current through it under certain conditions, and which, for the time, transforms the iron into a magnet, which will attract iron in the same way in which the earth attracts heavy bodies: and, in like manner, a piece of amber or of sealing-wax may be made to attract straws, pieces of paper, and other light substances, by being briskly rubbed. The phenomena of the gyroscope seem to show that gravity takes an appreciable time to act. If a heavy wheel set on the end of a horizontal shaft, which is sustained by two suitable supports, be put into rapid rotation, the support nearest the wheel may be taken away without the wheel falling down, from which it appears that the pulsations which produce gravity may be so confounded together by the rapid change in the position of the wheel, and consequently in the rapid change in the direction of the attracting pulses or waves, that the phenomena of gravity are no longer exhibited, or what remains of them is manifested in a horizontal direction instead of in a vertical—the wheel having shifted into or towards that direction before the pulsations have had time

to be completed. We know from experience that conflicting sounds may be made to produce silence, and that conflicting lights may be made to produce darkness; and in like manner, it would appear, that a conflict in the pulsations which are the cause of gravity may sensibly impair or destroy that gravity. It has long been known that sunlight consists of light of those different colours which are exhibited in the rainbow, and that the phenomena of colours in natural objects is produced by the property those objects have of absorbing some rays, and reflecting others, so that in a red object the whole of the rays except the red rays are absorbed—and they are reflected; and in a blue object the whole of the rays except the blue rays are absorbed—and they are reflected; and, as only the reflected rays meet the eye, the objects appear of a red or blue colour. It has also long been known that in black objects the whole of the rays are absorbed, and none reflected; and in white objects that the whole are reflected and none absorbed. But the resources of photography also enable us to know that there is a species of light which is *invisible*—which has no colour, and no illuminating power, but which reveals its existence by the effect it produces on photographic preparations. The use of these photographic preparations is consequently equivalent to the acquisition of a distinct sense; and one of the most important problems in philosophy is to discover how we may acquire the use of artificia. senses, whereby we may more effectually interrogate nature. There may be rays in sunlight, and modes of communication between one body and another, of which we have no distinct conception yet; but there must be a mode of communication, of *some* kind or other, in every case in which cause and effect are known to exist.

The force of gravity, like the force of light or of sound, varies in strength with the extension of the orb of propagation; or, in other words, it diminishes in intensity according to a given law with the distance from the earth's surface. Nor is this force precisely the same in all parts of the world, as near the equator it is partly counteracted by the operation of the centrifugal force due to the earth's rotation. But all these disturbing causes

are of too little effect to be worth noticing further in a work of this kind; and for all practical purposes we may reckon the force of gravity as *uniform* in all ages, and at all parts of the earth's surface. Now, as power is pressure acting through space, a falling body just before it reaches the earth must have a certain proportion of mechanical power stored up in it which, if again used to raise the weight, would carry it up once more to its original position. This action we observe in a pendulum. If we raise the ball of a pendulum sideways through any given elevation, it will accumulate so much power or momentum in its descent through the arc in which it swings, as to carry it up to the same height on the opposite side of the arc, or at least it will do so *nearly*, and would do so *wholly* but for the friction of the suspending point and of the atmosphere, which will cause some slight diminution in the amount of elevation at each successive beat. If a hole could be made through the centre of the earth, and a ball were suffered to drop down it, the velocity would go on accelerating—supposing there were no resisting atmosphere —until the centre of the earth were reached; and the ball would then pursue its course with a velocity gradually diminishing until it reached the surface at the antipodes, when it would come to rest, and return—circulating on for ever from surface to surface, in a manner similar to that in which a pendulum beats in its arc. If we suppose an atmosphere to be introduced into the hole or tunnel, then the ball would go on accelerating only until the resistance of the atmosphere balanced the weight, after which no further acceleration would take place. This is the same action that exists when a railway-train or a steam-vessel is put into motion by an engine. In each case the train, or steamer, continues to accelerate until the resistance of the air or of the water balances the propelling force, after which, an equipoise being established, no further acceleration takes place.

The velocity which bodies acquire by falling freely by gravity proceeds according to a known law, and it is consequently easy, when we know the height from which a body has fallen, to determine its velocity; or conversely, when we know its velocity, we can easily tell from what height it must have descended.

Since, too, power is measurable by the distance through which a given weight is lifted, or through which it descends, it becomes easy to tell when we know the weight and velocity of any body, how much power there is stored up in it, since this power will, in fact, be represented by the weight multiplied by the height through which the body must have fallen to acquire its velocity.

If the successive additions of velocity which a fallen body receives in each second of its fall—namely, $32\frac{1}{6}$ feet—be represented by the letter g, then the different relations of the time of falling, the ultimate velocity, and the height fallen through, will be as follows:—

MOTION OF A HEAVY BODY FALLING IN VACUO.

Time in seconds............	0	1	2	3	4	5	6	7	8	9	10
Ultimate velocity..........	0	$1g$	$2g$	$3g$	$4g$	$5g$	$6g$	$7g$	$8g$	$9g$	$10g$
Height fallen through......	0	$1\frac{g}{2}$	$4\frac{g}{2}$	$9\frac{g}{2}$	$16\frac{g}{2}$	$25\frac{g}{2}$	$36\frac{g}{2}$	$49\frac{g}{2}$	$64\frac{g}{2}$	$81\frac{g}{2}$	$100\frac{g}{2}$
Spaces in each second......	0	$1\frac{g}{2}$	$3\frac{g}{2}$	$5\frac{g}{2}$	$7\frac{g}{2}$	$9\frac{g}{2}$	$11\frac{g}{2}$	$13\frac{g}{2}$	$15\frac{g}{2}$	$17\frac{g}{2}$	$19\frac{g}{2}$

The same relations are shown more in detail in the following table:

MOTION OF A BODY FALLING IN VACUO.

Time of falling in seconds.	Height fallen in feet.	Velocity acquired in feet per second.
0 rest	0	0
$\frac{1}{4}$	$1\frac{1}{192}$	$8\frac{1}{24}$
$\frac{1}{2}$	$3\frac{4}{48}$	$16\frac{1}{12}$
$\frac{3}{4}$	$9\frac{3}{4}$	$24\frac{1}{8}$
1	$16\frac{1}{12}$	$32\frac{1}{6}$
$1\frac{1}{4}$	$25\frac{25}{192}$	$40\frac{5}{24}$
$1\frac{1}{2}$	$36\frac{3}{16}$	$48\frac{1}{4}$
$1\frac{3}{4}$	$49\frac{49}{192}$	$56\frac{7}{24}$
2	$64\frac{1}{3}$	$64\frac{1}{3}$
$2\frac{1}{4}$	$81\frac{27}{64}$	$72\frac{3}{8}$
$2\frac{1}{2}$	$100\frac{25}{48}$	$80\frac{5}{12}$
$2\frac{3}{4}$	$121\frac{121}{192}$	$88\frac{11}{24}$
3	$144\frac{3}{4}$	$96\frac{1}{2}$
4	$257\frac{1}{3}$	$128\frac{2}{3}$
5	$402\frac{1}{2}$	$160\frac{5}{8}$
6	579	193
7	$788\frac{1}{2}$	$225\frac{1}{6}$
8	$1029\frac{1}{3}$	$257\frac{1}{3}$
9	$1302\frac{3}{4}$	$289\frac{1}{2}$

RULES.

VELOCITY FROM HEIGHT.

TO FIND THE VELOCITY ACQUIRED BY A HEAVY BODY IN FALLING THROUGH ANY GIVEN HEIGHT.

RULE.—*Multiply the square root of the height in feet through which the body has fallen by the constant number* 8·021. *The result will be the velocity in feet per second which the body will have attained.*

Example.—Suppose a leaden bullet to be dropped from a height of 400 feet: with what velocity will it strike the ground?

Here the square root of 400 is 20, and 20, multiplied by 8·021=160·42, which is the velocity in feet per second which the bullet will have acquired on reaching the ground.

The same result is attained by multiplying the space fallen through in feet by 64·333, and extracting the square root of the product, which will be the velocity in feet per second.

VELOCITY FROM TIME.

TO FIND THE VELOCITY IN FEET PER SECOND WHICH A BODY WILL ACQUIRE BY FALLING FREELY DURING ANY GIVEN NUMBER OF SECONDS.

RULE.—*Multiply the number of seconds occupied in falling by* 32·166. *The result is the velocity of the body in feet per second.*

Example.—Suppose a stone to be dropped from such a height that it requires four seconds to reach the ground, what velocity will the stone have acquired at the end of its descent?

Here four seconds multiplied by 32·166=128·664, which is the velocity in feet per second that the stone will have acquired on reaching the ground.

HEIGHT FROM VELOCITY.

TO FIND FROM THE VELOCITY ACQUIRED BY A FALLING BODY THE HEIGHT FROM WHICH IT MUST HAVE FALLEN, AND ALSO THE TIME OF THE DESCENT.

RULE.—*Divide the square of the acquired velocity in feet per second by* 64·333, *which will give the height in feet from which the body must have fallen; and divide the height fallen by the constant number* 16·083, *and extract the square root of the quotient, which will be the time of descent in seconds.*

Example.—If a stone dropped from the summit of a tower strike the ground with a velocity of 120 feet per second, what will be the height of the tower, and what the time occupied by the stone in its descent?

Here 120 squared=14400 and 144400 divided by 64·33= 223·84, which is the height of the tower. Further, 223·84 divided by the constant number 16·083=13·9, the square root of which is 3·72, which will be the time in seconds that the stone will have taken to fall 223·84 feet.

HEIGHT FROM TIME.

TO FIND FROM THE TIME OCCUPIED IN THE DESCENT OF A FALLING BODY WHAT THE HEIGHT IS FROM WHICH IT MUST HAVE DESCENDED.

RULE.—*Multiply the square of the time occupied in the descent in seconds by the constant number* 16·083. *The product is the height in feet from which the body must have fallen.*

Example.—If a stone when suffered to fall into a well strikes the surface of the water in four seconds, what is the depth of the well to the surface of the water?

Here 4 seconds squared=16 seconds, and 16 multiplied by 16·083=257½ feet, which is the depth of the well to the surface of the water.

TIME FROM VELOCITY.

TO FIND THE TIME IN SECONDS DURING WHICH A HEAVY BODY MUST HAVE CONTINUED TO FALL TO ATTAIN ANY GIVEN VELOCITY.

RULE.—*Divide the velocity in feet per second by the constant number* 32·166. *The quotient is the number of seconds during which the body must have continued to fall to attain its velocity.*

Example.—If a stone in falling has attained a velocity on reaching the ground of 128·664 feet per second, how many seconds must it have occupied in its descent?

Here 128·664 divided by 32·166=4, which is the number of seconds that the stone must have continued to fall to attain its velocity.

TIME FROM HEIGHT.

TO FIND THE TIME IN WHICH A HEAVY BODY WILL FALL THROUGH A GIVEN HEIGHT.

RULE.—*Divide the height expressed in feet by the constant number* 16·083, *and extract the square root of the quotient, which will give the time in seconds in which the heavy body will fall through the given height.*

Example.—Suppose a stone to be let fall from a tower 400 feet high, in what time will it reach the ground?

Here 400 divided by 16·083=24·87, and the square root of 24·87 is 4·986, or very nearly 5 seconds, which is the time that would elapse before the stone reached the ground.

TO FIND THE NUMBER OF FEET PASSED THROUGH BY A FALLING BODY IN ANY GIVEN SECOND OF ITS DESCENT.

RULE.—*Multiply the number of the second by* $32\frac{1}{6}$ *and subtract from the product* $16\frac{1}{12}$. *The remainder will be the number of feet passed through in the second given.*

Example.—To find the number of feet passed through by a falling body in the ninth second of its descent.

Here we have $9 \times 32\frac{1}{6} = 289\frac{1}{2} - 16\frac{1}{12} = 273\frac{5}{12}$, which is the number of feet passed through in the ninth second of the descent.

MOTION OF FLUIDS.

The velocity with which water will flow out of a hole at the side or in the bottom of a cistern, will be the same as that which a heavy body will acquire in falling from the level of the water surface to the level of the orifice, and may easily therefore be computed by a reference to the laws of falling bodies. The atmosphere exerts a pressure of about 14·7 lbs. per square inch, or 2116·4 lbs. per square foot, on all bodies on the earth's surface; and if the atmosphere be pumped out of the space beneath a piston, while suffered to press on its upper surface, the piston will be forced downward in its cylinder with a pressure of 14·7 lbs. on each square inch of the piston's area. In a common sucking pump the water is drawn up after the piston, in consequence of the production of a partial vacuum beneath the piston; and the water in the well being subjected to the pressure of the atmosphere while the pressure is removed from the water in the pump barrel, the water rises in the suction pipe, and would continue to do so if the pump were raised further and further up, until a column of water had been interposed between the pump-barrel and the well sufficiently high to balance the weight of the atmosphere. The water will cease to rise any higher after this altitude has been attained.

When we know the weight of a cubic inch or cubic foot of water, it is easy to tell the number of cubic inches or cubic feet that must be piled upon one another to produce a weight of 14·7 lbs. on the square inch or 2116·4 lbs. on the square foot; and it will be found to be 408 cubic inches in the case of the cubic inches, or a column 1 inch square and 34 feet high, or 34 cubic feet in the case of the cubic feet. Mercury being about 13·6 times heavier than water, a column of mercury 1 inch square and 30 inches high will weigh about 15 lbs. A column

of air high enough to weigh 15 lbs., will be 773·29 times higher than a column of water of the same weight—water being 773·29 times heavier than air at the ordinary barometric density of 29·9 inches of mercury. In other words, the height of a column of air 1 inch square and the same density as that on the earth's surface, that will weigh 15 lbs., will be $34 \times 773{\cdot}29 = 25521{\cdot}86$ feet, or taking the atmospheric pressure at 14·7 lbs., the height will be 26214 feet. The velocity therefore with which water will rush into a vacuum, will be equal to that which a heavy body will acquire in falling through a height of 34 feet. The velocity with which mercury will flow into a vacuum, will be equal to that which a heavy body will acquire by falling through a height of $2\frac{1}{2}$ feet; and the velocity with which air will flow into a vacuum, will be equal to that which a heavy body will acquire by falling through a height of 26214 feet. Now the velocity which a heavy body will acquire in falling through 34 feet will be equal to the square root of 34, which is 5·8 multiplied by the constant number 8·021; or it will be 46·5218 feet per second, which consequently will be the velocity with which water will flow into a vacuum. The velocity with which mercury will flow into a vacuum will be 12·83 feet per second, for the square root of $2\frac{1}{2}$ is 1·6 nearly, and 1·6 multiplied by $8{\cdot}021 = 12{\cdot}8336$. The velocity with which air weighing 0·080728 lbs. per cubic foot will flow into a vacuum will be 1298.5999 feet per second; for the square root of 26214 is 161·9 nearly, which multiplied by $8{\cdot}021 = 1298{\cdot}5999$ feet per second. The density of the air here supposed is the density at the temperature of melting ice. At the ordinary atmospheric temperatures the density will be somewhat less; and if the density be taken so that the height of the homogeneous atmosphere, as it is called, or of that imaginary atmosphere which produces the pressure—and which is supposed to be of uniform density throughout its depth—is 27,818 feet, then the velocity of the air rushing into a vacuum will be a little greater than what it has been here reckoned at, or it will be 1338 feet per second. These velocities it will be understood are the *theoretical* velocities, which can in no case be exceeded; but which are fallen

short of in practice to a greater or less extent, depending on the size and form of the orifice through which the air enters, and other analogous circumstances.

The velocity with which steam or any vapour or gas whatever will rush into a vacuum, can easily be determined when we know its pressure and density; for taking into account the density, or the weight of one cubic foot, we have merely to see how many of these cubic feet must be piled upon one another to produce the given pressure or weight upon the square foot of base; and the velocity will be in every case the same as that which a heavy body would acquire in falling through the height of the column required to produce the weight. Thus it is found that the density of steam of the atmospheric pressure is about 1700 times less dense than water. Mr. Watt reckoned that a cubic inch of water produced a cubic foot or 1728 cubic inches of steam, having the same pressure as the atmosphere; and if the pressure of the atmosphere be equal to the pressure produced by 34 feet of water, then, if we reckon steam as 1700 times less dense than water, it would require 1700 columns of steam, each 34 feet high, placed on top of one another, to exert the same weight or pressure as one column of water 34 feet high. Now 1700 hundred times 34 is 57800, which therefore is the height a column of steam 1700 times less dense than water would require to have in order to balance the pressure of the atmosphere or of 34 feet of water. The velocity which a body would acquire in falling through a height of 57800 feet, is 1926·6 feet per second; for the square root of 57800 is 240·2 nearly, and 240·2 multiplied by 8·021=1926·6442 feet per second, which is consequently the velocity with which steam of this pressure would rush into a vacuum. The velocity with which steam of a greater pressure than that of the atmosphere will rush into a vacuum, will not be sensibly greater than that of steam of the atmospheric pressure. For as the density of the steam increases in nearly the same ratio as its pressure, the column will require to be as much lower, by virtue of the increased density, as it requires to be higher to give the increased pressure. In other words, the height of the theoretical column of steam required

to produce the pressure, will be nearly the same at all pressures; since a low column of dense steam will produce the same pressure as a high column of rare, and the density and pressure advance in nearly the same ratio. It may hence be concluded that steam of all pressures will rush into a vacuum with a velocity of about 2,000 feet per second, if the vacuum be perfect and the flow unimpeded.

If steam, instead of being suffered to escape into a vacuum, be made to issue into a vessel containing steam of a lower pressure, the velocity of efflux will be the same as that which a heavy body would acquire in falling from the top of the column of steam required to produce the greater pressure, to the top of a lower column of the same steam adequate to produce the lesser pressure. Thus if we have steam with a pressure of two atmospheres, flowing into steam with a pressure of one atmosphere, then, inasmuch as the density or weight of the steam increases very nearly in the same proportion as its pressure, a cubic inch of steam with a pressure of two atmospheres will be about twice as heavy as a cubic inch of steam with a pressure of one atmosphere. Such steam, therefore, instead of being 1700 times less dense than water, will be the half of this or only 850 times less dense than water. A column of this steam, therefore, 850 times 34 feet=28900 feet high, will exert a pressure of one atmosphere, or about 15 lbs. on each square inch; and a column of twice this height, or 57800 feet, will exert a pressure of two atmospheres or 30 lbs. on each square inch. The velocity with which the steam will rush from one vessel to the other, will be the same as that which a heavy body would acquire in falling from the height of the column of the denser steam required to produce the higher pressure to the top of the column of the same steam of such height as would produce the less pressure; and as in this case the heights of such columns will be 1700 × 34 feet, and 850 × 34 feet, or 57800 and 28900 feet, the *difference* of height will be 28900 feet; and the velocity of efflux from one vessel into the other will be equal to that which a heavy body would acquire by falling through a height of 28900 feet. Now the square root of 28900 is 170; and 170

multiplied by 8·021=1363·57 feet per second, which is the velocity with which steam with a pressure of two atmospheres would rush into steam with a pressure of one atmosphere. This consequently may be reckoned as the velocity with which steam of 15 lbs. pressure above the atmosphere would rush into the atmosphere. Such velocities at different pressures are exhibited in the following table:—

VELOCITY OF EFFLUX OF HIGH-PRESSURE STEAM INTO THE ATMOSPHERE.

Pressure of steam above the atmosphere.	Velocity of free efflux in feet per second.	Pressure of steam above the atmosphere.	Velocity of free efflux in feet per second.
lbs.	feet.	lbs.	feet.
1	482	50	1791
2	663	60	1838
3	791	70	1877
4	890	80	1919
5	973	90	1936
10	1241	100	1957
20	1504	110	1972
30	1643	120	1990
40	1729	130	2004

This table is computed by taking the difference of the two pressures for the effective pressure, which effective pressure is expressed in pounds per square inch, divided by the weight of a cubic foot of the denser fluid in pounds, and the square root of the quotient is multiplied by 96. The denser the fluids are the less, it is clear, will be the velocity of efflux which a given difference of pressure will create; for the heights of the columns, and also the difference of their heights, will be small in the proportion of the density of the denser fluid. The more dense the fluid is, the larger becomes the mass of matter which a given pressure has to move. With steam of 16 lbs. pressure flowing into steam or air of 15 lbs. pressure, the moving pressure is 1 lb., and the velocity of efflux is 482 feet per second. With steam of 101 lbs. pressure flowing into steam or air of 100 lbs. pressure,

the moving pressure is the same, but the velocity of efflux will only be 207 feet per second.

INERTIA AND MOMENTUM.

When a body is moved from a state of rest to a state of motion, or from a slow motion to a faster, power is absorbed by the body; and when a body is brought from a state of motion to rest, or from a fast motion to a slow one, power is liberated by the body. The quality which enables a body to resist the sudden communication of motion is termed its Inertia; and the quality which enables a body to resist the sudden extinction of motion is termed its Momentum. Whatever power a body absorbs in being put into motion, it afterwards surrenders in being brought to a state of rest; and the amount of power existing in any moving body is measurable by its weight multiplied by the square of its velocity, or by the height through which it must have fallen by gravity to attain its velocity.

A railway carriage of ten tons' weight, therefore, moving at a speed of 20 miles an hour, will have as great a momentum as 4 railway carriages weighing 10 tons each moving at the rate of 10 miles an hour. In like manner the momentum of a cannon ball moving at a velocity of 1,700 feet a second, will be 28,900 times greater than if it moved at a speed of 10 feet per second, since the square of 1,700 is to the square of 10 as 28,900 to 1. Josephus mentions that some of the battering-rams employed by the Romans in Judea were 90 feet long, and weighed 1,500 talents of 114 lbs. to the talent, or 76·3392 tons. The weight of a cannon ball which has the same amount of mechanical power stored up in it, or which will give the same force of impact when moving at a speed of 1,800 feet per second, as the battering-ram will do when moving at a velocity of 10 feet per second, can easily be determined; for we have only to multiply 76·3392 tons by the square of 10 and divide by the square of 1,800, which will give ·0023561 tons, or 5·12776 lbs., as the weight of the ball required.

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TO FIND THE QUANTITY OF MECHANICAL POWER REQUIRED TO COMMUNICATE DIFFERENT VELOCITIES OF MOTION TO HEAVY BODIES.

RULE.—*Multiply the mass of matter by the height due to the velocity it has acquired, supposing that it attained its velocity by falling by gravity. The product is the mechanical power communicated in generating that velocity of motion in the body.*

Example 1.—Suppose a waggon on a railway to weigh 2,500 pounds, what mechanical power must be communicated to it to urge it from rest into motion with a velocity of 3 miles an hour, or 4·4 feet per second?

Now here the height in feet from which a body must have fallen to acquire any given velocity will be the square of the velocity in feet per second divided by 64⅓; or it will be the square of the quotient obtained by dividing the velocity in feet per second by the square root of 64⅓, or 8·021. Now 4·4 ÷ 8·021 =·5487, the square of which is ·301 feet, the height that a body must fall to acquire a velocity of 3 miles an hour. Hence the mechanical power communicated is 2,500 lbs. × ·301 ft. = 752·5 lbs. descending through 1 foot.

Example 2.—Required the mechanical effect treasured up in a cast-iron fly-wheel, the mean diameter of which is 30 feet with a sectional area of rim of 60 square inches, and making 20 turns in the minute.

The diameter of the wheel being 30 feet, the circumference will be 94·248 feet, and, as the wheel makes 20 revolutions in the minute, the velocity of the rim will be 94·248 × 20 = 1884·96 feet per minute, or 31·416 feet per second. Again the cubical content of the rim in cubic feet being 60 × 94·248 ÷ 144 = 39·27 cubic feet, and the weight of a cubic foot of cast-iron being 453¼ lbs., we have 39·27 × 453¼ = 17794·22 lbs. as the weight of the rim. Hence the mechanical effect treasured up in the rim of this wheel is $17794{\cdot}22 \times (31{\cdot}416 \div 8{\cdot}021)^2 = 268{,}650$ lbs. raised one foot high. This it will be observed is about eight actual horse-power. The mechanical energy with which the fly-wheel of an engine is generally endowed, is equal to the power exerted

in from four to six half strokes of the engine, or two to three complete revolutions; so that the fly-wheel above particularized is such as would be suitable for an engine which exerts a power of four actual horses, or four times 33,000 pounds raised one foot high in each revolution, or 80 horses' power.

BODIES REVOLVING IN A CIRCLE.

When bodies revolve in circles round fixed axes of motion, the different particles can have no motion except in circles described round such fixed axes; and the velocities of the particles composing the body must be greater or less, depending upon their distance from the centre round which the body revolves. To apply the laws of falling bodies to this case we must imagine the particles composing such revolving bodies to be divided and collected into several small bodies situated at different distances from the centre, and therefore moving with different velocities; and then we may determine the power which must be communicated to each of the supposed separate bodies to give it the velocity which it actually possesses. The sum of all the powers so determined is the total power which must be communicated to the body, to give to it the velocity of motion with which it actually revolves. Thus a rod moving about one of its extremities may be supposed to be compounded of a number of balls, like a string of beads strung on a wire. The velocity of each of these balls can then be ascertained, which will enable us to compute the mechanical power resident in it, and which will be the same as if it moved in a straight line. The sum of the quantities thus ascertained will be the total mechanical power resident in the revolving body.

CENTRIFUGAL FORCE.

The centrifugal force of a body which revolves in any circle in a given time, is proportional to the diameter of the circle in which it revolves. Thus, in the case of two fly-wheels of the same weight but one of twice the diameter of the other, the

larger wheel will have twice the amount of centrifugal force that the small one has.

The centrifugal force of a body moving with different velocities in the same circle is proportional to the square of the velocities with which it moves in that circle; or, what is the same thing, to the square of the number of revolutions performed in a given time. Thus, the fly-wheel of any engine will have four times the amount of centrifugal force it possessed before, if driven at twice the speed. In Mr. Watt's engines with sun and planet wheels, in which the fly-wheel made twice the number of revolutions made by the engine, the fly-wheel had four times the centrifugal force that would be possessed by the *same fly-wheel* if coupled immediately to the crank.

The centrifugal force of a body of a given weight, revolving with a certain uniform velocity in a circle of a given diameter, was investigated by the Marquis de l'Hôpital, who gave the rule for ascertaining this force that is now generally followed. It is founded on the consideration of the height from which the body must have fallen by gravity to have acquired the velocity with which its centre of gyration moves in the circle which it describes. Then as the radius of that circle is to double the height due to the velocity, so is the weight of the body to its centrifugal force.

TO FIND THE CENTRIFUGAL FORCE OF A BODY OF A GIVEN WEIGHT REVOLVING IN A CIRCLE OF A GIVEN DIAMETER.

RULE.—*Divide the velocity in feet per second by* 4·01, *and the square of the quotient is four times the height in feet due to the velocity. Divide this quadrupled height by the diameter of the circle, and the quotient is the centrifugal force when the weight of the body is* 1; *consequently, multiplying it by the weight of the body gives the actual centrifugal force in pounds or tons.*

Example 1.—Suppose that the rim of a fly-wheel 30 feet diameter and weighing 15718 lbs., moves at the rate of 27·49 feet per second, what will be its centrifugal force? Here we have

the velocity 27·49÷4·01=6·85, which, squared, is 46·9225; and this, divided by 30, is 1·564: so that the centrifugal force is 1·564 times the weight of the body. or 10·97 tons.

Example 2.—Suppose that the rim of a fly-wheel which is 20 feet diameter moves with a velocity of 32⅙ feet per second: then 32·16÷4·01=8·02, the square of which is 64·32 feet, which is the quadrupled height due to the velocity, and this divided by 20 feet diameter gives 3·216 times the weight of the rim as the centrifugal force.

ANOTHER RULE.—*Multiply the square of the number of revolutions per minute by the diameter of the circle of revolution in feet, and divide the product by the constant number* 5870*; the quotient is the centrifugal force of the body in terms of its weight, which is supposed to be* 1.

Example 1.—Suppose a stone of 2 lbs. weight is placed in a sling, and whirled round in a circle of 4 feet diameter, at the rate of 120 revolutions per minute: then 120 squared=14400 × 4 feet diameter=57600÷5870=9·81 which is the ratio of the centrifugal force to the weight; and, the weight being 2 lbs., the centrifugal force acting to break the string and escape is 19·6 lbs.

Example 2.—In the case of the first fly-wheel 30 feet diameter, referred to above, we multiply the square of the number of revolutions per minute (17½) by the diameter of the circle in feet (30), and divide the product by 5870; which gives the centrifugal force in terms of the weight of the body, and $17\frac{1}{2}^2 \times 30 \div 5870 = 1·564$ as before.

TO FIND THE RATE AT WHICH A BODY MUST REVOLVE IN ANY CIRCLE, THAT ITS CENTRIFUGAL FORCE MAY BE EQUAL TO ITS WEIGHT.

RULE.—*Divide the constant number* 5870 *by the diameter of the circle in feet, and the square root of the quotient is the number of revolutions it will make per minute, when the centrifugal force is equal to the weight.*

Example.—In a circle of 6·5 feet diameter, a body must revolve about 30 times a minute that its centrifugal force may be

equal to its weight; for $5870 \div 6{\cdot}5 = 903$, the square root of which is 30·05 revolutions per minute.

The mechanical power which must be communicated to a solid disc of uniform density, to make it revolve on its axis, is the same as that which must be communicated to one-half of its weight of matter, to give it motion in a straight line with the same velocity with which the circumference of the disc moves in a circle.

TO DETERMINE THE BURSTING STRAIN OF A FLY-WHEEL.

If we suppose half of a fly-wheel to be securely attached to the axis, while the other half is held only by the rim or by bolts which it tends to break by its centrifugal force, then there will be a velocity at which the centrifugal force of half the rim will overcome the cohesion of the metal of the rim, or of the bolts, and the wheel will be burst by its centrifugal force.

In mechanical works it has been usual to reckon the cohesive strength of wrought-iron within the limits of elasticity at 17,800 lbs. per square inch of section, and of cast-iron at 15,300 lbs. per square inch of section; by which is meant that a bar of wrought-iron one inch square might be stretched by a weight of 17,800 lbs. without injury, and a bar of cast-iron might be stretched by a weight of 15,300 lbs. without injury, and though somewhat drawn out by such weights, would, like a spiral spring, again return to the original length on the weight being removed. This estimate for cast-iron is much too high; and in machinery wrought-iron should not be loaded with more than 4,000 lbs. per square inch of section, and cast-iron should not be loaded with more than 2,000 lbs. per square inch of section. The breaking tensile strength of good wrought-iron is about 60,000 lbs. per square inch of section, and of good cast-iron about 15,000 lbs. per square inch of section. But both wrought and cast-iron will be broken gradually with much less strain than would be required to break them at once; and if the limit of elasticity be exceeded, they will undergo a gradual deterioration, and will be broken in the course of time. If the velocity

of rotation of a cast-iron fly-wheel be so great that its centrifugal force becomes greater than 15,000 lbs. in each square inch of the section of the rim, it will necessarily burst, as a wrought-iron one would also do if the centrifugal force exceeded 60,000 lbs. per square inch of section. But to be within the limits of safety, a strain of 4,000 lbs. per square inch of section should not be exceeded for wrought-iron, and 2,000 lbs. per square inch of section for cast.

TO DETERMINE THE MECHANICAL POWER RESIDENT IN A REVOLVING DISC.

RULE.—*Multiply one-half of the weight of the revolving disc by the height due to the velocity with which the circumference of the wheel or disc moves; the product is the mechanical power communicated.*

Example 1.—Suppose that a grindstone 4·375 feet diameter, weighing 3,500 lbs., makes 270 revolutions per minute; what power must be communicated to it to give it that motion?

The velocity of the circumference will be 61·83 feet per second, and the height due to this velocity is 59·4 feet. The mechanical power is 1,750 lbs. (half the weight) × 59·4 feet = 103·950 lbs. raised one foot.

If the revolving-wheel is not an entire disc or solid circle, but only a ring or annulus, it must first be considered as a disc, and the effect of the part which is wanting must then be calculated and deducted.

Example 2.—Suppose the rim of a cast-iron fly-wheel to be 22 feet diameter outside, and 20 feet inside, and that the thickness of the rim is 6 inches, and that the wheel makes 36 revolutions per minute, what power must be communicated to the rim to give it that motion, the weight of the arms being left out of the account?

A solid wheel 22 feet diameter and 6 inches thick would contain 190 cubic feet, from which, if we deduct 157 cubic feet, which would be the capacity of a solid wheel 20 feet diameter and 6 inches thick, we have 33 cubic feet as the cubical contents

of the annulus. Now in the case of a solid wheel of 22 feet diameter, the velocity of the circumference at 36 revolutions per minute would be 41·47 feet per second, the height due to which would be 26·8 feet, which multiplied by 95 cubic feet (or half the mass) gives 2,546 cubic feet of cast-iron, raised 1 foot for the power communicated. Then supposing another solid wheel 20 feet diameter, we shall find by a like mode of computation that the power communicated is equivalent to 1,735 cubic-feet of cast-iron raised through 1 foot. This deducted from 2,546 leaves 811 cubic feet raised through 1 foot as the power resident in the annulus; and if we take the weight of a cubic foot of cast-iron in round numbers as 480 lbs., we have 389,280 lbs. raised 1 foot, for the mechanical power which must be communicated to the rim of the fly-wheel in question to give it a velocity of 36 revolutions per minute.

The mechanical power which must be communicated to solid discs of different diameters, but of the same thickness and density, to make them revolve in the same time, is as the fourth powers of their diameters.

CENTRES OF GYRATION AND PERCUSSION.

The centre of gyration is a point in bodies which revolve in circles in which the momentum, or energy of the moving mass, may be supposed to be collected. It is in the same point as the centre of percussion of revolving bodies, because a revolving body, if suffered to strike another body that is either at rest or that moves with a different velocity in the same orbit, will neither be deflected to the right nor to the left, but will act just as if the whole mass of matter were collected in that point. In bodies moving forward in a straight line, the centre of percussion is in the centre of gravity; but, in bodies revolving in circles, the part of the body most remote from the centre of the circle moves with a different velocity from the part nearest to the centre of the circle. The centre of percussion, therefore, cannot be in the centre of gravity in such a case, but at some point nearer the circumference of the circle; and the line traced

by that point will divide the body into two parts, each having the same amount of mechanical power treasured in them, or each requiring the same amount of mechanical power to put them into revolution at their existing velocity. If the body, therefore, could be divided instantly, and without violence, through the line traced by the centre of gyration, each portion of the body would continue to revolve with its former velocity. The point tracing the line which thus divides the body is the centre of percussion, and also the centre of gyration, and in revolving bodies these centres are identical. If a given pressure act, through a given space, upon a body at its centre of gyration, in the direction of a tangent to the circle which that centre must describe round the fixed centre of motion, such an amount of power will move the centre of gyration with the same velocity in its circle of revolution, as it would move an equal mass of matter in a right line by acting at the centre of gravity of the mass. If the whole mass of the revolving body could be collected into its centre of gyration, the mechanical power resident in the body would be represented by multiplying the total weight of the body by the square of the velocity of the centre of gyration.

TO FIND THE DISTANCE OF THE CENTRE OF GYRATION OF ANY REVOLVING BODY FROM THE CENTRE OR AXIS OF MOTION.

RULE.—*Multiply the weight of each particle, or equal small portion of the body, by the square of its distance from the axis, and divide the sum of all these products by the weight of the whole mass; the square root of the quotient will be the distance of the centre of gyration from the axis of motion.*

Example.—Suppose three cannon balls to be fixed on a straight rod which is assumed to be without weight; one ball, weighing 2 lbs., is fixed at a distance of 10 inches from the axis of motion; another, which weighs 4 lbs., at 6 inches' distance; and the third, which weighs 6 lbs., at 4 inches' distance; then the distance of the centre of gyration from the axis of motion will be found thus: 10 inches squared $=100$; $\times$ 2 lbs. $=200$;

6 inches squared × 4 lbs. = 144; and 4 inches squared × 6 lbs. = 96. The sum of these products is 440, which divided by the sum of the weights, or 12 lbs. = 36·66, the square root of which, 6·05 inches, is the distance of the centre of gyration from the axis of motion; therefore, a single ball of 12 lbs. weight, placed at 6·05 inches from the axis of motion, and making the same number of revolutions in any given time, would have the same amount of mechanical power resident in it as the three balls in their several places, as at first supposed.

The mechanical power which must be communicated to a straight uniform rod or lever, to put it in motion, about one of its extremities, as a fixed centre or axis, is the same as that which must be communicated to an equal weight of matter to give it motion in a straight line, with ·57,735 of the velocity with which the extremity of the lever moves in its circle. The point in the revolving lever which moves with that velocity is the centre of gyration.

The mechanical power which must be communicated to a solid circular wheel to make it revolve upon its axis, is the same as that which must be communicated to an equal weight of matter to give it motion in a straight line with ·7071 of the velocity with which the periphery of the wheel moves within its circle, and the point in the radius of the wheel which moves with ·7071 of the velocity of the circumference is the centre of gyration. The weight of the revolving body, multiplied into the height due to the velocity with which the centre of gyration moves in its circle, in all cases represents the mechanical power which must be expended upon the body to give it the velocity of rotation that it possesses.

THE PENDULUM.

The point from which the pendulum is hung is termed the centre of suspension. The effective centre of the ball is an imaginary point called the centre of oscillation, and which is so situated that the distance from the centre of suspension to the centre of oscillation is the same as if the rod of the pendulum were destitute of weight, and the whole matter of the ball were

collected into the centre of oscillation. The centre of oscillation is situated in a line passing between the centre of suspension and the centre of gravity.

The number of vibrations made by pendulums of different lengths is inversely as the square roots of their lengths. The length of the pendulum which will make one vibration every second is somewhat different at different parts of the earth's surface, but in the latitude of London its length is variously stated at 39·1393 inches and 39·1386 inches.

TO FIND THE HEIGHT THROUGH WHICH A BODY WILL FALL IN THE TIME THAT A PENDULUM MAKES ONE VIBRATION.

RULE.—*Multiply the length of the pendulum by* 4·9348 *and it will give the height.*

Example.—If we take the length of the seconds pendulum at 39·1386 in., then $39{\cdot}1386 \times 4{\cdot}9348 = 193{\cdot}141$ in., which is the height that a body will fall by gravity in a second.

TO FIND THE LENGTH OF A PENDULUM WHICH WILL PERFORM A GIVEN NUMBER OF VIBRATIONS IN A MINUTE.

RULE.—*Divide the constant number* 375·36 *by the number of vibrations to be made per minute, and the square of the quotient is the length of the pendulum in inches.*

Example.—If the pendulum has to make 60 vibrations per minute, then $375{\cdot}36 \div 60 = 6{\cdot}256$, the square of which is 39·1386. The length 39·1393 is probably still more nearly the correct length of the seconds pendulum in London.

TO FIND THE NUMBER OF VIBRATIONS PER MINUTE WHICH A PENDULUM OF A GIVEN LENGTH WILL MAKE.

RULE.—*Multiply the square root of the length of the seconds pendulum by the number of vibrations it makes per minute, and divide the product by the square root of the length of the pendulum whose rate of vibration has to be found. The quotient is the number of vibrations per minute that the pendulum will make.*

Example.—If the length of a pendulum in the latitude of London be 28·75 inches, what will be the number of vibrations that it will make per minute?

Here the square root of 39·1393 multiplied by 60, and divided by the square root of 28·75=70 vibrations per minute.

TO FIND THE LENGTH OF A PENDULUM WHICH SHALL MAKE A GIVEN NUMBER OF VIBRATIONS IN A GIVEN TIME IN THE LATITUDE OF LONDON.

RULE.—*Multiply the square of the number of seconds in the given time by the constant number* 39·1393, *and divide the product by the square of the number of vibrations; the quotient will be the required length of pendulum in inches.*

Example.—What must be the length of a pendulum in order to give 35 vibrations per minute?

The number of seconds in the given time is 60, hence 60 multiplied by 60 multiplied by 39·1393 gives 140901·48, which divided by 1225 (the square of 35) gives 115·021 inches, the length of pendulum required.

TO FIND THE NUMBER OF VIBRATIONS WHICH WILL BE MADE IN A GIVEN TIME BY A PENDULUM OF A GIVEN LENGTH.

RULE.—*Multiply the square of the number of seconds in the given time by the constant number* 39·1393, *divide the product by the given length of the pendulum in inches, and the square root of the quotient will be the number of vibrations in the given time.*

Example.—The length of a pendulum being 64 inches, what number of vibrations will it make in 60 seconds?

In this case the square of 60 multiplied by 39·1393 gives 140901·48, which being divided by 64 gives 2201·5856, the square root of which 46·09 is the number of vibrations required.

THE GOVERNOR.

The governor is a centrifugal pendulum; and its proportions may be fixed by the same rules which are employed to deter-

mine the rates of vibration of pendulums. If we suppose a pendulum, in the act of vibration, to be at the same time pushed sideways by a suitable force, it will nevertheless perform its vibration in the same period of time; and if during its return it be again pushed sideways in the opposite direction, it will, during this double vibration, have pursued a curvilinear course, which, if the deflection be sufficient, will be a circle. A pendulum, therefore, of the same vertical height as the cone described by the arms of a governor, will perform a double vibration in the same time as the governor performs one revolution. The rules, however, according to which governors are usually proportioned are as follow:—

TO DETERMINE THE PROPER HEIGHT OF THE POINT OF SUSPENSION OF THE BALLS OF A GOVERNOR, ABOVE THE PLANE IN WHICH THEY REVOLVE WHEN MOVING WITH MEAN VELOCITY.

RULE.—*Divide the number* 35,225 *by the square of the main number of revolutions which the governor makes per minute. The quotient is the proper vertical height in inches of the point of suspension of the balls above the plane in which they revolve, when moving with mean velocity.*

Example.—What is the proper vertical height of the point of suspension above the plane of revolution in the case of a governor making 30 revolutions per minute?

Here 35225÷900 (the square of 30) = 39·139, which is the same height as that of the seconds pendulum.

If we have already the vertical height, and wish to know the proper time of revolution, we must proceed as follows:—

TO DETERMINE THE PROPER TIME OF REVOLUTION OF A GOVERNOR OF WHICH THE VERTICAL HEIGHT IS KNOWN.

RULE.—*Multiply the square root of the height by the constant fraction* 0·31986, *and the product will be the proper time of revolution in seconds.*

Example.—In what time should a governor be made to revolve upon its axis when the vertical height of the cone in which

the arms are required to revolve when in their mean position is 39·1393 inches? Here 6·256 × 0·31986 = 2 seconds.

FRICTION.

When two bodies are rubbed together they generate heat, and consume thereby an amount of power which is the mechanical equivalent of the heat produced. Clean and smooth iron drawn over clean and smooth iron without the interposition of a film of oil, or other lubricating material, requires about one-tenth of the force to move it that is employed to force the surfaces together. In other words, a piece of iron 10 lbs. in weight would require a weight of 1 lb. acting on a string passing over a pulley to draw the 10 lb. weight along an iron table. But if the surfaces are amply lubricated, the friction will only be from $\frac{1}{40}$th to $\frac{1}{60}$th of the weight. The friction of cast-iron surfaces in sandy water is about one-third of the weight. The extent of the rubbing surface does not affect the amount of the friction.

The experiments of General Morin on the friction of various bodies without an interposed film of lubricating liquid, but with the surfaces wiped clean by a greasy cloth have been summarised by Mr. Rankine in the following table:—

GENERAL MORIN'S EXPERIMENTS ON FRICTION.

No.	SURFACES.	Angle of repose.	Friction in terms of the weight.
1	Wood on wood, dry	14° to 26½°	·25 to 5
2	" " soaped	11½° to 2°	·2 to ·04
3	Metals on oak, dry	26½° to 31°	·5 to ·6
4	" " wet	13½° to 14½°	·24 to ·26
5	" " soapy	11½°	·2
6	Metals on elm, dry	11½° to 14°	·2 to ·25
7	Hemp on oak, dry	28°	·53
8	" " wet	18½°	·33
9	Leather on oak	15° to 19½°	·27 to ·38
1°	Leather on metals, dry	29½°	·56
11	" " wet	20°	·36
12	" " greasy	13°	·23
13	" " oily	8½°	·15
14	Metals on metals, dry	8½° to 11½°	·15 to ·2
15	" " wet	16½°	·3
16	Smooth surfaces, occasionally greased	4° to 4½°	·07 to ·08
17	" " continually greased	3°	·05
18	" " best results	1¼° to 2°	·03 to ·036
19	Bronze on lignum vitæ, constantly wet	3° ?	·05?

The 'Angle of repose,' given in the first column, is the angle which a flat surface will make with the horizon when a weight placed upon it just ceases to move by gravity. The column of 'Friction in terms of the weight' means the proportion of the weight which must be employed to draw the body by a string in order to overcome its friction; and the proportional weight is sometimes called the *Co-efficient of Friction.*

In a paper, of which an abstract has appeared in the *Comptes Rendus* of the French Academy of Sciences for the 26th of April, 1858, M. H. Bochet describes a series of experiments which have led him to the conclusion, that the friction between a pair of surfaces of iron is not, as it has hitherto been believed, absolutely independent of the velocity of sliding, but that it diminishes slowly as that velocity increases, according to a law expressed by the following formula. Let

R denote the friction;

Q, the pressure;

v, the velocity of sliding, in mètres per second = velocity in feet per second $\times$ 0·3048;

f, a, γ, constant co-efficients; then

$$\frac{R}{Q}=\frac{f+\gamma a v}{1+a v}$$

The following are the values of the co-efficients deduced by M. Bochet from his experiments, for iron surfaces of wheels and skids rubbing longitudinally on iron rails:—

f, for dry surfaces, 0·3, 0·25, 0·2; for damp surfaces, 0.14.

a, for wheels sliding on rails, 0·03; for skids sliding on rails, 0·07.

γ, not yet determined, but treated meanwhile as inappreciably small.

The friction of a bearing or machine *per revolution*, is nearly the same at all velocities, the pressure being supposed to be uniform; but as every revolution absorbs a definite quantity of power, and generates a corresponding quantity of heat, it will be necessary to enlarge the rubbing surfaces at high velocities, both to prevent the wear from being inconveniently rapid, and also to enable the bearing to present a larger cooling surface to

the atmosphere, so as to disperse the increments of heat which in the case of high velocities it will rapidly receive. With the same object the lubrication should be ample. The oil should overflow the bearing, in the same manner as the oil in a carcel or moderator lamp overflows the wick to prevent carbonisation; and, to prevent waste, the oil should be returned by an oil pump so as to maintain a circulation that will both cool and lubricate the rubbing parts.

It was found by General Morin in his experiments, that the 'Friction of Rest' was considerably more than the 'Friction of Motion,' or, in other words, that it took a greater force to move a rubbing body from a state of rest than it afterwards took to continue the motion, some of the softer bodies being in fact slightly indented with the weight. But in determining the friction of machinery, it is the friction of motion alone that has to be considered, so that the other need not be here taken into account.

In the case of rubbing surfaces which are amply lubricated, the amount of the friction depends more on the nature of the lubricant than upon the material of which the rubbing bodies are composed; and hard lubricants, such as tallow, are more suited for heavy pressures; and thin lubricants, such as almond oil, are best suited for mechanisms moving with considerable velocity, but on which the strain is small. If too heavy a pressure be applied to a bearing, the oil will be forced out and the surfaces will heat; and this will be liable to take place when the pressure on the bearing is much more than 800 lbs. per square inch on the longitudinal section of the bearing, though in practice the pressure is sometimes half as much again, or about 1,200 lbs. per square inch in the longitudinal section of the bearing, but such bearings will be liable to heat. Thus in a marine engine with a cylinder of 74½ inches diameter, the crank pin is 9½ inches diameter, and the length of the bearing is 10 inches, which makes the area of the longitudinal section of the bearing 95 square inches. The area of the cylinder is 4,359 square inches, and if we take the pressure upon the piston—including steam and vacuum—at 25 lbs. per square inch, we

shall have a total pressure on the piston of 108,975 lbs., and, consequently, this amount of pressure on the crank pin bearing. Now 108,975 lbs., the total pressure, divided by 95 square inches, the total surface, gives 1,147 lbs. for each square inch of the parallelogram which forms the longitudinal section of the bearing. In the engines of Messrs. Maudslay, Messrs. Seaward, and most of the London engineers, the pressure per square inch put upon the crank pin is less. Thus in their 120-horse engines, the diameter of the cylinder is 57½ inches, giving an area of 2,597 square inches, which multiplied by a pressure of 25 lbs. per square inch, gives 64,925 lbs. as the total pressure upon the piston. The crank pin is 8 inches diameter, and the bearing is 8½ inches long, giving 68 square inches as the area of the longitudinal section; and 64,925 lbs., the total pressure, divided by 68 square inches, the total area, gives a pressure of 954·77 lbs. per square inch of section. This is still in excess of the 800 lbs. per square inch to which it is expedient to limit the pressure. But the assumed pressure on the piston is rather large in the case of these engines, and the actual pressures will be found to agree pretty well with the limit of 800 lbs. on each square inch of the longitudinal section of bearings which it is proper to fix as a general rule in the case of engines moving slowly. In the case of fast-moving engines, however, the surface should be greater. The proportion in which the surface should vary with the speed is pretty accurately expressed by the following rule:—

TO FIND THE PRESSURE PER SQUARE INCH THAT MAY BE PUT UPON A BEARING MOVING WITH ANY GIVEN VELOCITY.

RULE.—*To the constant number* 50 *add the velocity of the bearing in feet per minute, and reserve the sum for a divisor. Divide the constant number* 70,000 *by the divisor found as above. The quotient will be the number of pounds per square inch that may be put upon the bearing.*

Example 1.—An engine with a cylinder 74½ inches diameter, has a crank pin 10 inches diameter. At 220 feet of the piston

per minute, and with a stroke of 7½ feet, the number of revolutions per minute will be about 15; and as the circumference of the crank pin will be about 30 inches or 2½ feet, the surface of the bearing will travel 15 times 2½, or 37½ feet per minute. Adding to this the constant number 50, we have 87½, and 70,000 divided by 87½ = 800, which, at this speed, is the proper pressure to put on each square inch of the longitudinal section of the bearing. If it is found on trial that this pressure is exceeded, the length or diameter of the pin must be increased or both.

Example 2.—An engine with a cylinder 42 inches diameter, has a crank pin 8½ inches diameter, the circumference of which is 26·7 inches or 2·225 feet. When the engine makes 54·8 revolutions per minute, the surface of the crank pin will move with a speed of 54·8 times 2·225 feet per minute, or 121·8 feet per minute. Now 50 + 121·8 = 171·8, and 70,000 divided by 171·8 = 407·3, which, at this speed of revolution, is the proper load to place upon each square inch of section in the line of the axis. The pressure of steam and vacuum in this engine was 40 lbs. per square inch; and as the area of a piston 42 inches diameter is 1385·4 square inches, the pressure urging the piston will be 40 times 1385·4 or 55,416 lbs. Now 55,416 divided by 407·3 is 136, which must be the number of square inches in the longitudinal section of the bearing in order that there may not be more than 407·3 lbs. on each square inch. To obtain this area, the bearing must be 16 inches long, since 8½ inches multiplied by 16 inches is 136 square inches. At 60 revolutions, the speed of the bearing surface per minute is 60 times 2·225 feet or 133·5 feet. Now 50 + 133·5 = 183·5, and 70,000 divided by 183·5 = 377·4, which is the proper load in lbs. for each square inch in the longitudinal section of the bearing. At 70 revolutions the speed of the bearings is 70 times 2·225 feet, or 155·75 feet per minute. Now 50 + 155·75 = 205·75, and 70,000 divided by 205·75 = 340·2, which is the proper load in pounds to put upon each square inch of the longitudinal section of the bearing at this speed of rotation.

TO FIND THE PROPER VELOCITY FOR THE SURFACE OF A BEARING WHEN THE PRESSURE PER SQUARE INCH ON ITS LONGITUDINAL SECTION IS GIVEN.

RULE.—*Divide the constant number* 70,000 *by the pressure per square inch on the longitudinal section of the bearing. From the quotient subtract the constant number* 50. *The remainder is the proper velocity of the surface of the bearing in feet per second.*

Example 1.—What is the proper velocity of the surface of a bearing which has the pressure of 800 lbs. on each square inch of its longitudinal section? Here 70,000 divided by 800=87·5; from which if we take 50 there will remain 37·5, which is the proper velocity of the bearing in feet per second.

If we take a hypothetical pressure of 1,400 lbs. per square inch of section, we get 70,000 divided by 1,400 = 50, and 50−50=0; so that with such a pressure there should be no velocity. Even in cases, however, in which there is very little motion, such as in the top eyes of the side rods of marine engines, it is not advisable to have so great a pressure upon the bearing as 1,400 or even 1,200 lbs. per square inch of section.

Example 2.—What is the proper velocity of the bearing of an engine which has a pressure upon it of 407·3 lbs. per square inch of section? Here 70,000 divided by 407·3=171·8, which diminished by 50 is 121·8, which is the proper speed of the surface of the bearing with this pressure per square inch of section. If the diameter of the bearing be 8½ inches, its circumference will be 2·225 feet, and 121·8 divided by 2·225=54·8 revolutions, which will be the speed of the engine with these data. These proportions allow a good margin, which may often be availed of in practice, either in driving the engine faster than is here indicated, or in putting more pressure upon the bearing. But to obviate inconvenient heating and wear, it will be found preferable to adhere, as nearly as practicable, to the proportion of surface which these rules prescribe.

STRENGTH OF MATERIALS.

The various kinds of strain to which materials are exposed in machines and structures may be all resolved into strains of extension and strains of compression; and in investigating the strength of materials there are three fixed points, varying in every material, to which it is necessary to pay special regard—the ultimate or breaking strength, the elastic or proof strength, and the safe or working strength. The tensile or breaking strength of wrought-iron, is about 60,000 per square inch of section, whereas the crushing strength of wrought-iron is about 27,000 per square inch of section. In steam-engines where the parts are alternately compressed and extended, it is not proper to load the wrought-iron with more than 4,000 lbs. per square inch of section; or the cast-iron with more than 2,000 lbs. per square inch of section. But in boilers where the strain is constantly in one direction, the load of 4,000 lbs. per square inch of section may be somewhat exceeded. The elastic strength is the strength exhibited by any material without being permanently altered in form, or crippled; for as a piece of iron is finally broken by being bent backward and forward, so by applying undue strains to any material, it will be finally broken with a much less strain than would suffice to break it at once. The elastic tensile strength of wrought-iron is between one-third and one-fourth of its ultimate tensile strength, and to this point the material might be proved without injury. But in proving boilers, and many other objects, it is not usual to make the proving pressure more than twice or three times the working pressure, such proof it is considered involving no risk of straining the material while it is adequate to the detection of accidental flaws if such exist. The following table exhibits the tenacity or tensible strength, and the resistance to compression or crushing strength of various materials:—

TENSILE AND CRUSHING STRENGTHS OF VARIOUS MATERIALS PER SQUARE INCH OF SECTION.

MATERIAL.	Tensile strength in lbs. per square inch of section.	Crushing strength in lbs. per square inch of section.
METALS.		
Wrought-iron bars	60,000	27,000 to 37,000
Wrought-iron plates	52,000	varies as cube of thickness nearly.
Wrought-iron hoops (best best)	64,000	
Wrought-iron wire*	70,000 to 100,000	
Cast-iron (average)	16,500	100,000
Cast-iron (toughened)	25,764	130,000
Steel	100,000 to 130,000	260,000
Cast brass	18,000	10,000
Gun metal	36,000	
Brass wire	50,000	
Cast copper	19,000	
Copper sheets	30,000	
Copper bolts	38,000	
Copper wire	60,000	
Silver (cast)	40,997	
Gold	20,490	
Tin (cast)	4,736	
Bismuth (cast)	3,187	
Zinc	7,000	
Antimony	1,062	
Lead (sheet)	3,000	
WOODS.		
Ash	17,000	9,000†
Beech	12,000	9,300
Birch	15,000	6,400
Box	20,000	10,300
Elm	13,000	10,300
Fir (red pine)	10,000 to 14,000	5,375 to 6,200
Hornbeam	20,000	7,300
Lance-wood	23,000	
Lignum Vitæ	12,000	9,900
Locust	16,000	
Mahogany	8,000 to 16,000	8,200
Oak	10,000 to 19,000	10,000
Pear	9,800	"
Teak	15,000	12,000
STONES.		
Granite	.....	5,500 to 11,000
Limestone	.....	4,000 to 5,000
Slate	10,000 to 12,000	
Sandstone	.....	4,000 to 5,000
Brick (weak)	.....	550 to 800
Brick (strong)	.....	1,100
Brick (fire)	.....	1,700
Glass	9,500	
Mortar	50	

* Mr. Pole found the German steel wire used for pianofortes to bear as much as 268,800 lbs. per square inch.

† These values are for dry wood. In wet wood the crushing strength is only half as great.

It will be remarked that there are very large variations in the amount of the strength recorded in this table; and there are so many varieties in the quality of the materials experimented upon that it is hopeless to expect any absolute agreement in the results of different experiments. It will be useful, under these circumstances, to set down the main results arrived at by a few of the principal experimentalists, leaving the reader to select such value as he may consider most nearly agrees with the circumstances with which he has to deal, The following are the strengths of various metals ascertained by Mr. George Rennie, in 1817:—

TENSILE STRENGTHS OF METALS BY RENNIE.

KIND OF METAL.	Tearing weight in lbs. of a bar one inch square.	Length of bar one inch square in feet that would break by its own weight.
Cast steel........................	134,256	39,455
Swedish malleable iron............	72,064	19,740
English malleable iron.............	55,872	16,938
Cast-iron	19,096	6,110
Cast copper......................	19,072	5,003
Yellow brass.....................	17,958	5,180
Case tin..........................	4,736	1,496
Cast lead........................	1,824	348

Professor Leslie, in his *Natural Philosophy*, thus explains the law of the extension of iron by weights:—

'A bar of soft iron will stretch uniformly by continuing to append to it equal weights till it can be loaded with half as much as it can bear; beyond that limit, however, its extension will become doubled by each addition of the eighth part of the disruptive force. Suppose the bar to be an inch square and 1,000 inches in length; 36,000 lbs. will draw it out 1 inch, but 45,000 will stretch it 2 inches; 54,000 lbs. 4 inches; 63,000 8 inches; and 72,000 16 inches, where it would finally break.' This popular explanation of the law agrees pretty nearly

with the subsequent deductions of Hodgkinson and other enquirers.

The cohesive strength of woods varies still more than that of metals in different specimens, and varies even in different parts of the same tree. Thus in Barlow's experiments he found the cohesive strength of fir to vary from 11,000 to 13,448 lbs. per square inch of section; of ash from 15,784 to 17,850; oak from 8,889 to 12,008; pear from 8,834 to 11,537, and other woods in the same proportions. The following fair average values may be adopted:—

TENSILE STRENGTHS OF WOODS BY BARLOW.

KIND OF WOOD.	Tearing weight in lbs. of a rod one inch square.	Length in feet of a rod one inch square that would break by its own weight.
Teak	12,915	36,049
Oak	11,880	32,900
Sycamore	9,630	35,800
Beech	12,225	38,940
Ash	14,130	42,080
Elm	9,720	39,050
Memel Fir	9,540	40,500
Christiana Deal	12,346	55,500
Larch	12,240	42,160

The crushing strength of wood, as of most other materials, is very different from its tensile strength, and is greatly affected by its dryness. The following table exhibits the results of the experiments made by Mr. Hodgkinson, to ascertain the crushing strengths of different woods per square inch of section. The specimens crushed were short cylinders, 1 inch diameter and 2 inches long, flat at the ends. The results given in the first column are those obtained when the wood was moderately dry. Those in the second column were obtained from similar speeimens which had been kept two months longer in a warm place:—

CRUSHING STRENGTHS OF WOODS BY HODGKINSON.

KIND OF WOOD.	Crushing strength per square inch of section.		
Alder	6831	to	6960
Ash	8683	"	9363
Bay	7518	"	7518
Beech	7733	"	7363
English Birch	3297	"	6402
Cedar	5674	"	5863
Red Deal	5748	"	6586
White Deal	6781	"	7293
Elder	7451	"	9973
Elm	..	"	10331
Fir (Spruce)	6499	"	6819
Mahogany	8198	"	8198
Oak (Quebec)	4231	"	5982
Oak (English)	6484	"	10058
Pine (Pitch)	6790	"	6790
Pine (Red)	5395	"	7518
Poplar	3107	"	5124
Plum (Dry)	8241	"	10493
Teak	..	"	12101
Walnut	6063	"	7227
Willow	2898	"	6128

The crushing strength of cast-iron is 98,922 lbs., or, say 100,000 per square inch of section.

The strength of wooden columns of different lengths and diameters to sustain weights has not been conclusively determined, and the longer a column is the weaker it is. But, however short it may be, the load upon it should not be above one-third of the crushing load, as given above.

LAW OF THE STRENGTH OF PILLARS.

The theory of the strength of pillars propounded by Euler is that the strength varies as the fourth power of the diameter divided by the square of the length; and the recent investigations of Hodgkinson and others show that this doctrine is nearly correct. Thus, in the case of hollow cylindrical columns of cast-iron, it is found experimentally that the 3·55th power of the internal diameter subtracted from the 3·55th power of the externa

diameter, and divided by the 1·7th power of the length, will give the strength very nearly. In the case of hollow cylindrical columns of malleable iron, it is found that the 3·59th power of the internal diameter, subtracted from the 3·59th power of the external diameter, and divided by the square of the length, will represent the strength; but this rule only holds when the load does not exceed 8 or 9 tons per square inch of section. The power of plates to resist compression varies as the cube or more nearly as the 2·878th power of their thickness. But this law only holds so long as the pressure applied does not exceed 9 to 12 tons per square inch of section. If the load is made greater than this, the metal is crushed and gives way. It has been found experimentally that in malleable iron tubes of the respective thicknesses of ·525, ·272 and ·124 inches, the resistances to compression per square inch of section are 19·17, 14·47, and 7·47 tons respectively. Moreover, in wrought-iron tubes 1½ inches diameter and ⅛th of an inch thick, the crushing strength is only 6·55 tons per square inch of section, while in tubes of nearly the same length and thickness, but about 6 inches diameter, the crushing strength is 16 tons per square inch of section. The strength of a pillar fixed at both ends is twice as great as if it were rounded at both ends. The crushing strength of a single square cell or tube of wrought-iron of large size, with angle-irons at the corners, of the construction adopted in tubular bridges, is when the thickness of the plate is not less than one-thirtieth of the diameter of the cell, about 27,000 lbs. per square inch of section; and where a number of such cells are grouped together so as to prevent deflection, the crushing strength rises to nearly 36,000 lbs. per square inch of section, which is also the crushing strength of short wrought-iron struts. The length of independent pillars should not be more than 25 times the diameter.

The weight in lbs. which a square post of oak of any length will with safety sustain may be determined as follows:—

TO DETERMINE THE PROPER LOAD FOR OAK POSTS.

RULE.—*To 4 times the square of the breadth in inches add half the square of the length in feet, and reserve the sum for*

a divisor. Multiply the cube of the breadth in inches by 3,960 times the length in feet, and divide the product by the divisor found as above. The quotient is the weight in lbs. which the oak post or pillar will with safety sustain.

Example.—What weight will a column of oak 6 inches square and 12 feet long sustain with safety?

Here the breadth of the post is 6 inches, the square of which is 36; and 4 times 36 is 144. The length being 12 feet, the square of the length is 144, half of which is 72; and 72 added to 144 gives 216 for the divisor. The breadth being 6 inches, the cube of the breadth is 216, and the length being 12 feet, we get 12 times 3,960 which is 47,520. Then 216 times 47,520 is 10,-264,320, which divided by 216 gives 47,520, which is the weight in lbs. that the post will with safety sustain.

The following table is computed from the rule given above:—

SCANTLINGS OF SQUARE POSTS OF OAK.

With the weights they will support and the extent of surface of flooring they will safely sustain, allowing 1 cwt., 1½ cwt., or 2 cwts. to the superficial foot of flooring, and calculated for a height of 10 feet.

NOTE.—*These Scantlings may be safely used up to 12 feet in height; but above that a little extra thickness should be allowed.*

Scantlings.	Weight.		Extent of surface of flooring supported.		
			1 cwt. per foot.	1½ cwt. per foot.	2 cwt. per foot.
Inches.	Tons.	Cwts.	Square feet.	Square feet.	Square feet.
3×3	5	10	110	82½	55
4×4	9	18	198	148½	99
5×5	14	14	294	220½	147
6×6	19	12	392	294	196
7×7	24	12	492	369	246
8×8	29	10	590	442½	295
9×9	34	8	688	516	344
10×10	39	4	784	588	392
11×11	44	0	880	660	440
12×12	48	16	976	732	488
13×13	53	10	1070	802½	535
14×14	58	4	1164	873	582
15×15	62	16	1256	942	628

Similar calculations of the dimensions and loads proper for rectangular columns of other woods may be determined by a reference to their relative crushing strengths given in page 128.

The formula given by Mr. Hodgkinson for determining the breaking weight of square oak posts where the length exceeds 30 times the thickness is

$$W=2452\frac{d^4}{l^2};$$

where W is the breaking weight in lbs.; d the side of the square base in inches; and l the length of the post in feet.

TO DETERMINE THE PROPER LOAD TO BE PLACED UPON SOLID PILLARS OF CAST-IRON.

The load which may be safely placed upon round posts, or solid pillars of cast-iron, may be ascertained by the following rule:—

RULE.—*To* 4 *times the square of the diameter of the solid pillar in inches, add* 0·18 *times the square of the length of the pillar in feet, and reserve the sum for a divisor. Multiply the fourth power of diameter of the pillar in inches by the constant number* 9562 *and divide the product by the divisor found as above. The quotient is the weight in lbs. which the solid cylinder or post of cast-iron will with safety sustain.*

Mr. Hodgkinson's formula for the breaking strength in tons of solid pillars of cast-iron in the case of pillars with rounded ends is—

$$\text{Strength in tons}=14{\cdot}9\frac{d^{3\cdot6}}{l^{1\cdot7}};$$

and in pillars with flat ends—

$$\text{Strength in tons}=44{\cdot}16\frac{d^{3\cdot6}}{l^{1\cdot7}}$$

where d is the diameter in inches, and l the length in feet.

The loads in cwts. which may be put upon solid cylinders or columns of cast-iron of different diameters and lengths are exhibited in the following table:—

WEIGHT IN CWTS. SUSTAINABLE WITH SAFETY BY SOLID CYLINDERS OR COLUMNS OF CAST-IRON OF DIFFERENT DIAMETERS AND LENGTHS.

Diameter of column in inches.	LENGTH OF COLUMN IN FEET.					
	6	8	10	12	14	16
	cwts.	cwts.	cwts.	cwts.	cwts.	cwts.
2	61	50	40	32	26	22
$2\frac{1}{2}$	105	91	77	65	55	47
3	163	145	128	111	97	84
$3\frac{1}{2}$	232	214	191	172	156	135
4	310	288	266	242	220	198
$4\frac{1}{2}$	400	379	354	327	301	275
5	501	479	452	427	394	365
6	592	573	550	525	497	469
7	1013	989	959	924	887	848
8	1315	1289	1259	1224	1185	1142

In hollow pillars nearly the same laws obtain as in solid. Thus in the case of hollow pillars, with rounded ends or movable ends, like the cast-iron connecting-rod of a steam-engine, the formula is—

$$\text{Strength in tons}=13\frac{D^{3\cdot6}-d^{3\cdot6}}{l^{1\cdot7}}$$

and in the case of hollow pillars, with flat ends—

$$\text{Strength in tons}=44\cdot3\frac{D^{3\cdot6}-d^{3\cdot6}}{l^{1\cdot7}};$$

where D is the external and d the internal diameter. The strength of a pillar with a cross section of the form of a cross was found to be only about half as great as that of a cylindrical hollow pillar. It was also found that in pillars of the same dimensions, but of different materials, taking the strength of cast-iron at 1,000, that of wrought-iron was 1,745, cast steel 2,518, Dantzic oak 108·8, and red deal 78·5.

Mr. Hodgkinson's rule for the breaking weight of cast-iron beams is as follows:—

STRENGTH OF CAST-IRON BEAMS.

RULE.—*Multiply the sectional area of the bottom flange in square inches by the depth of the beam in inches, and divide the product by the length between the supports also in inches. Then* 514 *times the quotient will be the breaking weight in cwts.*

STRENGTH OF SHAFTS.

44 lbs. acting at a foot radius will twist off the neck of a shaft of lead 1 inch diameter, and the relative strengths of other materials, lead being 1, is as follows:—Tin, 1·4; copper 4·3; yellow brass, 4·6; gun metal, 5; cast-iron, 9; Swedish iron, 9·5; English iron, 10·1; blistered steel, 16·6; shear steel, 17; and cast steel, 19·5. The strength of a shaft increases as the cube of its diameter.

CHAPTER III.

THEORY OF THE STEAM-ENGINE.

THE Steam-Engine is a machine for extracting mechanical power from heat through the agency of water.

Heat is one form of mechanical power, or more properly, a given quantity of heat is the equivalent of a determinate amount of mechanical power; and as heat is capable of producing power, so contrariwise power is capable of producing heat. The nature of the medium upon which the heat acts in the production of the power—whether it be water, air, metal, or any other substance—is immaterial, except in so far as one substance may be more convenient and manageable in practice than another. But with any given extremes of temperature, and any given expenditure of heat, the amount of power generated by any given quantity of heat will be the same, whatever be the nature of the substance on which the heat is made to act in the generation of the power. And just in the proportion in which power is generated so will the heat disappear. We cannot have both the heat and the power; but as the one is transformed into the other, so it will follow that the acquisition of the one entails a proportionate loss of the other, and this loss cannot possibly be prevented. It has been already explained that, as in all cases in which power is produced in a steam-engine, there must be a difference of pressure on the two sides of the piston, or between the boiler and the condenser; so in all cases in which power is pro-

duced in any species of caloric engine, there must be a difference of temperature between the source of heat and the atmosphere or refrigerator. The amount of this difference will determine the amount of power, up to a certain limit, which a unit of heat will generate in any given engine. But as it has been already explained that the mechanical equivalent of the heat consumed in heating 1 lb. of water 1° Fahrenheit would, if utilised without loss, raise a weight of 772 lbs. 1 foot high, it will follow that in no engine whatever can a greater performance be obtained than this, whatever difference of temperature we may assume between the extremes of heat and cold. A weight of 772 lbs. raised 1 foot for 1° Fahrenheit is equivalent to a weight of 1389·6 lbs. raised 1 foot for 1° Centigrade; and for convenience the term foot-pound is now very generally employed to denote the dynamical unit, or measure of power, expressed by a weight of 1 lb. raised through 1 foot. A horse-power, or as it is now commonly termed an *actual* or *indicator* horse-power—to distinguish it from a nominal horse-power, which is a mere measure of capacity—is a dynamical unit expressed by 33,000 lbs. raised 1 foot high in a minute; or it is 550 foot-pounds per second; 33,000 foot-pounds per minute; or 1,980,000 foot-pounds per hour. This unit takes into account the *rate of work* of the machine.

Heat, like light, is believed to be a species of motion, and there are three forms of heat of which a work of this nature requires to take cognisance—Sensible Heat, Latent Heat, and Specific Heat.

Sensible Heat is heat that is sensible to the touch, or measurable by the thermometer. *Latent Heat* is the heat which a body absorbs in changing its state from solid to liquid, and from liquid to aëriform, without any rise of temperature, or it is the heat absorbed in expansion. And *Specific Heat* is an expression for the relative quantity of heat in a body as compared with that in some other standard body of the same temperature. There is a constant tendency in hot bodies to cool, or to transfer part of their heat to surrounding colder bodies; and contiguous bodies are said to be of equal temperatures when there ceases to be any

transfer of heat from one to the other. The most prominent phenomena of heat are *Dilatation*, *Liquefaction*, *and Vaporisation.*

Difference between temperature and quantity of heat.—It is quite clear that two pounds of boiling water have just twice the quantity of heat in them that is contained in one pound of boiling water. But it does not by any means follow, nor is it the case, that two pounds of boiling water at 212° contain twice the quantity of heat that is contained in two pounds of water at 106°. Experiment indeed shows, that when equal quantities of water at different temperatures are mixed together, the resulting temperature is the mean of the two, so that if a pound of water at 200° be mixed with a pound of water at 100°, we have a resulting two pounds of water of 150°. But before we could suppose that a pound of water at 200° has twice the quantity of heat in it that is contained in a pound of water at 100°, it would be necessary to conclude that water at 0° or zero, has no heat in at whatever. This, however, is by no means the case; and temperatures much below zero have been experimentally arrived at, and even naturally occur in northern latitudes. A pound of ice at a temperature below zero, rises in temperature by each successive addition of heat, until it attains the temperature of 32°, when it begins to melt; and, notwithstanding successive additions being made to its heat, its temperature refuses to rise above 32° until liquefaction has been completed. So soon as all the ice has been melted, the temperature of the resulting water will continue to rise with each successive increment of heat, until the temperature of 212° has been attained, when the water will boil, and all subsequent additions to the heat will be expended in evaporating the water or in converting it into steam. Although, therefore, a pound of water in the form of steam has only the same temperature as a pound of boiling water, it has a great deal more heat in it, as is shown by the fact that it will heat to a given temperature a great many more pounds of cold water than a pound of boiling water would do.

Absolute zero.—The foregoing considerations lead naturally to the inquiry whether, although bodies at the zero of Fahren-

heit's scale are still possessed of some heat, there may not, nevertheless, be a point at which there would be no heat whatever, and which point therefore constitutes the true and absolute zero. Such a point has never been practically arrived at. But the law of the elasticity of gases and their expansion by heat, leads to the conclusion that there is such a point, and that it is situated 461·2° Fahrenheit below the zero of Fahrenheit's scale, or in other words that it is —461·2° Fahrenheit, —274° Centigrade, or —219·2° Reaumur. Mr. Rankine has shown, that by reckoning temperatures from this theoretical zero, at which there is supposed to be no heat and no elasticity, the phenomena dependent upon temperature are more readily grouped and more simply expressed than would otherwise be possible.

Fixed Temperatures.—The circumstance of the temperatures of liquefaction and ebullition being fixed and constant, enables us to obtain certain standard or uniform temperatures, to which all others may easily be referred. One of these standard temperatures is the melting-point of ice, and another is the boiling-point of pure water under the average amospheric pressure of 14·7 lbs. on the square inch, 2116·8 lbs. on the square foot; or under the pressure of a vertical column of mercury 29·922 inches high, the mercury being at the density proper to the temperature of melting ice.

Thermometers.—Thermometers measure temperatures by the dilatation which a certain selected body undergoes from the application of heat. Sometimes the selected body is a solid, such as a rod of brass or platinum; at other times it is a liquid, such as mercury or spirits of wine; and at other times, again, it is a gas, such as air or hydrogen. In a perfect gas the elasticity is proportionate to the compression, whereas in an imperfect gas, such as carbonic acid, which may be condensed into a liquid, the rate of elasticity diminishes as the point of condensation is approached. Every gas approaches more nearly to the condition of a perfect gas the more it is heated and rarefied, but an absolutely perfect gas does not exist in nature. Common air, however, approaches sufficiently to the condition of a perfect gas, to be a just measure of temperatures by its expansion.

Air and all other gases expand equally with equal increments of temperature; and it is found experimentally that a cubic foot of air at the temperature of melting ice, or 32°, will form 1·365 cubic feet of the same pressure at the temperature of boiling water, or 212°. Thermometers, however, are not generally constructed with air as the expanding fluid, except for the measurement of very high temperatures. The most usual species of thermometer consists of a small glass bulb filled with mercury, and in connection with a capillary tube. The bulb is immersed in the substance the temperature of which it is desired to ascertain; and the amount of the dilatation is measured by the height to which the mercury is forced up the capillary tube. The thermometer commonly used in this country is Fahrenheit's thermometer, of which the zero or 0 of the scale is fixed at the temperature produced by mixing salt with snow; and which temperature is 32° below the freezing-point of water. The Centigrade thermometer is that commonly used on the continent of Europe; and it is graduated by dividing the distance between the point where the mercury stands at the freezing-point of water, and the point where it stands at the boiling-point of water, into 100 equal parts. Of this thermometer the zero is at the freezing point of water. Another thermometer, called Reaumur's thermometer, has its zero also at the freezing-point of water; and the distance between that and the boiling-point of water is divided into eighty equal parts. Hence 80° Reaumur are equal to 100° Centigrade, and 180° Fahrenheit. The corresponding degrees of these thermometers are shown in the following table:—

CENTIGRADE, REAUMUR'S, AND FAHRENHEIT'S THERMOMETERS.

Cent.	Reau.	Fahr.	Cent.	Reau.	Fahr.	Cent.	Reau.	Fahr.	Cent.	Reau.	Fahr.
100	80·	212·	64	51·2	147·2	29	23·2	84·2	—6	—4·8	21·2
99	79·2	210·2	63	50·4	145·4	28	22·4	82·4	7	5·6	19·4
98	78·4	208·4	62	49·6	143·6	27	21·6	80·6	8	6·4	17·6
97	77·6	206·6	61	48·8	141·8	26	20·8	78·8	9	7·2	15·8
96	76·8	204·8	60	48·	140·	25	20·	77·	10	8·	14·
95	76·	203·	59	47·2	138·2	24	19·2	75·2	11	8·8	12·2
94	75·2	201·2	58	46·4	136·4	23	18·4	73·4	12	9·6	10·4
93	74·4	199·4	57	45·6	134·6	22	17·6	71·6	13	10·4	8·6
92	73·6	197·6	56	44·8	132·8	21	16·8	69·8	14	11·2	6·8
91	72·8	195·8	55	44·	131·	20	16·	68·	15	12·	5·
90	72·	194·	54	43·2	129·2	19	15·2	66·2	16	12·8	3·2
89	71·2	192·2	53	42·4	127·4	18	14·4	64·4	17	13·6	1·4
88	70·4	190·4	52	41·6	125·6	17	13·6	62·6	18	14·4	—0·4
87	69·6	188·6	51	40·8	123·8	16	12·8	60·8	19	15·2	2·2
86	68·8	186·8	50	40·	122·	15	12·	59·	20	16·	4·
85	68·	185·	49	39·2	120·2	14	11·2	57·2	21	16·8	5·8
84	67·2	183·2	48	38·4	118·4	13	10·4	55·4	22	17·6	7·6
83	66·4	181·4	47	37·6	116·6	12	9·6	53·6	23	18·4	9·4
82	65·6	179·6	46	36·8	114·8	11	8·8	51·8	24	19·2	11·2
81	64·8	177·8	45	36·	113·	10	8·	50·	25	20·	13·
80	64·	176·	44	35·2	111·2	9	7·2	48·2	26	20·8	14·8
79	63·2	174·2	43	34·4	109·4	8	6·4	46·4	27	21·6	16·6
78	62·4	172·4	42	33·6	107·6	7	5·6	44·6	28	22·4	18·4
77	61·6	170·6	41	32·8	105·8	6	4·8	42·8	29	23·2	20·2
76	60·8	168·8	40	32·	104·	5	4·	41·	30	24·	22·
75	60·	167·	39	31·4	102·2	4	3·2	39·2	31	24·8	23·8
74	59·2	165·2	38	30·2	100·4	3	2·4	37·4	32	25·6	25·6
73	58·4	163·4	37	29·6	98·6	2	1·6	35·6	33	26·4	27·4
72	57·6	161·6	36	28·8	96·8	1	0·8	33·8	34	27·2	29·2
71	56·8	159·8	35	28·	95·	0	0·	32·	35	28·	31·
70	56·	158·	34	27·2	93·2	—1	—0·8	30·2	36	28·8	32·8
69	55·2	156·2	33	26·4	91·4	2	1·6	28·4	37	29·6	34·6
68	54·4	154·4	32	25·6	89·6	3	2·4	26·6	38	30·4	36·4
67	53·6	152·6	31	24·8	87·8	4	3·2	24·8	39	31·2	38·2
66	52·8	150·8	30	24·	86·	5	4·	23·	40	32·	40·
65	25·	149·									

Water, in common with molten cast-iron, molten bismuth, and various other fluid substances, the particles of which assume a crystalline arrangement during congelation, suffers an increase of bulk as the point of congelation is approached, and expands in solidifying. But so soon as any of these substances has become solid, it then contracts with every diminution of temperature. Water in freezing bursts by its expansion any vessel in which it may be confined, and ice, being lighter than water, floats upon water. So also for a like reason solid cast-iron floats on molten cast-iron. *The point of maximum density of water is* 39·1° *Fahrenheit*, and between that point and 32° the bulk of water in-

creases by cold. A cubic foot of water at 32° weighs 62·425 lbs., whereas a cubic foot of ice at 32° weighs only 57·5 lbs. There is consequently a difference of nearly 5 lbs. in each cubic foot, between the weight of ice and the weight of water.

DILATATION.

Dilatation of Solids.—A solid body of homogeneous texture will dilate uniformly throughout its entire bulk by the application of heat. Thus, if it be found that a bar of zinc is increased one 340th part of its length by being raised in temperature from 32° to 212°, its breadth will also be increased one 340th part, and its thickness will be increased one 340th part. It is found, moreover, that equal increments of heat produce equal augmentations of volume in nearly all bodies, at all temperatures, until the melting-point is approached, when irregularities occur. Different solids dilate to different amounts when subjected to the same increase of temperature, and advantage is taken of this property in the arts in the construction of time-keepers and other instruments. Thus, in Harrison's gridiron pendulum, the ball is composed of bars of different metals, some of which expand more than the others at the same temperature; and as the bars which expand the most are fixed at the lower ends and expand upwards, they compensate for the expansion of the pendulum rod in the opposite direction, and maintain the centre of oscillation in the same place. The following table exhibits the rates of dilatation of various solids, as ascertained by the best authorities:—

DILATATION OF SOLIDS BY HEAT.

Bodies.	Dilatation in Fractions.	
	Decimal.	Vulgar.
Dilatation from 32° *to* 212°, *according to Lavoisier and Laplace.*		
Flint Glass (English)	0·00081166	$\frac{1}{1232}$
Platinum (according to Borda)	0·00085655	$\frac{1}{1167}$
Glass (French) with lead	0·00087199	$\frac{1}{1147}$
Glass tube without lead	0·00087572	$\frac{1}{1142}$
Ditto	0·00089694	$\frac{1}{1115}$
Ditto	0·00089760	$\frac{1}{1114}$
Ditto	0·00091750	$\frac{1}{1090}$
Glass (St. Gobain)	0·00089089	$\frac{1}{1122}$
Steel (untempered)	0·00107880	$\frac{1}{927}$
Ditto	0·00107915	$\frac{1}{927}$
Ditto	0·00107960	$\frac{1}{926}$
Steel (yellow temper) annealed at 65°	0·00123956	$\frac{1}{807}$
Iron, soft forged	0·00122045	$\frac{1}{819}$
Iron, round wire-drawn	0·00123504	$\frac{1}{810}$
Gold	0·00146606	$\frac{1}{682}$
Gold (French standard) annealed	0·00151361	$\frac{1}{661}$
Gold (ditto) not annealed	0·00155155	$\frac{1}{645}$
Copper	0·00171220	$\frac{1}{584}$
Ditto	0·00171733	$\frac{1}{582}$
Ditto	0·00172240	$\frac{1}{581}$
Brass	0·00186670	$\frac{1}{535}$
Ditto	0·00187821	$\frac{1}{532}$
Ditto	0·00188970	$\frac{1}{529}$
Silver (French standard)	0·00190868	$\frac{1}{524}$
Silver	0·00190974	$\frac{1}{524}$
Tin, Indian	0·00193765	$\frac{1}{516}$
Tin, Falmouth	0·00217298	$\frac{1}{460}$
Lead	0·00284836	$\frac{1}{351}$
According to Smeaton.		
Glass, white (barometer tubes)	0·00083333	$\frac{1}{1200}$
	0·00108333	$\frac{1}{923}$
Steel	0·00115000	$\frac{1}{870}$
Steel (tempered)	0·00122500	$\frac{1}{816}$
Iron	0·00125833	$\frac{1}{795}$
Bismuth	0·00139167	$\frac{1}{719}$
Copper	0·00170000	$\frac{1}{588}$
Copper 8 parts, tin 1	0·00181667	$\frac{1}{550}$
Brass cast	0·00187500	$\frac{1}{533}$
Brass 16 parts, tin 1	0·00190833	$\frac{1}{524}$

DILATATION OF SOLIDS BY HEAT—*continued.*

Bodies.		Dilatation in Fractions. Decimal.	Vulgar.
Brass wire		0·00193333	$\frac{1}{517}$
Telescope speculum metal		0·00193333	$\frac{1}{517}$
Solder (copper 2 parts, zinc 1)		0·00205833	$\frac{1}{486}$
Tin (fine)		0·00228333	$\frac{1}{438}$
Tin (grain)		0·00248333	$\frac{1}{403}$
Solder white (tin 1 part, lead 2)		0·00250533	$\frac{1}{399}$
Zinc 8 parts, tin 1, slightly forged		0·00269167	$\frac{1}{372}$
Lead		0·00286667	$\frac{1}{349}$
Zinc		0·00294167	$\frac{1}{340}$
Zinc lenthened $\frac{1}{12}$ by hammering		0·00310833	$\frac{1}{322}$
Palladium (*Wollaston*)		0·00100000	$\frac{1}{1000}$
According to Dulong and Petit.			
Platinum	32° to 212°	0·00088420	$\frac{1}{1131}$
	32° to 572°	0·00275482	$\frac{1}{363}$
Glass	32° to 212°	0·00086133	$\frac{1}{1161}$
	32° to 392°	0·00184502	$\frac{1}{542}$
	32° to 572°	0·00303252	$\frac{1}{329}$
Iron	32° to 212°	0·00118210	$\frac{1}{846}$
	32° to 572°	0·00440528	$\frac{1}{227}$
Copper	32° to 212°	0·00171820	$\frac{1}{582}$
	32° to 572°	0·00564972	$\frac{1}{177}$
According to Troughton.			
Platinum		0·00099180	$\frac{1}{1008}$
Steel		0·00118990	$\frac{1}{840}$
Steel wire, drawn		0·00144010	$\frac{1}{694}$
Copper		0·00191880	$\frac{1}{521}$
Silver		0·00208260	$\frac{1}{480}$
From 32° to 217° according to Roy.			
Glass (tube)		0·00077550	$\frac{1}{1289}$
Glass (solid rod)		0·00080833	$\frac{1}{1237}$
Glass cast (prism of)		0·00111000	$\frac{1}{901}$
Steel (rod of)		0·00114400	$\frac{1}{874}$
Brass (Hamburg)		0·00185550	$\frac{1}{539}$
Brass (English) rod		0·00189296	$\frac{1}{528}$
Brass (English) angular		0·00189450	$\frac{1}{528}$

Measure of the Force of Dilatation.—The force with which solid bodies dilate and contract is equal to that which would compress them through the space they have dilated, or to that which would stretch them through a space equal to the amount of their contraction. Now, as it has been shown to be a physical law that in every substance whatever, the same expenditure of heat, with the same extremes of temperature, will generate the same amount of mechanical power, it will follow that the less a body expands with any given increase of temperature, the more forcible will be the expansion, since the force, multiplied by the space passed through, must, in every case be a constant quantity.

Dilatation of Liquids.—The rate of expansion of liquids becomes greater as the temperature becomes higher, so that a mercurial thermometer, to be accurately graduated, should have the graduations at the top of the scale somewhat larger than at the bottom. It so happens, however, that there is a similar irregularity in the expansion of the glass bulb, but in an opposite direction; and one error very nearly corrects the other. Thermometers are accordingly graduated by immersing the bulb in melting ice, and marking the point at which the mercury stands. The point at which the mercury stands when the bulb is immersed in boiling water is next marked, and the space between the two marks is divided into 180 equal parts, and the graduation is extended above the boiling-point and below the freezing, by continuing the same lengths of division on the scale. The increment of volume which water receives on being raised from 32° to 212° is $\frac{1}{23}$rd of its bulk at 32°. Mercury at 32° expands $\frac{1}{55}$th of its bulk at 32° by being raised to 212°; and alcohol, by the same increase of temperature, increases in volume $\frac{1}{8}$th of its bulk at 32°.

Compression and Dilatation of Gases.—When a gas or vapour is compressed into half its original bulk, its pressure is doubled; when compressed into a third of its original bulk, its pressure is trebled; when compressed into a fourth of its original bulk, its pressure is quadrupled; and generally the pressure varies inversely as the bulk into which the gas is compressed. So, in

like manner, if the volume be doubled, the pressure is made one-half of what it was before—the pressure being in every case reckoned from 0, or from a perfect vacuum. Thus, if we take the average pressure of the atmosphere at 14·7 lbs. on the square inch, a cubic foot of air, if suffered to expand into twice its bulk by being placed in a vacuum measuring two cubic feet, will have a pressure of 7·35 lbs. above a perfect vacuum, and also of 7·35 lbs. below the atmospheric pressure; whereas, if the cubic foot be compressed into a space of half a cubic foot, the pressure will become 29·4 lbs. above a perfect vacuum, and 14·7 lbs. above the atmospheric pressure. This law, which was first investigated by Mariotte, is called *Mariotte's law*. It has already been stated that a cubic foot of air at 32° becomes 1·365 cubic feet at 212°, the pressure remaining constant; or if the volume be kept constant, then the pressure of one atmosphere at 32° becomes 1·365 atmospheres, or a little over 1⅓ atmospheres at 212°. These two laws, which are of the utmost importance in all physical researches, it is necessary fully to understand and remember. The rates of dilatation and compression for each gas are not precisely the same; but the departure from the law is so small as to be practically inappreciable. According to M. Regnault, the dilatation under the same pressure, and the increase of pressure with the same volume of different gases when heated from 32° to 212°, is as follows:—

CO-EFFICIENTS OF DILATATION OF DIFFERENT GASES.

	Pressure under constant voume.	Dilatation under constant pressure.
Hydrogen	0·3667	0·3661
Atmospheric air	0·3665	0·3670
Nitrogen	0·3668	"
Carbonic oxide	0·3667	0·3669
Carbonic acid	0·3688	0·3710
Protoxide of nitrogen	0·3676	0.3719
Sulphurous acid	0·3845	0·3903
Cyanogen	0·3829	0.3877

The rates of dilatation vary somewhat with the pressure and temperature, and in the case of gases, which are more easily

condensable into liquids, the rate of dilatation increases rapidly with the density; whereas the effect of heat is to remove these irregularities, and to maintain more completely the condition of a perfect gas.

If we take the dilatation of atmospheric air when heated 180°, or from 32° to 212°, at 0·367 as determined by M. Regnault, then the amount of expansion which it will undergo from each increase of one degree in temperature will be 180th of 0·367 = 180th of $\frac{367}{1000} = \frac{367}{180000} = \frac{1}{490}$. In other words, air will be enlarged $\frac{1}{490}$th part of its bulk at 32° by being raised one degree in temperature.

If the same quantity of air or gas be simultaneously submitted to changes of temperature and pressure, the relations between its volumes, pressures, and temperatures, will be expressed by the general formula—

$$\frac{V}{V'} = \frac{490 \pm T}{490 \pm T'} \times \frac{P'}{P};$$

where T and T′ express the number of degrees above or below 32° at which the temperature stands, + being used when *above* and — when *below* 32°, and the pressures being expressed in the usual manner by P and P′. By this formula, the volume of a gas at any proposed temperature and pressure may be found, if its volume at any other temperature and pressure be given, or the same thing may be done by the following rule:—

THE BULK OF A GAS AT 32° BEING KNOWN, TO DETERMINE ITS BULK AT ANY OTHER TEMPERATURE, THE PRESSURE BEING CONSTANT.

RULE.—*Divide the difference between the number of degrees in the temperature and* 32° *by* 490. *Add the quotient to* 1 *if the temperature be above* 32°, *and subtract it from* 1 *if it be below* 32°. *Multiply the volume of the gas at* 32° *by the resulting number, and the product will be the volume of the gas at the proposed temperature.*

Example 1.—What volume will 1000 cubic inches of air at 32° acquire by being heated to 1000° Fahrenheit?

EXPANSION OF DRY AIR BY HEAT.

[In the columns V. of the following table are expressed in cubic inches the volumes which a thousand cubic inches of air at 32º will have at the temperatures expressed in the columns T., the air being supposed to be maintained under the same pressure.]

T.	V.	T.	V.	T.	V.	T.	V.	T.	V.
—50	832·7	8	951·0	66	1069.4	124	1187·8	182	1306·1
—49	834·7	9	953·1	67	1071·4	125	1189·8	183	1308·2
—48	836·7	10	955·1	68	1073·5	126	1191·8	184	1310·2
—47	838·8	11	957·1	69	1075·5	127	1193·9	185	1312·2
—46	840·8	12	959·2	70	1077·6	128	1195·9	186	1314·3
—45	842·8	13	961·2	71	1079·6	129	1198·0	187	1316·3
—44	844·9	14	963·3	72	1081·6	130	1200·0	188	1318·4
—43	846·9	15	965·3	73	1083·7	131	1202·0	189	1320·4
—42	849·0	16	967·3	74	1085·7	132	1204·1	190	1322·4
—41	851·0	17	969·4	75	1087·8	133	1206·1	191	1324·5
—40	853·1	18	971·4	76	1089·8	134	1208·2	192	1326·5
—39	855·1	19	973·5	77	1091·8	135	1210·2	193	1328·6
—38	857·1	20	975·5	78	1093·9	136	1212·2	194	1330·5
—37	859·2	21	977·6	79	1095·9	137	1214·3	195	1332·6
—36	861·2	22	979·6	80	1098·0	138	1216·3	196	1334·7
—35	863·3	23	981·6	81	1100·0	139	1218·4	197	1336·7
—34	865·3	24	983·7	82	1102·0	140	1220·4	198	1338·8
—33	867·3	25	985·7	83	1104·1	141	1222·4	199	1340·8
—32	869·4	26	987·8	84	1106·1	142	1224·5	200	1342·9
—31	871·4	27	989·8	85	1108·2	143	1226·5	201	1344·9
—30	873·5	28	991·8	86	1110·2	144	1228·6	202	1346·0
—29	875·5	29	993·9	87	1112·2	145	1230·6	203	1349·0
—28	877·6	30	995·9	88	1114·3	146	1232·7	204	1351·1
—27	879·6	31	998·0	89	1116·3	147	1234·7	205	1353·1
—26	881·6	32	1000·0	90	1118·4	148	1236·7	206	1355·1
—25	883·7	33	1002·0	91	1120·4	149	1238·8	207	1357·3
—24	885·7	34	1004·1	92	1122·4	150	1240·8	208	1359·3
—23	887·8	35	1006·1	93	1124·5	151	1242·9	209	1361·3
—22	889·8	36	1008·2	94	1126·5	152	1244·9	210	1363·4
—21	891·8	37	1010·2	95	1128·6	153	1246·9	211	1365·5
—20	893·9	38	1012·2	96	1130·6	154	1249·0	212	1367·6
—19	895·9	39	1014·3	97	1132·7	155	1251·0	213	1369·2
—18	898·0	40	1016·3	98	1134·7	156	1253·0	214	1371·4
—17	900·0	41	1018·4	99	1136·7	157	1255·1	215	1373·2
—16	902·0	42	1020·4	100	1138·8	158	1257·1	216	1375·5
—15	904·1	43	1022·4	101	1140·8	159	1259·2	217	1377·5
—14	906·1	44	1024·5	102	1142·0	160	1261·2	218	1379·6
—13	908·2	45	1026·5	103	1144·9	161	1263·3	219	1381·6
—12	910·2	46	1028·6	104	1147·0	162	1265·3	220	1383·7
—11	912·2	47	1030·6	105	1149·0	162	1267·3	230	1404·1
—10	914·3	48	1032·7	106	1151·0	164	1269·4	240	1424·5
— 9	916·3	49	1034·7	107	1153·1	165	1271·4	250	1444·9
— 8	918·4	50	1036·7	108	1155·1	166	1273·5	260	1465·3
— 7	920·4	51	1038·8	109	1157·1	167	1275·5	270	1485·7
— 6	922·5	52	1040·8	110	1159·2	168	1277·5	280	1506·1
— 5	924·5	53	1042·9	111	1161·2	169	1279·6	290	1526·5
— 4	926·5	54	1044·9	112	1163·3	170	1281·6	300	1546·9
— 3	928·6	55	1046·9	113	1165·3	171	1283·7	400	1751·0
— 2	930·6	56	1049·0	114	1167·3	172	1285·7	500	1955·1
— 1	932·7	57	1051·0	115	1169·4	173	1287·8	600	2159·2
0	934·7	58	1053·1	116	1171·4	174	1289·8	700	2363·3
1	936·7	59	1055·1	117	1173·5	175	1291·8	800	2567·3
2	938·8	60	1057·1	118	1175·5	176	1293·9	900	2771·4
3	940·8	61	1059·2	119	1177·6	177	1295·9	1000	2975·5
4	942·9	62	1061·2	120	1179·6	178	1298·0	1500	3997·9
5	944·9	63	1063·3	121	1181·8	179	1300·0	2000	5016·3
6	947·0	64	1065·3	122	1183·7	180	1302·0	2500	6036·7
7	949·0	65	1067·3	123	1185·7	181	1304·1	3000	7057·1

The difference between 1000° and 32° is 968, which divided by 490 = 1·9755, and this added to 1 = 2·9755. Then 1000 × 2·9755 = 2975·5, which will be the volume in cubic inches at 1000°.

Example 2.—What will be the volume of the above air at 2000°?

Here 2000 — 32 = 1968 which ÷ by 490 = 4·0163, and this added to 1 = 5·0163. Finally, 5·0163 × 1000 = 5016·3, which will be the volume of the air in cubic inches at 2000°.

The volume which 1000 cubic inches of air at 32° acquires at all the various temperatures between —50° and 3000° is shown in the preceding table:

ANOTHER RULE.—*To each of the temperatures before and after expansion add the constant number* 459: *divide the greater sum by the lesser, and multiply the quotient by the volume at the lower temperature, and the product will give the expanded volume.*

Example 1.—What will be the volume of 1000 cubic inches of air at 32° when heated to 212°, the pressure being without alteration?

Here $\frac{212 + 459}{32 + 459} = 1{\cdot}366$, which multiplied by 1000=1366, which will be the volume in cubic inches at 212°.

Example 2.—If the volume of steam at 212° be 1696 times the volume of the water which produced it, what will the volume be if the steam be heated to 250·3 degrees Fahrenheit, the pressure remaining constant?

Here by the rule 212+459=671 and 250·3+459=709·3°. Moreover, 709·3 divided by 671 and multiplied by 1696=1792·8, which will be the bulk which the 1696 measures of steam will acquire when heated to 250·3° out of contact with water, the pressure remaining the same as at first.

If we take the co-efficient of expansion of a perfect gas between 32° and 212° at 0·365 instead of 0·367, the expansion per degree Fahrenheit will be $\frac{1}{493\cdot2}$ of the total bulk=0·0020276 per degree Fahrenheit, instead of $\frac{1}{490}$th, as supposed by the rule from which the table is computed. This is equivalent to starting from the point of absolute zero, or 461·2° below the zero of Fahrenheit; as 461·2°+32°=493·2°

TABLE SHOWING THE MELTING POINTS OF VARIOUS BODIES, IN DEGREES OF FAHRENHEIT'S THERMOMETER.

Name of Substance.	Degrees Fahren.	Experimentalist.
Platinum	3082°	Clarke.
English wrought-iron	2912	Vauquelin.
French " "	2732	Pouillet.
Steel	2552	"
" another sample	2372	"
Cast-iron	2192	"
" manganese	2282	"
" brown, fusible	2192	"
" " very fusible	2012	"
" white, fusible	2012	"
" " very fusible	1922	"
Gold (very pure)	2282	"
Gold coin	2156	"
Copper	1922	"
Brass	1859	Daniell.
Silver (very pure)	1832	Pouillet.
Bronze	1652	"
Antimony	810	"
Zinc	700	Murray.
	705	G. Morveau.
	680	Pouillet.
Lead	629	Person.
	608	Pouillet.
	590	Irvine.
Bismuth	518	Person.
	509	Ermann.
	505	Pouillet.
	480	Crichton.
Tin	512	G. Morveau.
	455	Person.
	446	Pouillet.
	442	Crichton.
	433	Ermann.
Alloy, 5 parts tin, 1 part lead	381	Pouillet.
Alloy, 4 parts tin, 1 part lead	372	"
Alloy, 3 parts tin, 1 part lead	367	"
Alloy, 2 parts tin, 1 part lead	460	"
Alloy, 1 part tin, 3 parts lead	552	"

TABLE SHOWING THE MELTING POINTS OF VARIOUS BODIES, IN DEGREES OF FAHRENHEIT'S THERMOMETER—*continued.*

Name of Substance.	Degrees Fahren.	Experimentalist.
Alloy, 3 parts tin, 1 part bismuth	392	Pouillet.
Alloy, 2 parts tin, 1 part bismuth	333·9	"
Alloy, 1 part tin, 1 part bismuth	286·2	"
Alloy, 4 parts tin, 1 part lead, 5 parts bismuth	246	"
Sulphur	239	Person.
	237	Dumas.
Iodine	225	Pouillet.
Alloy, 2 parts lead, 3 parts tin, 5 parts bismuth	212	"
Alloy, 5 parts lead, 3 parts tin, 8 parts bismuth	212	"
Alloy, 1 part lead, 1 part tin, 4 parts bismuth	201	"
Soda	194	Gay-Lussac.
Potash	162	"
	136	Pouillet.
Phosphorus	111·6	Person.
	109	Pouillet.
	100	Murray.
Stearic acid	158	Pouillet.
Wax bleached	154	"
Wax unbleached	142	"
	143	Person.
Stearine	120	Pouillet.
	109	"
Spermaceti	120	"
Acetic acid	113	"
Tallow	92	"
Ice	32	"
Oil of turpentine	14	"
Mercury	—38.2	"

LIQUEFACTION.

Solidity is an accident of temperature, as there is every reason to believe that there is no substance in nature which may not be melted, and even vaporised, by the application of powerful heat.

There are two incidents attending liquefaction that are worthy of special attention: the *first* is that the liquefaction always takes place at the same temperature in the case of the same substance, so that the melting-point may in fact be used as an index of temperature; and the *second* is that during liquefaction the temperature remains fixed, the accession of heat which has been received during the process of liquefaction being consumed or absorbed in accomplishing the liquefaction, or in other words it has become latent. This heat is given out again in the process of solidification. Water deprived of air and covered with a thin film of oil may be cooled to 20° or 22° below the freezing-point. But on solidification the temperature will rise to the freezing-point. Each different substance has, under ordinary circumstances, its own particular melting-point; but it is found that the electrical condition of a body affects its melting-point, and that electricity will fuse bodies at a low temperature which commonly require for their fusion a very high degree of heat. Thus, platinum may be melted or vaporised by an electrical current, even although the heat generated is small; and a process for separating metals from their ores by the aid of electricity has been projected by using low temperatures, aided by electricity, instead of high degrees of heat. In Part XV. of Taylor's Scientific Memoirs, page 432, there is a paper 'On the Incandescence and Fusion of Metallic Wires by Electricity,' by Peter Riess, being the substance of a paper read before the Royal Society of Berlin; and in this paper it is shown that electrical fusion and vaporisation may take place at temperatures far below those at which metals are red hot. This property of electricity promises to be of service in the arts both in rendering refractory bodies fusible and in enabling bodies to be melted at low temperatures, which might be injured in their qualities by a subjection to high degrees of heat. Thus wrought-

iron if heated to a very high temperature, is liable to be burnt, unless carefully preserved from contact with the air; whereas by sending a current of electricity through it, fusion may be accomplished at a comparatively low temperature, and any injury to the metal may thus be prevented. The melting-points of some of the most important substances are given in the preceding table.

Latent Heat of Liquefaction.—Ice in melting absorbs as much heat as would raise the temperature of the same weight of water 142·65°, or as would raise 142·65 times that weight of water 1 degree; yet, notwithstanding this accession of heat, the ice, during liquefaction, does not rise above 32°. If the heat employed to melt ice was applied to heat the same weight of ice-cold water, it would heat it to the temperature of 142·65° + 32° = 174·65°. The following table shows the amount of heat which becomes latent in the liquefaction of various bodies—the unit of latent heat being the amount of heat necessary to raise the same weight of water 1 degree:

TABLE SHOWING THE HEAT WHICH BECOMES LATENT IN THE LIQUEFACTION OF VARIOUS SOLID BODIES, AS ASCERTAINED BY M. PERSON.

Names of Substances.	Points of Fusion Fahrenheit.	Latent Heat for Unity of Weight.
Chloride of lime	83·3	72·42
Phosphate of soda	97·5	120·24
Phosphorus	111·6	8·48
Bees'-wax (yellow)	143·6	78·32
D'Arcet's alloy	204·8	10·73
Sulphur	239·0	16·51
Tin	455·0	25·74
Bismuth	518·0	22·32
Nitrate of soda	590·9	113·36
Lead	629·6	9·27
Nitrate of potash	642.2	83·12
Zinc	793.4	49·43

By this table we see that the heat which becomes latent in melting a pound of bees' wax would raise the temperature of a

pound of water 78·32 degrees; and the heat which becomes latent in melting a pound of lead would raise the temperature of a pound of water 9·27 degrees.

When there is no external source of heat, from which the heat which becomes latent in liquefaction can be derived, and the circumstances are, nevertheless, such as to cause liquefaction to take place, the heat which becomes latent is derived from the substances themselves, and correspondingly lowers their temperatures. Thus, when snow and salt are mixed together, the snow and salt are dissolved. But, as in melting they absorb heat, and as there is no external source from which the heat is derived, the temperature of the mixture falls very much below that of either of the substances before mixing. So, also, when saltpetre and other salts are dissolved in water, cold is produced, and on this principle the freezing mixtures are compounded which are employed to produce artificial cold in warm climates. A more effectual process, however, is to compress air, which heats it; and the superfluous heat being got rid of by water, it will follow that when this air is again expanded, it will take back an amount of heat equal to that which it before lost, and which demand for heat may be made to cool surrounding bodies. A very effectual freezing machine is constructed on this principle. But it is material that the air in expanding should be made to generate power, else the friction consequent on its escape will generate heat.

VAPORISATION.

As the first phenomenon of the application of heat to a solid substance is to dilate it, and the next to melt it, so also the further application of heat converts it from a liquid into a vapour or gas. The point at which successive increments of heat, instead of raising the temperature, are absorbed in the generation of vapour, is called the *boiling-point* of the liquid. Different liquids have different boiling-points under the same pressure, and the same liquid will boil at a lower temperature in a vacuum, or under a low pressure, than it will do under a high pressure. As the pressure of the atmosphere varies at different

altitudes, liquids will boil at different temperatures at different altitudes, and the height of a mountain may be approximately determined by the temperature at which water boils at its summit.

Difference between Gases and Vapours.—Vapours are saturated gases, or gases are vapours surcharged by heat. Ordinary steam is the saturated vapour of water, and if any of the heat be withdrawn from it, a portion of the water is necessarily precipitated. This is not so in the case of a gas under ordinary conditions. But if the gas be forced into a very small bulk, so that much of the heat is squeezed out of it, then it will follow that any diminution of the temperature will cause a portion of the gas to condense into a liquid. Surcharged or superheated steam resembles gas in its qualities, and a portion of the heat may be withdrawn from such steam, without producing the precipitation of any part of its constituent water.

Liquefaction of the gases.—Many of the gases have already been brought into the liquid state, by the conjoint agency of cold and compression, and all of them are probably susceptible of a similar reduction by the use of means sufficiently powerful for the required end. They must, consequently, be regarded as the superheated steams, or vapours, of the liquids into which they are compressed. The pressures exerted by some of these steams or gases are given in the following table:—

TABLE SHOWING THE TEMPERATURE AND PRESSURE AT WHICH THE SEVERAL GASES NAMED ARE LIQUEFIED.

Names of Gases condensed.	Temperature in degrees Fahrenheit.	Pressure in Atmospheres.	Temperature in degrees Fahrenheit.	Pressure in Atmospheres.
Sulphurous acid........	32°	1·5	46·4°	2·5
Cyanogen gas..........	32	2·3		
Hydriodic acid.........	32	4·0		
Ammoniacal gas........	32	4·4	50	5
Hydrochloric acid.......	32	8·0		
Protoxide of azote.....	32	37·0	51·8	43
Carbonic acid..........	32	32·0	50	45

Latent heat of Evaporation.—It has already been stated, that when a liquid begins to boil, the subsequent accessions of heat

which it receives go not to increase the temperature, but to accomplish the vaporisation. The heat which thus ceases to be discoverable by the thermometer is called the *Latent heat of Vaporisation;* and experiments have shown, that if the heat thus consumed had been employed to raise the temperature of the water, instead of boiling it away, the temperature of the water would have been raised about 1,000 degrees Fahrenheit, or it would have raised about 1,000 times the same weight of water that is boiled off 1 degree Fahrenheit.

The heat consumed in evaporating the same weight of different liquids varies very much, but it does not follow that any of them would, therefore, be better than water as an agent for the generation of power, as the bulk of the resulting vapour in those which require least heat is small, in the proportion of the smaller quantity of heat expended in accomplishing the evaporation. Under the pressure of one atmosphere, or 14·7 lbs. per square inch, the latent heat of steam from water has been found to be 966·1. Alcohol, which boils at 172·2, has a latent heat of evaporation of 364·3. Ether, which boils at 95°, has a latent heat of evaporation of 162·8°, and sulphuret of carbon, which boils at 114·8°, has a latent heat of evaporation of 156°.

The most important of the researches in connection with this subject are those which have reference to the *Latent heat of Steam*, and this topic has been illustrated by the researches of various experimentalists. At the atmospheric pressure, and starting at the temperature of 212°, the following estimates of the latent heat of steam have been formed by the best authorities:—

Watt	950·°	Despretz	955·8°
Southern	945·	Regnault	966·1
Lavoisier	1000·	Fabre and Silbermann	964·8
Rumford	1008·8		

The experiments which are generally considered to be the most correct in connection with this subject are those of M. Regnault. The following table, taken from his results, show that there is a difference of about 150° between the total heat of the vapour of water at the pressures corresponding to 32° and 446° respectively.

SENSIBLE AND LATENT HEAT OR STEAM. BY M. REGNAULT.

Temperature in degrees Fahrenheit.	Latent Heat.	Sum of Sensible and Latent Heats.	Temperature in degrees Fahrenheit.	Latent Heat.	Sum of Sensible and Latent Heats.
32	1092·6	1124·6	248	936·6	1187·6
50	1080·0	1130·0	266	927·0	1193·0
68	1067·4	1135·4	284	914.4	1198·4
86	1054·8	1140·8	302	901·8	1203·8
104	1042·2	1146·2	320	889·2	1209·2
122	1029·6	1151·6	338	874·8	1212·8
140	1017·0	1157·0	356	862·2	1218·2
158	1004·4	1162·4	374	849·6	1223·6
176	991·8	1167·8	392	835·2	1227·2
194	979·2	1173·2	410	822·6	1232·6
212	966·6	1178·6	428	808·2	1236·2
230	952·2	1182·2	446	795·6	1241·6

Rules for connecting the temperature and elastic force of saturated steam.—Various formulæ have been at different times propounded for deducing the elastic force of saturated steam from its temperature, and the temperature from the elastic force. The experiments of Mr. Southern, which were made at the instance of Boulton and Watt, led to the adoption of the following rules, which, though not quite so accurate as some others which have since been arrived at, are sufficiently so for practical purposes, and being intimately identified with engineering practice, it appears desirable to retain them.

THE TEMPERATURE OF SATURATED STEAM BEING GIVEN IN DEGREES FAHRENHEIT, TO FIND THE CORRESPONDING ELASTIC FORCE IN INCHES OF MERCURY BY SOUTHERN'S RULE.

RULE.—*To the given temperature add* 51·3 *degrees. From the logarithm of the sum subtract the logarithm of* 135·767, *which is* 2·1327940. *Multiply the remainder by* 5·13, *and to the natural number answering to the sum, add the constant fraction* ·1. *The sum will be the elastic force in inches of mercury.*

Example.—If the temperature of saturated steam be 250·3° Fahrenheit, what will be the corresponding elastic force in inches of mercury?

Here 250·3 × 51·3 = 301·6	Log.	2.4794313	
135·767	Log.	2·1327940	subtract.
remainder		0·3466373	
multiply by		5·13	
Natural number 60·013	Log.	1·7782493	

This natural number increased by ·1 gives us 60·113 inches of mercury, as the measure of the elastic force sought.

THE ELASTIC FORCE OF SATURATED STEAM BEING GIVEN IN INCHES OF MERCURY, TO FIND THE CORRESPONDING TEMPERATURE IN DEGREES FAHRENHEIT BY SOUTHERN'S RULE.

RULE.—*From the given elastic force subtract the constant fraction* ·1; *divide the logarithm of the remainder by* 5·13, *and to the quotient add the logarithm* 2·1327940. *Find the natural number answering to the sum of the logarithms, and from the number thus found subtract the constant* 51·3. *The remainder will be the temperature sought in degrees Fahrenheit.*

Example.—If the elastic force of saturated steam balances a vertical column of mercury 238·4 inches high, what is the temperature of that steam?

Here 238·4 — 0·1 = 238·3

Log. 238·3 = 2·3771240 ÷ 5·13 = 0·4633770

2·1327940 add

Natural number 394·61 Log. 2·5961710

Constant...... 51·3 subtract

Required temperature 343·31 degrees Fahrenheit.

The temperature of the steam which will balance such a column of mercury, has been ascertained by observation to be 343·6 degrees.

Experiments have been made by the French Academy, the Franklin Institute in America, and various other experimentalists, to determine the elastic force of steam at different temperatures; but of all these experiments, the most elaborate and the most widely accepted are those of M. Regnault. The results obtained by the French Academy are given in the following table, and those obtained by the Franklin Institute are very similar:—

PRESSURE OF STEAM AT DIFFERENT TEMPERATURES.

Results of Experiments made by the French Academy.

An atmosphere is reckoned as being equal to 29·922 inches of mercury.

Pressure in Atmospheres.	Temperature in degrees of Fahrenheit.	Pressure in Atmospheres.	Temperature in degrees of Fahrenheit.
1	212°	13	386·66°
1½	234	14	386·94
2	250·5	15	392·86
2½	263·8	16	398·48
3	275·2	17	403·83
3½	285	18	408·92
4	293·7	19	413·78
4½	300·3	20	418·46
5	307·5	21	422·96
5½	314·24	22	427·98
6	320·36	23	431·42
6½	326·26	24	435·56
7	331·7	25	439·34
7½	336·86	30	457·16
8	341·78	35	472·73
9	350·78	40	486·59
10	358·88	45	499·14
11	366·85	50	510·6
12	374		

Formulæ for connecting the temperature and elastic force of steam have been given by Young, Tredgold, Prony, Biot, Roche, Magnus, Holtzmann, Rankine, Regnault, and many others—all more or less complicated. Regnault employs different formulæ

for different parts of the thermometric scale, as appears from the following recapitulation in which all the degrees are degrees centigrade :—

REGNAULT'S FORMULA FOR THE TEMPERATURE AND ELASTIC FORCE OF STEAM.

Between 0° and 100° the formula is

$$\text{Log. } F = a + b\, a^t{}_1 - c\beta^t{}_1,$$

which resembles the formula previously given by M. Biot. In this formula t is counted from 0° centigrade. $a = 4{\cdot}7384380$; Log. $a_1 = 0{\cdot}006865036$; Log. $\beta_1 = \bar{1}{\cdot}9967249$; Log. $b = \bar{2}{\cdot}1340339$, and Log. $c = 0{\cdot}6116485$.

Between 100° and 230°, the formula he used is

$$\text{Log. } F = a - b\, a^\tau - c\beta^\tau,$$

in which $\tau = t + 20$, t being the centigrade temperature reckoned from 0°. Hence $a = 6{\cdot}2640348$; Log. $a = \bar{1}{\cdot}994049292$; Log. $\beta = \bar{1}{\cdot}998343862$; Log. $b = 0{\cdot}1397743$, and Log. $c = 0{\cdot}6924351$.

The principal properties of saturated steam as deduced from the experiments of M. Regnault, exhibiting the pressure, the relative volume, the temperature, the total heat, and the weight of a cubic foot of steam of different densities, are given by Mr. Clark in the following tables :—

PROPERTIES OF SATURATED STEAM.

BY M. REGNAULT.

Total Pressure per Square Inch.	Relative Volume.	Temperature.	Total Heat.	Weight of one Cubic Foot.	Total Pressure per Square Inch.	Relative Volume.	Temperature.	Total Heat.	Weight of One Cubic Foot.
Lbs.		*Fahr.*	*Fahr.*	*Lbs.*	*Lbs.*		*Fahr.*	*Fahr.*	*Lbs.*
15	1669	213·1	1178·9	·0373	48	573	278·4	1198·8	·1087
16	1572	216·3	1179·9	·0397	49	562	279·7	1199·2	·1108
17	1487	219·5	1180·9	·0419	50	552	281·0	1199·6	·1129
18	1410	222·5	1181·8	·0442	51	542	282·3	1200·0	·1150
19	1342	225·4	1182·7	·0465	52	532	283·5	1200·4	·1171
20	1280	228·0	1183·5	·0487	53	523	284·7	1200·8	·1192
21	1224	230·6	1184·3	·0510	54	514	285·9	1201·1	·1212
22	1172	233·1	1185·0	·0532	55	506	287·1	1201·5	·1232
23	1125	235·5	1185·7	·0554	56	498	288·2	1201·8	·1252
24	1082	237·9	1186·5	·0576	57	490	289·3	1202·2	·1272
25	1042	240·2	1187·2	·0598	58	482	290·4	1202·5	·1292
26	1005	242·3	1187·9	·0620	59	474	391·6	1202·9	·1314
27	971	244·4	1188·5	·0642	60	467	292·7	1203·2	·1335
28	939	246·4	1189·1	·0664	61	460	293·8	1203·6	·1356
29	909	248·4	1189·7	·0686	62	453	294·8	1203·9	·1376
30	881	250·4	1190·3	·0707	63	447	295·9	1204.2	·1396
31	855	252·2	1190·8	·0729	64	440	296·9	1204·5	·1416
32	830	254·1	1191·4	·0751	65	434	298·0	1204·8	·1436
33	807	255·9	1192·0	·0772	66	428	299·0	1205·1	·1456
34	785	257·6	1192·5	·0794	67	422	300·0	1205·4	·1477
35	765	259·3	1193·0	·0815	68	417	300·9	1205·7	·1497
36	745	260·9	1193·5	·0837	69	411	301·9	1206·0	·1516
37	727	262·6	1194·0	·0858	70	406	302·9	1206·3	·1535
38	709	264·2	1194·5	·0879	71	401	303·9	1206·6	·1555
39	693	265·8	1195·0	·0900	72	396	304·8	1206·9	·1574
40	677	267·3	1195·4	·0921	73	391	305·7	1207·2	·1595
41	661	268·7	1195·9	·0942	74	386	306·6	1207·5	·1616
42	647	270·2	1196·3	·0963	75	381	307·5	1207·8	·1636
43	634	271·6	1196·8	·0983	76	377	308·4	1208·0	·1656
44	621	273·0	1197·2	·1004	77	372	309·3	1208·3	·1675
45	608	274·4	1197·6	·1025	78	368	310·2	1208·6	·1696
46	595	275·8	1198·0	·1046	79	364	311·1	1208·9	·1716
47	584	277·1	1198·4	·1067	80	359	312·0	1209·1	·1736

PROPERTIES OF SATURATED STEAM—*continued.*

BY M. REGNAULT.

M. Regnault extended his researches to the pressure of other vapours, beside that of water. The following are the results he obtained with alcohol, ether, sulphuret of carbon, chloroform, and essence of turpentine:

TEMPERATURE AND ELASTIC FORCE OF THE VAPOURS OF DIFFERENT LIQUIDS. BY M. REGNAULT.

[A millimètre is one thousandth part of a mètre, or 0·03937 of an inch.]

Tension of the Vapour of Alcohol.		Tension of the Vapour of *Æther*.		Tension of the Vapour of Sulphuret of Carbon.		Tension of Vapour of Chloroform by Tension in *Vacuo*.		Tension of the Vapour of Essence of Turpentine.	
Temperature in Degrees Centigrade.	Pressure in Millimètres of Mercury.	Temperature in Degrees Centigrade.	Pressure in Millimètres of Mercury.	Temperature in Degrees Fahrenheit.	Pressure in Millimètres of Mercury.	Temperature in Degrees Fahrenheit.	Pressure in Millimètres of Mercury.	Temperature in De-	[illegible]
—21°	3·12	—20	69·2	—16°	58·8	+10°	130·4	0°	2·1
—20	3·34	—10	113·2	—10	79·0	20	190·2	10	2·3
—10	6·50	0	182·3	0	127·3	30	276.1	20	4·3
0	12·73	10	286·5	10	199·3	36	342.2	30	7·0
10	24·08	20	434·8	20	298·2			40	11·2
20	44·0	30	637·0	30	434·6	by the method of ebullition.		50	17·2
30	78·4	40	913·6	40	617·5			60	26·9
40	134·1	50	1268·0	50	852·7			70	41·9
50	220·3	60	1730·6	60	1162·6	....	....	80	61·2
60	350·0	70	2309·5	70	1549·0	36	313·4	90	91·0
70	539·2	80	2947·2	90	2030·5	40	364·0	100	134·9
80	812·8	90	3899·0	80	2628·1	50	524·3	110	187·3
90	1190·4	100	4920·4	100	3321·3	60	738·0	120	257·0
100	1685·0	101	7076·2	110	4136·3	70	976·2	130	347·0
110	2351·8	....	....	120	5121·6	[illegible]	1367·8	140	462·3
120	3207·8	....	....	130	6260·6	90	1811·5	150	604·5
130	4331·2	....	....	136	7029·2	[illegible]	2354·6	[illegible]	777·2
140	5637·7	....	....	....	....	110	3020·4	170	989·0
150	7257·8	....	....	....	....	120	3818·0	180	1225·0
152	7617·8	....	....	....	....	130	4721·0	190	1514·7
....	....	....	....	....	....	....	....	200	1865·6
....	....	....	....	...	....	....	....	210	2251·2
....	....	....	....	....	....	...	....	220	2690·3
....	....	....	....	....	....	. .	....	222	2778·5

Unit of heat.—It is convenient with the view of enabling us to compare the quantities of heat in different bodies to fix upon some *thermal unit*, by which quantities of heat may be measured; and the thermal unit employed in this country is the quan

tity of heat which is required to raise a pound of pure water at its point to maximum density, through one degree Fahrenheit. In France the thermal unit employed is the quantity of heat required to raise a kilogramme of pure water at its point of greatest density through one degree Centigrade. A kilogramme is 2·20462 lbs. avoirdupois, or a pound avoirdupois is 0·453593 of a kilogramme. A degree Centigrade is 1·8 degrees Fahrenheit; and a degree Fahrenheit is 0·555 of a degree Centigrade. There are 3·96832 British thermal units in a French thermal unit, and there is 0·251996 of a French thermal unit in a British thermal unit.

SPECIFIC HEAT.

The *specific heat* of a substance is an expression for the quantity of heat in any given weight of it at a certain temperature, just as its *specific gravity* is an expression for the quantity of matter in a given bulk. Specific heat is most conveniently expressed by a reference to the number of thermal units consumed in producing a given elevation of temperature in the body under consideration; or, if the weight of a heated body immersed in water be multiplied by the temperature it loses, and the weight of water be multiplied by the temperature it gains, the quotient obtained by dividing the latter product by the former, will be the specific heat of the body. The specific heats of various substances have been experimentally ascertained and recorded in tables, in which the specific heat of water is reckoned as unity. Thus, the specific heat of air is ·2379, or it is 4·207 times less than that of water. An amount of heat, therefore, which would raise a pound of water 1 degree, would raise a pound of air 4·207 degrees.

The following tables of specific heats are derived from the experiments of the best authorities, and chiefly from those of M. Regnault. The specific heat of ice is given on the authority of M. Person.

SPECIFIC HEATS OF SOLIDS.

The specific heat of water being reckoned as unity.

NAME OF SUBSTANCE.	Specific Heat.	NAME OF SUBSTANCE.	Specific Heat.
Iron	0·11379	Gold	0·03244
Cast-iron (white)	0·12983	Platinum	0·03243
Steel, soft	0·11650	Glass	0·19768
" tempered	0·11750	Sulphur	0·20259
Copper	0·09515	Silicia	0·19132
Brass	0·09391	Carbon	0·24111
Zinc	0·09555	Coke	0·20200
Lead	0·03140	Diamond	0·14687
Tin	0·05623	Phosphorus	0·18870
Silver	0·05701	Ice	0·50400

SPECIFIC HEATS OF GASES AND VAPOURS.

The specific heat of water being reckoned as unity.

NAME OF GAS OR VAPOUR.	Specific Heat.		Densities.
	For equal Weights.	For equal Volumes.	
Oxygen	0·2182	0·2412	1·1056
Nitrogen	0·2440	0·2370	0·9713
Hydrogen	3·4046	0·2356	0·0692
Chlorine	0·1214	0·2962	2·4400
Protoxide of nitrogen	0·2238	0·3413	1·5250
Binoxide of nitrogen	0·2315	0·2406	1·0390
Carbonic oxide	0·2479	0·2399	0·9674
Carbonic acid	0·2164	0·3308	1·5290
Sulphuret of carbon	0·1575	0·4146	2·6325
Sulphurous acid	0·1553	0·3489	2·2470
Ammonia	0·5080	0·2994	0·5894
Protocarburet of hydrogen—(marsh gas)	0·5929	0·3277	0·5527
Bi-carburet of hydrogen	0·3694	0·3572	0·9672
Water vapour, or steam	0·4750	0·2950	0·6210
Alcohol vapour	0·4513	0·7171	1·5890
Æther vapour	0·4810	1·2296	2·5563
Chloroform vapour	0·1568	0·8310	5·3000
Turpentine vapour	0·5061	2·3776	4·6978

SPECIFIC HEATS OF LIQUIDS.

The specific heat of water being reckoned as unity.

NAME OF LIQUID.	Specific Heat.	NAME OF LIQUID.	Specific Heat.
Mercury............	0·0333	Petroleum..........	0·4684
Turpentine..........	0·4672	Solution Chlo. Lime..	0·6448
Gin	0·4770	Spirit of Wine at 97..	0·6588
Olive Oil	0·3096	Acetic Acid.........	0·6501

It will be observed from the foregoing tables that the specific heat of steam is nearly the same as the specific heat of ice. The specific heat of water, and also of air, occupying the same volume, is found to be the same at all temperatures between boiling and freezing, and the specific heat of air under a constant pressure may be taken at 0·2379. In other words, it requires just the same amount of heat to raise water and air one degree in temperature at any one part of the thermometric scale as at any other; and the heat required to heat a pound of air 1 degree is only ·2379, or less than one-fourth of the quantity required to heat a pound of water one degree. If therefore a pound of water at 60° has transferred to it the heat in a pound of air at 1000°, the water will not acquire as much elevation of temperature as the air loses, but only ·2379 of that temperature.

RATIO OF SPECIFIC HEATS OF GASES UNDER CONSTANT PRESSURE TO THE SPECIFIC HEATS UNDER CONSTANT VOLUME.

When air is compressed it generates heat, as is shown in the syringe in which a piece of tinder is lighted by the heat produced by the sudden compression of air; and, contrariwise, when air or any other gas is expanded it produces cold. When, therefore, a cubic foot of air of the atmospheric pressure is heated until its pressure is doubled, it will have a certain temperature which will fall if the air is suffered to expand into a volume of two cubic feet, and to restore the previous temperature more heat must be added. It will take more heat, therefore, to heat

a cubic foot of air to a given temperature, if it be suffered to expand, than if it be not suffered to expand; and only that part of the heat is, properly speaking, *specific heat*, which is shown by the rise of temperature, that which is absorbed in enlarging the volume being, in point of fact, *latent heat.* Both kinds of heat, however, are very generally called specific heat, but as the quantities are very different, it follows that there are two kinds of specific heat—the one the specific heat under a constant volume, and the other the specific heat under the increased volume to which the body naturally enlarges. It is only in the case of gases that there is a material difference between these specific heats. But in the case of gases the difference is very considerable, and it is found that the specific heat under a constant pressure divided by the specific heat under a constant volume, is equal, in the case of air, to 1·408; or, in other words, the specific heat of air under a constant pressure is 1·408 times greater than that of air under a constant volume. The specific heat of air under a constant pressure, may be taken at ·2379, which makes the specific heat under a constant volume ·169. The following tables of the specific heats, and some other properties of solids, liquids, and gases are given by Mr. Rankine:—

SPECIFIC HEATS AND SPECIFIC GRAVITIES OF METALS.

Name of Metal.	Weight of a cubic foot in lbs. Do.	Specific Gravity. S. G.	Expansion from 32° to 212°. E.	Specific Heat. C.	Specific Heat in foot-pounds. K.
Brass..........	487 to 533	7·8 to 8·5	·00216		
Bronze.........	524	8·4	·00181		
Copper.........	537 to 556	8·6 to 8·9	·00184	·0951	73·3
Gold	1186 to 1224	19· to 19·6	·0015	·0298	23·0
Iron, cast......	444	7·11	·0011		
Iron, wrought..	480	7·69	·0012	·1138	87·8
Lead	712	11·4	·0029	·0293	22·6
Platinum	1311 to 1373	21 to 22	·0009	·0314	24·2
Silver	655	10·5	·002	·0557	43·0
Steel	490	7·85	·0012		
Tin............	462	7·4	·0022	·0514	39·7
Zinc...........	436	7·2	·00294	·0927	71·6
Ice	57·5	0·92		·504	389

SPECIFIC HEATS AND SPECIFIC GRAVITIES OF LIQUIDS.

Name of Liquid.	Do.	S. G.	E.	C.	K.
Water, pure at 39·1°	62·425	1·000	0·04775	1·000	772·0
" sea, ordinary	64·05	1·026	0·05		
Alcohol, pure	49·38	0·791	0·1112		
" proof spirit	57·18	0·916			
Æther	44·70	0·716		0·517	399·1
Mercury	848·75	13·596	0·018153	0·033	25·5
Naphtha	52·94	0·848			
Oil, Linseed	58·68	0·940	0·08		
" Olive	57·12	0·915	0·08		
" Whale	57·62	0·923			
" of Turpentine	54·31	8·870	0·07		
Petroleum	54·81	0·878			

DENSITIES, VOLUMES, RATES OF EXPANSION, AND SPECIFIC HEATS OF GASES.

Name of Gas.	Weight of a cubic foot in lbs. Do.	Volume in cubic feet of 1 lb. Vo.	Expansion from 32° to 212° E.	Specific Heat in degrees Fahr. Under constant volume. Cv.	Specific Heat in degrees Fahr. Under constant pressure Cp.	Specific Heat in foot-pounds. Under constant volume. Kv.	Specific Heat in foot-pounds. Under constant pressure. Kp.
Air	0·080728	12·387	·365	0·169	0·238	130·3	183·45
Oxygen	0·089256	11·204	·367	0·156	0·218	120·2	168·3
Hydrogen	0·005592	178·83	·366	2·410	3·405	1860·6	2628·7
Steam	0·05022*	19·913*	·365*	0·365*	0·475	281·3*	366·7
Æther vapour	0·2093*	4·777*	..	..	0·481	..	371·3
Bisulph. carbon vapour	0·2137*	4·679*	..	..	0·1575	..	121·6
Carbonic acid (ideal)	0·12259*	8·157*	·365*				
Ditto (actual)	0·12344	8·101	·370	..	0·217	..	167·0
Olefiant gas	0·0795	12·58	..	..	0·369	..	284·9
Nitrogen	0·078411	12·753	..	0·173	0·244	133·6	188·4
Vapour of Mercury	0·563*	1·7762*					

An asterisk (*) is affixed to the results computed for the ideal condition of a perfect gas.

In these tables the volumes are taken at the temperature of melting ice, or 32°; except in the case of water, which is taken at the temperature of maximum density, or 39·1°. The pressure is taken at the usual atmospheric pressure of 2116·4 lbs. upon the square foot.

D_o is the density or weight of 1 cubic foot of the substance in

bs. avoirdupois under the pressure of one atmosphere, or 2116·4 bs. on the square foot.

V_0 is the volume in cubic feet of 1 pound avoirdupois of the substance at the foregoing temperature and pressure. S.G. is the specific gravity, water being taken as unity. E is the expansion of unity of volume for fluids, and unity of length for solids, at the temperature of melting ice, in being raised from the temperature of melting ice to the temperature of boiling water under the pressure of one atmosphere. C is the specific heat in degrees Fahrenheit, the specific heat of water being reckoned as unity, and C_v is the specific heat under a constant volume, while C_p is the specific heat under a constant pressure. K is the specific heat, reckoned not in degrees of temperature, but in the equivalent value of pounds raised 1 foot high. It has already been explained that there is as much power in the form of heat expended in raising a pound of water 1 degree in temperature as would raise 772 lbs. to the height of 1 foot; and 772 foot-pounds is, consequently, the mechanical equivalent of a pound of water raised 1 degree. Now as the specific heats of all bodies are determinable by the temperature to which a pound of the substance will raise a pound of water, and as the accession of heat which a pound of water receives is transformable into its equivalent amount of mechanical power, it follows that the specific heats of all bodies may be represented by the amount of mechanical power in foot-pounds, which is the equivalent of the heat consumed in raising a pound of any of these bodies through one degree of temperature. Such specific heats, accordingly, are those represented in the tables by the letter K; the expression K_v being the specific heat in foot-pounds of unity of weight under a constant volume, and K_p the specific heat of the same weight under a constant pressure. The value of $K_p \div K_v$, Mr. Rankine states, is in the case of air, 1·408; oxygen, 1·4; hydrogen, 1·413; nitrogen, 1·409; and steam, considered as a perfect gas, 1·304; or, in other words, the specific heat under a constant volume is to the specific heat under a constant pressure as 1 to 1·4 in the case of oxygen, differing slightly in the case of the other gases.

PHENOMENA OF EBULLITION.

Influence of Viscosity or Molecular Attraction.—Salts dissolved in water will raise the temperature of its boiling-point. The attraction of a salt for water being greater than the attraction of the particles of the water for one another, will resist the repellent force of the heat to some extent. Mechanical pressure applied to the water has the same operation. Hence, water boils in a vacuum at a lower temperature than under the pressure of the atmosphere, and it also boils at a lower temperature under the pressure of one atmosphere than under a pressure of several atmospheres. Water, which has been well purged of air by boiling, does not pass into the state of steam when heated in clean glass vessels, until it has attained a temperature considerably higher than its ordinary boiling point; and when the steam finally forms, it forms rather by a jumping motion, or by a sudden shock, than by a gradual and silent disengagement. M. Magnus found that water well cleared of air may be raised to a temperature of 105° or 106° Centigrade before boiling, if the glass vessel in which it was heated were perfectly clean; but if the vessel were soiled, or if dust or other foreign particles were suffered to enter it, the temperature would fall to the usual boiling point of 100° Centigrade. The sides of metallic vessels, or sawdust, metal filings, or insoluble particles of almost any kind, introduced into a liquid, lower its boiling-point. These particles are not at every point completely moistened by the water, and they have a less attraction for the particles of the fluid than the particles of the fluid have for one another. In the process of ebullition, therefore, the steam chiefly forms around those particles and seems to come out of them, and the boiling-point is lowered by the greater facility they occasion to the disengagement of the steam. M. Donny, by freeing water carefully from air, succeeded in raising it to a temperature of 135° without boiling; but at this temperature steam was suddenly formed, and a portion of the water was projected forcibly from the tube. M. Donny concludes, from his experiments, that the mutual force of cohesion of the parti-

cles of water is equal to a pressure of about three atmospheres, and to this strong cohesive force he attributes the irregular jumping motion observed in ebullition, and also some of those explosions of steam-boilers which heretofore have perplexed engineers. It is well known that cases have occurred in which an open pan of boiling water has exploded, producing fatal results, and such explosions cannot be explained on the usual hypothesis. M. Donny says that liquids by boiling lose the greater part of the air which they hold in solution, and therefore the molecular attraction begins to manifest itself in a sensible manner. The liquid consequently attains a temperature considerably above its normal boiling-point, which determines the appearance of new air-bubbles, when the liquid separates abruptly, a quantity of vapour forms, and the equilibrium is for the moment restored. The phenomenon then recurs again with increased violence, and an explosion may eventually ensue.

Spheroidal Condition of Liquids on Hot Surfaces.—If a drop of water or other liquid be thrown upon a hot metal plate or other highly heated surface, it does not moisten the surface or diffuse itself over it, but forms a flattened ellipsoidal mass; and if the drop be sufficiently small, it forms a minute spheroid, which revolves rapidly round a shifting axis, and evaporates very slowly without entering the state of ebullition. From Church's experiments it appears that it is necessary for the liquid to emit vapour before it can assume the spheroidal state. Molten lead dropped upon a very hot platinum plate did not assume the spheroidal state, whereas mercury dropped upon this plate assumed the spheroidal state at once. The most remarkable experiments, however, which have been made in illustration of the phenomena of the spheroidal state are those of M. Boutigny, and to him engineers are mainly indebted for calling their attention to the subject. One of the most singular results obtained by M. Boutigny is the power of making ice in a red hot crucible. A small crucible or capsule of platinum being made white hot, some anhydrous sulphurous acid in the liquid state is poured into it. The boiling-point of this liquid is as low as 14° Fahrenheit; but as it immediately on being projected into the

capsule assumes the spheroidal state, it remains upon the white hot metal without touching it; and if a few drops of water be now let fall upon the liquid acid, the water will be immediately frozen, and a piece of ice may be turned out of the crucible. M. Boutigny has also shown that if acids and alkalies in solution be poured into a clean red hot platinum crucible they will not unite, but both will assume the spheroidal state and roll about the bottom of the crucible without entering into combination. Not merely the gravitation of the liquid, therefore, but also its chemical affinity, appears to be superseded by the causes which make it assume the spheroidal state.

When a liquid assumes the spheroidal state it does not wet the surface, but appears to avoid touching it, like water sprinkled upon grease. Instead of entering into violent ebullition when it reaches the hot surface, its temperature will rise very little, and the drops of liquid will either remain at rest or will acquire a gyratory motion. When the surface is cooled down to 400° to 500°, depending on the nature of the surface and also on the nature of the liquid, the liquid will begin to diffuse itself, and will be suddenly scattered in all directions. The requisite temperature of a platinum plate to make water at the boiling-point assume the spheroidal state is 120° Centigrade, or 248° Fahrenheit; but if glass be used instead of platinum, the temperature must be raised to 180° Centigrade, or 324° Fahrenheit. For water at 0° Centigrade, the temperatures required are 400° and 800° respectively.

When water assumes the spheroidal state, it is possible by placing the eye on the level of the hot surface to see between the surface and the liquid. The electric circuit, moreover, is interrupted, showing that there is no actual contact between the liquid and the plate. The repulsion existing between the liquid and the plate is usually imputed to the existence of an atmosphere of vapour upon which, as upon a cushion, the spheroids are supposed to rest. There is no reason to conclude, however, because vapour is raised from a liquid, that therefore its gravity must be suspended, and the cause is rather to be sought for in the motion of the spheroid, or of its internal particles, whereby the motion to which gravity is due is partially counteracted.

Spheroidal State of the Water in Boilers.—There can be no doubt that the water of boilers is sometimes repelled from the metal in the same manner as would be done if it were in the spheroidal state, and explosions have, no doubt, frequently had their origin in this phenomenon. Land boilers, whether of the cylindrical or waggon form, frequently bend down in the bottom where the strongest heat of the furnace impinges, and lead rivets, inserted in them for purposes of safety, are sometimes melted out. The water appears to be repelled from the iron in those parts of the boiler bottom where the heat is greatest, and the iron becomes red hot, and is bagged or bent out by the pressure of the steam. In some boilers the bottom can at any time be made red hot by very heavy firing, and in most factory boilers the bottom will be more or less injured if the stoker urges the fire very much. If gauge cocks be inserted at different levels, in a small upright cylindrical boiler, so that one cock is near the top, another near the bottom, and the rest in intermediate positions, it will follow, that if sufficient water be introduced into the boiler to show at the lowest gauge cock, it will continue to show there so long as a moderate heat is maintained. So soon, however, as the fire is made to burn fiercely, so as to impart a strong heat to the bottom, the water will disappear from the bottom cock and show in the top cock, thus proving that the water has been repelled by the heat until it occupies the top part of the boiler instead of the bottom part.

COMMUNICATION OF HEAT.

Heat may be communicated from a hot body to a cold one in three ways—by Radiation, by Conduction, and by Circulation.

The rapidity with which heat radiates varies, other things being equal, as the square of the temperature of the hot body in excess of the temperature of the cold one; so that a body if made twice as hot will lose a degree of temperature in one-fourth of the time; if made three times as hot, it will lose a degree of temperature in one-ninth of the time; and so on, in all other proportions. This explains how it comes that a very small proportion of surface in a boiler of which the furnace is maintained

at a high temperature is equivalent to a much larger proportion of surface when the temperature is somewhat lower. Radiant heat may be concentrated into a focus by a reflector, in the same manner as light, and, like light, it may likewise be made to undergo refraction and polarisation.

The conduction of heat through different substances varies very nearly in the same proportion as their conducting powers for electricity. Taking the conducting power of silver as 100, the following are the conducting powers of metals according to the best authorities:—

CONDUCTING POWERS OF METALS.

Name of Body.	Conductivity for Electricity.			Conductivity for Heat.
	Riess.	Becquerel.	Lenz.	Wiedemann and Franz.
Silver	100·0	100·0	100·0	100·0
Copper	66·7	91·5	73·3	73·6
Gold	59·0	64·9	58·5	53·2
Brass	18·4	....	21·5	23·6
Tin	10·0	14·0	22·6	14·5
Iron	12·0	12·35	13·0	11·9
Steel	....	....	....	11·6
Lead	7·0	8·27	10·7	8·5
Platinum	10·5	7·93	10·3	8·4
German Silver	5·9	....	....	6·3
Bismuth	....	....	1·9	1·8

The conducting power of marble is about the same as the conducting power of bismuth; and the conducting powers of porcelain and bricks are each about half that of marble. The conducting power of water is very low, and hence heat is transmitted *downwards* through water only very slowly. But *upwards* it is transmitted rapidly by virtue of the circulation which then takes place.

The efficiency of the heating surface of a boiler will depend very much upon the efficiency of the arrangements which are in force for maintaining or promoting a rapid circulation of the water. In like manner, the rapidity of the circulation which is maintained in the water used for refrigeration in surface con-

densers will mainly determine the weight of steam condensed in the hour by each square foot of refrigerating surface. Peclet found by a number of experiments that water, when used as the refrigerating fluid, was about ten times more effectual than air; and he further found that when water was used for refrigeration, each square foot of copper surface was able to condense about 21¼ lbs. of steam in the hour. Mr. Joule, however, found that a square foot of copper surface might, by maintaining a rapid circulation of the cooling water, be made to condense 100 lbs. of steam in the hour—the cooling water being contained in a pipe concentric with that containing the steam, and flowing *in the opposite direction*. With this rapidity of refrigeration, the cooling surface of a condenser need only be about one sixteenth of the heating surface of the boiler which supplies the engine with steam. In ordinary land boilers 10 square feet of heating surface will boil off a cubic foot, or 62½ lbs. of water in the hour; and one square foot of heating surface will therefore boil off one-tenth of this, or 6·25 lbs. of water in the hour. To boil off 100 lbs. in the hour would at this rate require 16 square feet of heating surface. But the 100 lbs. of steam thus boiled off will, according to Mr. Joule, be condensed by one square foot of cooling surface; so that, if this authority be accepted, the surface of a well-constructed condenser need only be about one-sixteenth of the heating surface of the boiler, the steam of which it condenses.

The importance of maintaining a rapid circulation in the water of boilers has not yet been sufficiently recognised. It is desirable that solid water and not steam should be in contact with the heating surface, else the metal plating will be liable to become overheated, and any given area of heating surface will be much less effective. The species of boiler invented by Mr. David Napier, called the haystack boiler, and in which the water is contained in vertical tubes, is about the best species of boiler for keeping up a rapid circulation of the water. But it necessary to apply large return pipes or a wide water space all round the exterior of the boiler, with a diaphragm to permit ascending and descending currents, in order that the water carried upward by the steam may be immediately returned.

COMBUSTION.

Combustion is energetic chemical combination between the oxygen of the air and the constituents of the combustible. The combustibles chiefly used to generate the heat consumed by steam-engines are coal, wood, and sometimes charcoal.

Coal consists chiefly of carbon and hydrogen, but the proportions in which these elements enter into the composition of different coals is very various. Cannel coal consists of about 60 per cent. of volatile matter, and 40 per cent. of coke and earthy matter, whereas splint coal consists of about 65 per cent. of coke, and 35 per cent. of volatile matter. Air consists of oxygen and nitrogen, mixed in the proportions of 8 lbs. of oxygen to every 28 lbs. of nitrogen, or 1 lb. of oxygen to every 3½ lbs. of nitrogen. To accomplish the combustion of 6 lbs. of carbon, 16 lbs. of oxygen are necessary, forming 22 lbs. of carbonic acid, which will have the same volume as the oxygen, and, therefore, a greater density. To accomplish the combustion of 1 lb. of hydrogen, 8 lbs. of oxygen are necessary. When, therefore, we know the proportions of carbon and hydrogen existing in coal, it is easy to tell the quantity of oxygen, and, consequently, the quantity of air necessary for its combustion. As a general rule, it may be stated that, for every pound of coal burned in a furnace, about 12 lbs. of air will be necessary to furnish the oxygen required, even if every particle of it entered into combination. But from careful experiments it has been found, that in ordinary furnaces, where the draught is produced by a chimney, about as much more air will in practice be necessary, or about 24 lbs. per lb. of coal burned. In the case of furnaces, with a more rapid draught maintained either by a steam jet or a fan blast, a smaller excess of air will suffice, and in those cases about 18 lbs. of air will be required from the combustion of 1 lb. of coal. If a cubic foot of air weigh 1·291 oz., then 12 lbs. or 192 oz. will measure about 150 cubic feet, as 1·291 oz. bears the same proportion to 1 cubic foot, as 192 oz. bears to 150 cubic feet nearly. In ordinary furnaces, with a chimney therefor, which require 24 lbs. of air per lb. of coal,

the volume of air necessary for the combustion of 1 lb. of coal will be about 300 cubic feet, which is equal to the content of a room measuring about 6 feet 8½ inches every way.

The specific gravity of oxygen is a little more than that of air, being by the latest experiments 1·106, while that of air is 1. Now, as 16 lbs. of oxygen unite with 6 lbs. of carbon to form 22 lbs. of carbonic acid, and, as the volume of the carbonic acid at the same temperature remains only the same as that of the original oxygen, it follows that the density or specific gravity of the carbonic acid must be greater than that of the oxygen, in the same proportion in which 22 is greater than 16. Multiplying therefore 1·106, which is the specific gravity of oxygen, by 22, and dividing by 16, we get 1·521, which must be the specific gravity of carbonic acid, if the specific gravity of oxygen is 1·106. Formerly, the specific gravity of oxygen was reckoned at 1·111, but there is reason to believe that 1·106 is the more accurate determination.

Total Heat of Combustion.—The temperature to which a pound of fuel would raise a pound of water, or the total heat of combustion in thermal units, has been carefully investigated by MM. Favre and Silbermann, whose determinations are recapitulated and condensed by M. Rankine as follows:—

TOTAL HEAT OF COMBUSTION OF 1 lb. OF EACH OF THE COMBUSTIBLES ENUMERATED.

Combustible, 1 lb. of each being burned.	Lbs. of ~~Air~~ Oxygen required.	Lbs. of Air required.	Total Heat in Thermal Units.	Evaporative Power from 212°.
Hydrogen gas	8	36	62,032	64·2
Carbon, imperfectly burned, so as to make carbonic oxide	1⅓	6	4,400	4·55
Carbon, completely burned, so as to make carbonic acid	2⅔	12	14,500	15·0
Olefiant gas	3³⁄₇	15³⁄₇	21,344	22·1
Various liquid hydrocarbons	..	..	from 21,000 to 19,000	from 22 to 20
Carbonic oxide, as much as is made by the imperfect combustion of 1 lb. of carbon, viz. 2⅓ lbs	1⅓	6	10,100	10·45

With regard to the quantities stated as being the total heat of combustion respectively of carbon completely burned, carbon imperfectly burned, and carbonic oxide, Mr. Rankine says that the following explanation has to be made:—

The burning of carbon is always complete at first; that is to say, one pound of carbon combines with 2⅔ lbs. of oxygen, and makes 3⅔ lbs. of carbonic acid; and although the carbon is solid immediately before the combustion, it passes during the combustion into the gaseous state, and the carbonic acid is gaseous. This terminates the process when the layer of carbon is not so thick, and the supply of air not so small, but that oxygen in sufficient quantity can get direct access to all the solid carbon. The quantity of heat produced is 14,500 thermal units per lb. of carbon, as already stated.

But in other cases part of the solid carbon is not supplied directly with oxygen, but is first heated, and then dissolved into the gaseous state, by the hot carbonic acid gas from the other parts of the furnace. The 3⅔ lbs. of carbonic acid gas from 1 lb. of carbon, are capable of dissolving an additional lb. of carbon, making 4⅔ lbs. of *carbonic oxide* gas; and the volume of this gas is double of that of the carbonic acid gas which produces it. In this case, the heat produced, instead of being that due to the complete combustion of

1 lb. of carbon or	14,500
falls to the amount due to the imperfect combustion of 2 lbs. of carbon, or	2 × 4,400 × 8,800
Showing a *loss* of heat to the amount of . . .	5,700

which disappears in volatilising the second pound of carbon. Should the process stop here, as it does in furnaces ill supplied with air, the waste of fuel is very great, as the carbonic oxide—which is a species of invisible smoke—has a large quantity of carbon in it which is dissipated in the atmosphere without useful result. But when the 4⅔ lbs. of carbonic oxide gas, containing 2 lbs. of carbon, is mixed with a sufficient supply of fresh air, it burns with a blue flame, combining with an additional 2⅔ lbs. of oxygen, making 7⅓ lbs. of carbonic acid gas, and giving

additional heat of double the amount due to the combustion of $1\frac{1}{3}$ lb. of carbonic oxide; that is to say,

$$10{,}100 \times 2 = 20{,}200$$

to which being added the heat produced by the imperfect combustion of 2 lbs. of carbon, or . . . $\underline{8{,}800}$

there is obtained the heat due to the complete combustion of 2 lbs. of carbon, or . . . . $2 \times 14{,}500 = 29{,}000$

The evaporative powers of different kinds of coal in practice is given in the following table:—

TABLE SHOWING THE ECONOMIC VALUES OF DIFFERENT COALS.

BY DE LA BECHE AND PLAYFAIR.

	Names of Coal employed in the Experiments.	Economical evaporating power, or number of lbs., of Water evaporated from 212° by 1 lb. of Coal.	Weight of 1 cubic foot of the Coal as used for Fuel. — lbs.	Space occupied by 1 ton of the Coal in cubic feet.	Rate of evaporation, or number of lbs. of Water evaporated per hour. — Mean.
Welsh Coals.	Graigola	9·35	60·166	37·23	441·48
	Anthracite (Jones & Co.)	9·46	58·25	38·45	409·37
	Oldcastle Fiery Vein	8·94	50·916	43·99	464·30
	Ward's Fiery Vein	9·40	57·433	39	529·90
	Binea	9·94	57·08	39·24	486·95
	Llangennech	8·86	56·93	39·34	378·22
	Pentrepoth	8·72	57·72	38·80	381·50
	Pentrefellin	6·36	66·166	33·85	247·24
	Duffryn	10·14	53·22	42·09	409·32
	Mynydd Newydd	9·52	56·33	39·76	470·69
	Three-quarter Rock Vein	8·84	56·388	39·72	486·86
	Cwm Frood Rock Vein	8·70	55·277	40·52	379·80
	Cwm Nanty-gros	8·42	56·0	40·00	404·16
	Resolven	9·53	58·66	38·19	390·25
	Pontypool	7·47	55·7	40·216	250·40
	Bedwas	9·79	50·5	44·32	476·96
	Ebbw Vale	10·21	53·3	42·26	460·22
	Porthmawr	7·53	53·0	42·02	347·44
	Coleshill	8·00	53·0	42·26	406·41
Scotch.	Dalkeith Jewel Seam	7·08	49·8	44·98	355·18
	" Coronation Seam	7·71	51·66	43·36	370·08
	Wallsend Elgin	8·46	54·6	41·02	435·77
	Fordel Splint	7·56	55·0	40·72	464·98
	Grangemouth	7·40	54·25	40·13	380·49
Engl.	Broomhill	7·30	52·5	42·67	397·78
	Lydney (Forest of Dean)	8·52	54·444	41·14	487·19
	Slievardagh (Irish Anthracite)	9·85	62·8	35·66	473·18
	Wylam's Patent Fuel	8·92	65·08	34·41	418·89
	Warlich's "	10·36	69·05	32·44	457·84
	Bell's "	8·53	65·3	34·30	549·11

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Maximum Temperature of the Furnace.—When we know the total heat of a combustible in thermal units, the weight of the smoke and ashes or the products of combustion, as they are called, and their specific heat, it is easy to tell what is the highest temperature that the furnace can attain, supposing that the air is not artificially heated. Thus the chief products of combustion of coal being carbonic acid, steam, nitrogen, and ashes, with a certain proportion of residual air, which passes unchanged through the fire; then, if we reckon the specific heat of carbonic acid at 0·217, of steam at 0·475, of nitrogen at 0·245, of air at 0·238, and of ashes at 0·200, and take into account the quantities of each which are present, the mean specific heat of the products of combustion may be taken, without much error, as about equal to the specific heat of air. Now, as 12 lbs. of air are required for the combustion of a pound of carbon, even if every particle of the oxygen be supposed to enter into combination, the weight of the products of combustion will on that supposition be 12 lbs. + 1 lb., or 13 lbs. If we take the total heat of combustion of carbon or charcoal at 14,500, and the mean specific heat of the products of combustion at 0·238, then the specific heat multiplied by the weight will be 3·094; and 14,500 divided by 3·094 = 4689, which will be the temperature to which the furnace would be raised in degrees Fahrenheit, supposing every atom of oxygen that entered the furnace entered into combination. If, however, as will be the case in ordinary furnaces, twice that quantity of air necessarily enters, then the weight of the products of combustion of 1 lb. of coal will be 25 lb., which, multiplied by the specific heat = 5·95, and 14,500 divided by 5·95 = 2,437, which is the temperature in degrees Fahrenheit that, on this supposition, the furnace would have. If 18 lbs. of air be supplied per lb. of coal, as suffices in the case of furnaces with artificial draught, then the weight of the products of combustion will be 19 lbs., which, multiplied by the specific heat, gives 4·522, and 14,500 divided by 4·522, gives 3,207 as the temperature of the furnace in degrees Fahr. This in point of fact may be taken as a near approach to the temperature of hot furnaces, such as that of a locomotive boiler.

The increased volume which any given quantity of air at 32° will acquire, by raising its temperature through any given number of degrees, can easily be determined by the rule already given for that purpose. Mr. Rankine has computed the volume in cubic feet, which 12 lbs. of air, 18 lbs., and 24 lbs., will respectively acquire, when heated to different temperatures, by combining with 1 lb. of carbon in a furnace; the volume of 12 lbs. at 32°, and at the atmospheric pressure, being taken at 150 cubic feet, of 18 lbs. at 225 cubic feet, and of 24 lbs. at 300 cubic feet. The results are as follows:

TEMPERATURES OF COMBUSTION AND VOLUMES OF PRODUCTS.

Temperatures.	Supply of Air in pounds per lb. of fuel.		
	12 lbs.	18 lbs.	24 lbs.
	Volume of Air or Gases in cubic feet at each Temperature.		
32°	150	225	300
68°	161	241	322
104°	172	258	344
212°	205	307	409
392°	259	389	519
572°	314	471	628
752°	369	553	738
1112°	479	718	957
1472°	588	882	1176
1832°	697	1046	1395
2500°	906	1359	1812
3275°	1136	1704	
4640°	1551		

Rate of Combustion.—The rate of combustion, or the quantity of fuel burned in the hour upon each square foot of fire-grate, varies very much in different classes of boilers. In Cornish boilers it is 3½ lbs. per square foot; in the older class of land boilers, 10 lbs.; in more recent land boilers, 13 to 14 lbs.; in modern marine boilers, 16 to 24 lbs., and in locomotive boilers, 80 to 120 lbs. on each square foot of fire-grate in the hour.

THERMO-DYNAMICS.

It has been already stated that heat and power are mutually convertible, and that the power in the shape of heat which is necessary to raise a pound of water through one degree Fahrenheit, would, if utilised without waste in a thermo-dynamic engine, raise 772 lbs. through the height of 1 foot. A pound of water raised through a degree centigrade is equivalent to 1390 lbs. raised through the height of 1 foot. In every heat engine, the greater the extremes of temperature, or the hotter the boiler or source of heat and the colder the condenser or refrigerator, the larger will be the proportion of the heat utilised as power.

In a perfect steam engine, if a be the temperature of the boiler, reckoning from the point of absolute zero, and b be the temperature of the condenser, reckoning also from the point of absolute zero, the fraction of the entire heat communicated to the boiler which will be converted into mechanical effect, will be $\frac{a-b}{a}$. Now it is clear if $a = b$, or if the temperature of the boiler and condenser are the same, the value of $\frac{a-b}{a}$ becomes equal to 0, or there is none of the heat utilised as power, whereas, on the other hand, if a be taken larger and larger, the value of the fraction becomes continually greater, indicating that by increasing the difference of the temperatures of the boiler and condenser, a great quantity of the heat expended is converted into mechanical effect, and by taking $a = \infty$, or infinity, the limit to which the fraction approaches is found to be unity, showing that in such a case, if it were possible of realisation, the whole of the heat would be converted into power.

The formula given by Professor Thomson for determining the power generated by a perfect thermo-dynamic engine, is as follows:—

If S be the temperature of the source of heat, and T be the temperature of the refrigerator, both expressed in centigrade degrees; and if H denote the total heat in thermal units centigrade, entering the engine in a given time; and J be Joule's

equivalent of 1390 lbs. raised one foot high by a centigrade degree;—then the power produced, or W the work performed, is

$$W = JH\frac{S-T}{S+274}.$$

This formula may be expressed in words, as follows:—

TO FIND THE POWER GENERATED BY A PERFECT ENGINE IMPELLED BY THE MOTIVE POWER OF HEAT.

RULE.—*From the temperature of the source or boiler, subtract the temperature of the condenser; divide the remainder by the sum of the temperature of the source and* 274, *and multiply the quotient by the total heat communicated to the engine per minute, expressed in the number of degrees through which it would raise one pound of water. Finally, multiply this product by* 1390. *The result is the number of pounds that the engine will raise a foot high in the minute. The temperatures are all taken in degrees centigrade.*

Example.—In a steam-engine working with a pressure of 14 atmospheres, the temperature of the steam in the boiler will be 215° centigrade, and the temperature of the condenser may be taken at 44·44° centigrade. If a grain of coal be burned per minute, the heat imparted every minute to a pound of water will be ·905° centigrade. Now 215 − 44·44° = 170·56 and 215 + 274 = 489, and 170·56 divided by 489 = 0·35, which multiplied by ·905 and by 1390 = 440 lbs. raised 1 foot high every minute, which as a grain of coal is burned every minute, is very nearly the same result as that before indicated.

Cheapest Source of Motive Power.—The cheapest source of a mechanical power that will be available in all situations, is, so far as we yet know, the combustion of coal. Electricity and galvanism have been proposed as motive powers, and may be used as such, but they are much more expensive than coal. Mr. Joule has ascertained by his experiments that a grain of zinc, consumed in a galvanic battery, will generate sufficient power to raise a weight of 145·6 lbs. through the height of one foot;

whereas a grain of coal, consumed by combustion, will generate sufficient power to raise 1261·45 lbs. to the height of 1 foot.

Moreover, it appears certain that Mr. Joule's estimate of the heating power of coal is too small. A pound of coal will, under favourable circumstances, evaporate 12 lbs. of water, which is equivalent to a pound of water being heated 2 degrees Fahrenheit by a grain of coal, or it is equivalent to 1544 lbs. raised through 1 foot. This is more than ten times the power generated by a pound of zinc. But as thermo-electric engines, it is estimated, expend their energy about four times more beneficially than heat engines, the dynamic efficacy of a pound of zinc may be taken as about 4-10ths of that of a pound of coal. A ton of zinc, however, costs fifty or sixty times as much as a ton of coals, while it is not half so effective. There does not appear, therefore, to be the least chance of heat engines being superseded by electro-dynamic engines, of which zinc or some other metal supplies the motive force.

EXPANSION OF STEAM.

When air is compressed into a smaller volume, a certain amount of power is expended in accomplishing the compression, which power, as in the case of a bent spring, is given back again when the pressure is withdrawn. If, however, the air when compressed is suddenly dismissed into the atmosphere, the power expended in compression will be lost; and there is a loss of power, therefore, in dispensing with that power, which is recoverable by the expansion of the air to its original volume. Now the steam of an engine is in the condition of air already compressed; and unless the steam be worked in the cylinder expansively—which is done by stopping the supply from the boiler before the stroke is closed—there will be a loss of a certain proportion of the power which the steam would otherwise produce. If the flow of steam to an engine be stopped when the piston has performed one-half of the stroke, leaving the rest of the stroke to be completed by the expanding steam, then the efficacy of the steam will be increased 1·7 times beyond what it

would have been had the steam at half-stroke been dismissed without extracting more power from it; if the steam be stopped at one-third of the stroke, the efficacy will be increased 2·1 times; at one-fourth, 2·4 times; at one-fifth, 2·6 times; at one-sixth, 2·8 times; at one-seventh, 3 times; and at one-eighth, 3·2 times.

TO FIND THE INCREASE OF EFFICIENCY ARISING FROM WORKING STEAM EXPANSIVELY.

RULE.—*Divide the total length of the stroke by the distance (which call 1) through which the piston moves before the steam is cut off. The Neperian logarithm of the whole stroke expressed in terms of the part of the stroke performed with the full pressure of steam, represents the increase of efficiency due to expansion.*

Example 1.—Suppose that the steam be cut off at $\frac{2}{15}$ths of the stroke: what is the increase of efficiency due to expansion?

Here it is plain that $\frac{2}{15}$ths of the whole stroke is the same as $\frac{1}{7\cdot5}$ of the whole stroke. The hyperbolic logarithm of 7·5 is 2·015, which increased by 1, the value of the portion performed with full pressure, gives 3·015 as the relative efficacy of the steam when expanded to this extent, instead of 1, which would have been the efficacy if there had been no expansion.

If the steam be cut of at $\frac{1}{8}$, $\frac{2}{8}$, $\frac{3}{8}$, $\frac{4}{8}$, $\frac{5}{8}$, $\frac{6}{8}$, or $\frac{7}{8}$th of the stroke, the respective ratios of expansion will be 8, 4, 2·66, 2, 1·6, 1·33, and 1·14, of which the respective hyperbolic logarithms are 2·079, 1·386, 0·978, 0·693, 0·470, 0·285, and 0·131; and if the steam be cut off at $\frac{1}{10}$, $\frac{2}{10}$, $\frac{3}{10}$, $\frac{4}{10}$, $\frac{5}{10}$, $\frac{6}{10}$, $\frac{7}{10}$, $\frac{8}{10}$, or $\frac{9}{10}$ths of the stroke, the respective ratios of expansion will be 10, 5, 3·33, 2·5, 1·66, 1·42, 1·25, and 1·11, of which numbers the respective hyperbolic logarithms are 2·303, 1·609, 1·203, 0·916, 0·507, 0·351, 0·223, and 0·104. With these data it will be easy to compute the mean pressure of steam of any given initial pressure when cut off at any eighth part or any tenth part of the stroke, as we have only to divide the initial pressure of the steam in lbs. per square inch by the ratio of expansion, and to multiply the quo-

tient by the hyperbolic logarithm, increased by 1, of the number representing the ratio, which gives the mean pressure through out the stroke in lbs. per square inch. Thus, if steam of 100 lbs. be cut off at half stroke, the ratio of expansion is 2, and 100 divided by 2 and multiplied by 1·693 = 84·65, which is the mean pressure throughout the stroke in lbs. per square inch. The terminal pressure is found by dividing the initial pressure by the ratio of expansion; thus, the terminal pressure of steam of 100 lbs. cut off at half stroke will be 100 divided by 2 = 50 lbs. per square inch.

Example 2.—What is the mean pressure throughout the stroke of steam of 200 lbs. per square inch cut off at $\frac{1}{10}$th of the stroke?

Here 200 divided by 10 = 20, which, multiplied by 3·303 (the hyperbolic logarithm of 10 increased by 1) gives 66·04, which is the mean pressure exerted on the piston throughout the stroks in lbs. per square inch.

If the steam were cut off at $\frac{1}{8}$th of the stroke instead of $\frac{1}{10}$th, then we should have 200 divided by 8 = 25, which, multiplied by 3·079 (the hyperbolic logarithm of 8 increased by 1), gives 76·975 lbs., which would be the mean pressure on the piston throughout the stroke in such a case.

If the initial pressure of the steam were 3 lbs. per square inch, and the expansion took place throughout $\frac{7}{8}$ths of the stroke, or the steam were cut off at $\frac{1}{8}$th, then $3 \div 8 = \cdot 375$, which $\times$ by 3·079 = 1·154625 lbs. per square inch of mean pressure.

There are various expedients for stopping off the supply of steam to the engine at any desired point of the stroke, which are described in my 'Catechism of the Steam Engine,' and which, consequently, it would be superfluous to recapitulate here. One mode is by the use of an expansion valve, and another mode is by extending the length of the face of the ordinary slide valve by which the steam is let into and out of the cylinder, which extension of the face is called *lap* or *cover*. For the purposes of this work it will be sufficient to recapitulate the mean pressure of the steam on the piston of an engine throughout the whole stroke, supposing the steam to be cut off

at different successive points of the stroke, counting first by eighths, and next by tenths, and to explain what amount of lap answers to a given expansion, and what expansion follows the use of a given proportion of lap. The mean pressure of the steam throughout the stroke, with different initial pressures of steam and different rates of expansion, or, in other words, the equivalent constant pressure that would be exerted throughout the stroke if such a pressure were substituted for the varying pressures to which the piston is in reality subjected, are exhibited in the following tables, in one of which the pressures are those which would ensue if the expansion took place during so many eighths of the stroke, and in the other during so many tenths of the stroke:—

MEAN PRESSURE OF STEAM AT DIFFERENT DENSITIES AND RATES OF EXPANSION.

The column headed 0, contains the Initial Pressure in lbs., and the remaining columns contain the Mean Pressure in lbs., with different amounts of Expansion.

Proportion of the Stroke through which Expansion takes place.							
0	$\frac{1}{8}$	$\frac{2}{8}$	$\frac{3}{8}$	$\frac{4}{8}$	$\frac{5}{8}$	$\frac{6}{8}$	$\frac{7}{8}$
3	2·96	2·89	2·75	2·53	2·22	1·789	1·154
4	3·95	3·85	3·67	3·38	2·96	2·386	1·539
5	4·948	4·818	4·593	4·232	3·708	2·982	1·924
6	5·937	5·782	5·512	5·079	4·450	3·579	2·309
7	6·927	6·746	6·431	5·925	5·241	4·175	2·694
8	7·917	7·710	7·350	6·772	5·934	4·772	3·079
9	8·906	8·673	8·268	7·618	6·675	5·368	3·463
10	9·896	9·637	9·187	8·465	7·417	5·965	3·848
11	10·885	10·601	10·106	9·311	8·159	6·561	4·233
12	11·875	11·565	10·925	10·158	8·901	7·158	4·618
13	12·865	12·528	11·943	11·004	9·642	7·754	5·003
14	13·854	13·492	12·862	11·851	10·384	8·531	5·388
15	14·844	14·456	13·781	12·697	11·126	8·947	5·773
16	15·834	15·420	14·700	13·544	11·868	9·544	6·158
17	16·823	16·383	15·618	14·390	12·609	10·140	6·542
18	17·813	17·347	16·537	15·237	13·351	10·737	6·927
19	18·702	18·311	17·448	16·803	14·093	11·333	7·312
20	19·792	19·275	18·375	17·970	14·835	11·930	7·697
25	24·740	24·093	22·968	21·162	18·543	14·912	9·621
30	29·688	28·912	27·562	25·395	22·252	17·895	11·546
35	34·636	33·731	33·156	29·627	25·961	20·877	13·470
40	39·585	38·550	36·750	33·860	29·670	23·860	15·395
45	44·533	43·368	41·343	38·092	33·378	26·842	17·319
50	49·481	48·187	45·937	42·325	37·067	29·825	19·243

MEAN PRESSURE OF STEAM AT DIFFERENT DENSITIES AND RATES OF EXPANSION.

The column headed 0 contains the Initial Pressure in lbs., and the remaining columns contain the Mean Pressure in lbs., with different amounts of Expansion.

	Proportion of the Stroke through which Expansion takes place.								
0	$\frac{1}{10}$	$\frac{2}{10}$	$\frac{3}{10}$	$\frac{4}{10}$	$\frac{5}{10}$	$\frac{6}{10}$	$\frac{7}{10}$	$\frac{8}{10}$	$\frac{9}{10}$
3	2·980	2·930	2·830	2·710	2·539	2·299	1·981	1·668	0·990
4	3·974	3·913	3·780	3·614	3·386	3·065	2·642	2·087	1·320
5	4·968	4·892	4·725	4·518	4·232	3·832	3·303	2·609	1·651
6	5·961	5·870	5·670	5·421	5·079	4·598	3·963	3·130	1·981
7	6·955	6·848	6·615	6·325	5·925	5·364	4·624	3·652	2·311
8	7·948	7·827	7·560	7·228	6·772	6·131	5·284	4·174	6·641
9	8·942	8·805	8·505	8·132	7·618	6·897	5·945	4·696	2·971
10	9·936	9·784	9·450	9·036	8·465	7·664	6·606	5·218	3·302
11	10·929	10·762	10·395	9·939	9·311	8·430	7·266	5·739	3·632
12	11·923	11·740	11·340	10·843	10·158	9·196	7·927	6·261	3·962
13	12·856	12·719	12·285	11·746	10·994	9·963	8·587	6·783	4·292
14	13·910	13·967	13·230	12·650	11·851	10·729	9·248	7·305	4·622
15	14·904	14·676	14·175	13·554	12·697	11·496	9·909	7·827	4·953
16	15·897	15·654	15·120	14·457	13·544	12·262	10·569	8·348	5·283
17	16·891	16·632	16·065	15·361	14·051	13·028	11·230	8·870	5·613
18	17·884	17·611	17·010	16·264	15·237	13·795	11·890	9·392	5·944
19	18·878	18·589	17·955	17·168	16·083	14·561	12·551	9·914	6·273
20	19·872	19·568	18·900	18·072	16·930	15·328	13·212	10·436	6·600
25	24·840	24·460	23·625	22·590	21·162	19·100	16·515	13·040	8·255
30	29·808	29·352	28·350	27·108	25·395	22·992	19·818	15·654	9·906
35	34·776	34·244	33·075	31·626	29·627	26·824	23·121	18·263	11·557
40	39·744	39·136	37·800	36·144	33·860	30·656	26·224	20·872	13·208
45	44·912	44·028	42·525	40·662	38·092	34·888	29·727	23·481	14·859
50	49·680	48·920	47·250	45·180	42·325	38·320	33·030	26·090	16·510

Example.—If steam be admitted to the cylinder at a pressure of 3 lbs. per square inch, and be suffered to expand during $\frac{1}{10}$th of the stroke, the mean pressure during the whole stroke will be 2·96 lbs. per square inch. In like manner, if steam at the pressure of 3 lbs. per square inch were cut off after the piston had gone through the $\frac{1}{5}$th of the stroke, leaving the steam to expand through the remaining $\frac{4}{5}$ths, the mean pressure during the whole stroke would be 1·154 lbs. per square inch.

RELATIONS BETWEEN THE LAP OF THE VALVE AND THE AMOUNT OF EXPANSION.

The rules for determining the relations between the lap of the valve and the amount of the expansion are as follows:—

TO FIND HOW MUCH LAP MUST BE GIVEN ON THE STEAM SIDE, IN ORDER TO CUT THE STEAM OFF AT ANY GIVEN PART OF THE STROKE.

RULE.—*From the length of the stroke of the piston subtract the length of that part of the stroke that is to be made before the steam is cut off. Divide the remainder by the length of the stroke of the piston, and extract the square root of the quotient. Multiply the square root thus found by half the length of the stroke of the valve, and from the product take half the lead, and the remainder will be the amount of lap required.*

TO FIND AT WHAT PART OF THE STROKE ANY GIVEN AMOUNT OF LAP ON THE STEAM SIDE WILL CUT OFF THE STEAM.

RULE.—*Add the lap on the steam side to the lead: divide the sum by half the length of stroke of the valve. In a table of natural sines find the arc whose sine is equal to the quotient thus obtained. To this arc add* 90°, *and from the sum of these two arcs subtract the arc whose cosine is equal to the lap on the steam side divided by half the stroke of the valve. Find the cosine of the remaining arc, add* 1 *to it, and multiply the sum by half the stroke of the piston, and the product is the length of that part of the stroke that will be made by the piston before the steam is cut off.*

TO FIND HOW MUCH BEFORE THE END OF THE STROKE THE EXHAUSTION OF THE STEAM IN FRONT OF THE PISTON WILL BE CUT OFF.

RULE.—*To the lap on the steam side add the lead, and divide the sum by half the length of the stroke of the valve. Find the arc whose sine is equal to the quotient, and add* 90° *to it. Divide the lap on the exhausting side by half the stroke of the valve, and find the arc whose cosine is equal to the quotient. Subtract this arc from the one last obtained, and find the cosine of the remainder. Subtract this cosine from*

2, and multiply the remainder by half the stroke of the piston. The product is the distance of the piston from the end of the stroke when the exhaustion is cut off.

TO FIND HOW FAR THE PISTON IS FROM THE END OF ITS STROKE, WHEN THE STEAM THAT IS PROPELLING IT BY EXPANSION IS ALLOWED TO ESCAPE TO THE CONDENSER.

RULE.—*To the lap on the steam side add the lead; divide the sum by half the stroke of the valve, and find the arc whose sine is equal to the quotient. Find the arc whose cosine is equal to the lap on the exhausting side, divided by half the stroke of the valve. Add these two arcs together, and subtract* 90°. *Find the cosine of the residue, subtract it from* 1, *and multiply the remainder by half the stroke of the piston. The product is the distance of the piston from the end of its stroke, when the steam that is propelling it is allowed to escape to the condenser.*

NOTE.—*In using these rules all the dimensions are to be taken in inches, and the answers will be found in inches also.*

It will readily be perceived from a consideration of these rules that—supposing there is no lead—the point of the stroke at which the steam is cut off is determined by the proportion which the lap on the steam side bears to the stroke of the valve. Whatever the absolute dimensions of the lap may be, therefore, it will follow that, in every case in which it bears the same ratio to the stroke of the valve, the steam will be cut off at the same point of the stroke.

As some of the foregoing rules are difficult to be worked out by persons unacquainted with trigonometry, it will be convenient to collect the principal results into tables, which may be applied without difficulty to the solution of any particular example. This accordingly has been done in the three following tables, the mode of using which it will now be proper to explain.

I.—PROPORTION OF LAP REQUIRED TO ACCOMPLISH VARIOUS DEGREES OF EXPANSION.

Distance of the piston from the termination of its stroke, when the steam is cut off, in parts of the length of its stroke	$\frac{8}{24}$ or $\frac{1}{3}$	$\frac{7}{24}$	$\frac{6}{24}$ or $\frac{1}{4}$	$\frac{5}{24}$	$\frac{4}{24}$ or $\frac{1}{6}$	$\frac{3}{24}$ or $\frac{1}{8}$	$\frac{2}{24}$ or $\frac{1}{12}$	$\frac{1}{24}$
Lap on the steam side of the valve, in decimal parts of the length of its stroke.	·289	·270	·250	·228	·204	·177	·144	·102

Example.—In the first line of the first table will be found eight different parts of the stroke of the piston designated; and directly below each, in the second line, is given the quantity of lap requisite to cause the steam to be cut off at that particular part of the stroke. The different amounts of the lap are given in the second line in decimal parts of the length of the stroke of the valve; so that, to get the quantity of lap corresponding to any of the given degrees of expansion, it is only necessary to take the decimal in the second line, which stands under the fraction in the first, that marks that degree of expansion, and multiply that decimal by the length we intend to make the stroke of the valve. Thus suppose we have an engine in which we wish to have the steam cut off when the piston is a quarter of the length of its stroke from the end of it, we look in the first line of the table, and we shall find in the third column from the left, $\frac{1}{4}$. Directly under that, in the second line, we have the decimal, ·250. Suppose that we consider that 18 inches will be a convenient length for the stroke of the valve, we multiply the decimal ·250 by 18, which gives $4\frac{1}{2}$. Hence we learn, that with an 18-inch stroke for the valve, $4\frac{1}{2}$ inches of lap on the steam side will cause the steam to be cut off when the piston has still a quarter of its stroke to perform.

Half the stroke of the valve should always be *at least* equal to the lap on the steam side added to the breadth* of the port; consequently, as the lap in this case must be $4\frac{1}{2}$ inches, and as

* By the 'breadth' of the port, is meant its dimensions in the direction of the valve's motion: in short, its perpendicular depth when the cylinder is upright.

half the stroke of the valve is 9 inches, the efficient breadth of the port cannot be more than 9 — 4½ = 4½ inches, since half of it is covered over by the lap. If this breadth of port is not sufficient to give the required area to let the steam in and out, we must increase the stroke of the valve; by which means we shall get both the lap and the breadth of the port proportionally increased. Thus, if we make the length of valve-stroke 20 inches, we shall have for the lap ·250 × 20 = 5 inches, and for the breadth of the port 10 — 5 = 5 inches.

This table, as we have already intimated, is computed on the supposition that the valve is to have no lead; but, if it is to have lead, all that is necessary is to subtract half the proposed lead from the lap found from the table, and the remainder will be the proper quantity of lap to give to the valve. Suppose that, in the last example, the valve was to have ½ inch of lead, we should subtract ¼ inch from the 5 inches, found for the lap by the table. This would leave 4¾ inches for the quantity of lap that the valve ought to have.

II.—LAP IN INCHES REQUIRED ON THE STEAM SIDE OF THE VALVE TO CUT THE STEAM OFF AT ANY OF THE UNDER-NOTED PARTS OF THE STROKE.

Length of stroke of the valve in inches.	Proportion of the stroke at which the steam is cut off.							
	$\frac{1}{3}$	$\frac{7}{24}$	$\frac{1}{4}$	$\frac{5}{24}$	$\frac{1}{6}$	$\frac{1}{8}$	$\frac{1}{12}$	$\frac{1}{24}$
24	6·94	6·48	6·00	5·47	4·90	4·25	3·47	2·45
23½	6·79	6·34	5·88	5·36	4·79	4·16	3·39	2·39
23	6·65	6·21	5·75	5·24	4·69	4·07	3·32	2·34
22½	6·50	6·07	5·62	5·13	4·59	3·98	3·25	2·29
22	6·36	5·94	5·50	5·02	4·49	3·89	3·13	2·24
21½	6·21	5·80	5·38	4·90	4·39	3·80	3·10	2·19
21	6·07	5·67	5·25	4·79	4·28	3·72	3·03	2·14
20½	5·92	5·53	5·12	4·67	4·18	3·63	2·96	2·09
20	5·78	5·40	5·00	4·56	4·08	3·54	2·89	2·04
19½	5·64	5·26	4·87	4·45	3·98	3·45	2·82	1·99
19	5·49	5·13	4·75	4·33	3·88	3·36	2·74	1·94
18½	5·34	4·99	4·62	4·22	3·77	3·27	2·67	1·88
18	5·20	4·86	4·50	4·10	3·67	3·19	2·60	1·83
17½	5·06	4·72	4·37	3·99	3·57	3·10	2·53	1·78
17	4·91	4·59	4·25	3·88	3·47	3·01	2·45	1·73
16½	4·77	4·45	4·12	3·76	3·36	2·92	2·38	1·68
16	4·62	4·32	4·00	3·65	3·26	2·83	2·31	1·63
15½	4·48	4·18	3·87	3·53	3·16	2·74	2·24	1·58

TABLE—*Continued.*

Length of stroke of the valve in inches.	Proportion of the stroke at which the steam is cut off.							
	$\frac{1}{3}$	$\frac{7}{24}$	$\frac{1}{4}$	$\frac{5}{24}$	$\frac{1}{6}$	$\frac{1}{8}$	$\frac{1}{12}$	$\frac{1}{24}$
15	4·33	4·05	3·75	3·42	3·06	2·65	2·16	1·53
14½	4·19	3·91	3·62	3·31	2·96	2·57	2·09	1·48
14	4·05	3·78	3·50	3·19	2·86	2·48	2·02	1·43
13½	3·90	3·64	3·37	3·08	2·75	2·39	1·95	1·37
13	3·76	3·51	3·25	2·96	2·65	2·30	1·88	1·32
12½	3·61	3·37	3·12	2·85	2·55	2·21	1·80	1·27
12	3·47	3·24	3·00	2·74	2 45	2·12	1·73	1·22
11½	3·32	3·10	2·87	2·62	2·35	2·03	1 66	1·17
11	3·18	2·97	2·75	2·51	2·24	1·95	1·58	1·12
10½	3·03	2·83	2·62	2·39	2·14	1·86	1·51	1·07
10	2·89	2·70	2·50	2·28	2·04	1·77	1·44	1·02
9½	2·65	2·56	2·37	2·17	1·93	1·68	1·32	·96
9	2·60	2·43	2·25	2·05	1·84	1·59	1·30	·92
8½	2·46	2·29	2·12	1·94	1·73	1·50	1·23	·86
8	2·31	2·16	2·00	1·82	1·63	1·42	1·15	·81
7½	2·16	2·02	1·87	1·71	1·53	1·33	1·08	·76
7	2·02	1·89	1·75	1·60	1·43	1·24	1·01	·71
6½	1·88	1·75	1·62	1·48	1·32	1·15	·94	·66
6	1·73	1·62	1·50	1·37	1·22	1·06	·86	·61
5½	1·58	1·48	1·37	1.25	1·12	·97	·79	·56
5	1·44	1·35	1·25	1·14	1·02	·88	·72	·51
4½	1·30	1·21	1·12	1·03	·92	·80	·65	·46
4	1·16	1·08	1·00	·91	·82	·71	·58	·41
3½	1·01	·94	·87	·80	·71	·62	·50	·35
3	·86	·81	·75	·68	·61	·53	·44	·30

The above table is an extension of tho first, for the purpose of obviating, in most cases, the necessity of even the very small degree of trouble required in multiplying the stroke of the valve by one of the decimals in the first table. The first line of the second table consists, as in the first table, of eight fractions, indicating the various parts of the stroke at which the steam may be cut off. The first column on the left hand consists of various numbers that represent the different lengths that may be given to the stroke of the valve, diminishing by half inches from 24 inches to 3 inches. Suppose that we wish the steam to be cut

off at any of the eight parts of the stroke indicated in the first line of the table (say at $\frac{1}{8}$ from the end of the stroke), we find $\frac{1}{8}$ at the top of the 6th column from the left. We next look for the proposed length of stroke of the valve (say 17 inches) in the first column on the left. From 17, in that column, we run along the line towards the right, and in the sixth column, and directly under the $\frac{1}{8}$ at the top, we find 3·47, which is the amount of lap required in inches to cause the steam to be cut off at $\frac{1}{8}$ from the end of the stroke, if the valve has no lead. If we wish to give it lead (say $\frac{1}{4}$ inch), we subtract the half of that, or $\frac{1}{8}$=·125 inch, from 3·47, and we have 3·47—125=3·345 inches, the quantity of lap that the valve should have.

To find the greatest efficient breadth that we can give to the port in this case, we have, as before, half the length of stroke, $8\frac{1}{2}$—3·345=5·155 inches, which is the greatest efficient breadth we can give to the port with this length of stroke. It is scarcely necessary to observe that it is not at all essential that the port should be so broad as this; indeed, where great length of stroke in the valve is not inconvenient, it is always an advantage to make it travel further than is just necessary to make the port open fully; because, when it travels further, both the exhausting and steam ports are more quickly opened, so as to allow greater freedom of motion to the steam.

The manner of using this table is so simple, that we need not trouble ourselves with more examples, and may pass on, therefore, to explain the use of the third table.

Suppose that the piston of a steam-engine is making its downward stroke, that the steam is entering the upper part of the cylinder by the upper steam port, and escaping from below the piston by the lower exhausting port; if, as is generally the case, the slide-valve has some lap on the steam side, the upper port will be closed before the piston gets to the bottom of the stroke, and the steam above then acts expansively, while the communication between the bottom of the cylinder and the condenser still continues open, to allow any vapour from the condensed water in the cylinder, or any leakage past the piston, to escape into the condenser; but, before the piston gets to the

bottom of the cylinder, this passage to the condenser will also be cut off by the valve closing the lower port. Soon after the lower port is thus closed, the upper port will be opened towards the condenser, so as to allow the steam that has been acting expansively to escape. Thus, before the piston has completed its stroke, the propelling power is removed from behind it, and a resisting power is opposed before it, arising from the vapour in the cylinder, which has no longer any passage open to the condenser. It is evident, that if there is no lap on the exhausting side of the valve, the exhausting port before the piston will be closed, and the one behind it opened, at the same time; but, if there is any lap on the exhausting side, the port before the piston will be closed before that behind it is opened; and the interval between the closing of the one and the opening of the other, will depend on the quantity of lap on the exhausting side of the valve. Again, the position of the piston in the cylinder, when these ports are closed and opened respectively, will depend on the quantity of lap that the valve has on the steam side. If the lap is large enough to cut the steam off when the piston is yet a considerable distance from the end of its stroke, these ports will be closed and opened at a proportionably early part of the stroke; and in the case of engines moving at a high speed, it has been found that great benefit is obtained from allowing the steam to escape before the end of the stroke.

The third table is intended to show the parts of the stroke where, under any given arrangement of slide valve, the eduction ports close and open respectively, so that thereby the engineer may be able to estimate how much, if any, of the efficiency he loses, while he is trying to add to the power of the steam by increasing the expansion. In this table there are eight columns marked A, standing over eight columns marked B, and at the heads of these columns are eight fractions as before, representing so many different parts of the stroke at which the steam may be supposed to be cut off.

The columns marked A express *the distance of the piston—in parts of its stroke—from the end of the stroke when the eduction port before it is shut*, and the columns marked B, and which stand immediately under the columns marked A, express *the distance of the piston from the end of its stroke when the exhausting port behind it is opened*—also measured in parts of the stroke.*

III.—PROPORTION OF THE STROKE AT WHICH THE EDUCTION PORT IS SHUT AND OPENED.

Lap on the eduction side of the valve, in parts of the length of its stroke.	Proportion of the stroke at which the steam is cut off.							
	$\frac{1}{3}$	$\frac{7}{24}$	$\frac{1}{4}$	$\frac{5}{24}$	$\frac{1}{6}$	$\frac{1}{8}$	$\frac{1}{12}$	$\frac{1}{24}$
	A	A	A	A	A	A	A	A
1-8th	·178	·161	·143	·126	·109	·093	·074	·053
1-16th	·130	·118	·100	·085	·071	·058	·043	·027
1-32nd	·113	·101	·085	·069	·053	·043	·033	·024
0	·092	·082	·067	·055	·043	·033	·022	·011
	B	B	B	B	B	B	B	B
1-8th	·033	·026	·019	·012	·008	·004	·001	·001
1-16th	·060	·052	·040	·030	·022	·015	·008	·002
1-32nd	·073	·066	·051	·042	·033	·023	·013	·004
0	·092	·082	·067	·055	·044	·033	·022	·011

Suppose we have an engine in which the slide valve is made to cut the stem off when the piston is 1-3rd from the end of its

* In locomotive and other fast-moving engines it is very important to open the eduction passage before the end of the stroke, so as to give more time for the steam to escape, and in locomotive valves the lap of the valve is usually made a little over ¼th of the travel, and the lead is usually made $\frac{1}{24}$th of the travel. In engines moving slowly the same necessity for an early eduction does not exist, and in such engines there will be a loss from opening the eduction much before the end of the stroke, as the moving pressure urging the piston is thus removed before the stroke terminates. When the valve is closed before the piston previously to the end of the stroke, the attenuated vapour in the cylinder will be compressed, and sometimes the compression will be carried so far that the pressure resisting the piston at the end of the stroke will exceed the pressure of the steam in the boiler. The indicator diagram will in such cases appear with a loop at its upper corner, which shows that the pressure before the end of the stroke exceeds the pressure of the steam, and that the first effect of opening the communication between the cylinder and the boiler is to enable the cylinder to discharge its highly compressed vapour backward into the boiler. The act of compressing the steam is what is called *cushioning* and in all ordinary diagrams this action may be more or less perceived.

stroke, and that the lap on the eduction or exhausting side of the valve is 1-8th of the whole length of its stroke. Let the stroke of the piston be 6 feet, or 72 inches. We wish to know when the exhausting port before the piston will be closed, and when the one behind it will be opened. At the top of the left-hand column marked A, the given degree of expansion (1-3rd) is given, and in the extreme left column we have at the top the given amount of lap (1-8th). Opposite the 1-8th in the first column, marked A, we have ·178, and in the first column, marked B, ·033, which decimals, multiplied respectively by 72, the length of the stroke, will give the required positions of the piston: thus $72 \times \cdot 178 = 12\cdot 8$ inches = distance of the piston from the end of the stroke when the exhaustion-port *before* the piston is shut: and $72 \times \cdot 033 = 2\cdot 38$ inches = distance of the piston from the end of its stroke when the exhausting-port *behind* it is opened.

To take another example. Let the stroke of the valve be 16 inches, the lap on the exhausting side $\frac{1}{2}$ inch, the lap on the steam side $3\frac{1}{4}$ inches, and the length of the stroke of the piston 60 inches. It is required to ascertain all the particulars of the working of this valve. The lap on the exhausting side is evidently $\frac{1}{32}$ of the length of the valve stroke. Then, looking at 16 in the left-hand column of the table in page 190, we find in the same horizontal line, 3·26, or very nearly $3\frac{1}{4}$, under $\frac{1}{6}$ at the head of the column, thus showing that the steam will be cut off at one-sixth from the end of the stroke. Again, under $\frac{1}{6}$ at the head of the sixth column from the left in the table in page 194, and in a line with $\frac{1}{32}$ in the left-hand column, we have ·053 under A, and ·033 under B. Hence, $\cdot 053 \times 60 = 3\cdot 18$ inches = distance of the piston from the end of its stroke when the exhausting-port before it is shut, and $\cdot 033 \times 60 = 1\cdot 98$ inches = distance of the piston from the end of its stroke when the exhausting-port behind it is opened. If in this valve the lap on the exhausting side were increased say to 2 inches or $\frac{1}{8}$ of the stroke, the effect would be to cause the port before the valve to be shut sooner in the proportion of ·109 to ·053, and the port behind it later in the proportion of ·008 to ·003. Whereas, if the lap on

the exhausting side were removed entirely, the port before the piston would be shut and that behind it opened at the same time. The distance of the piston from the end of its stroke at that time would be $\cdot 043 \times 60 = 2 \cdot 58$ inches.

An inspection of the third table shows us the effect of increasing the expansion by the slide valve in augmenting the loss of power occasioned by the imperfect action of the eduction passages. Referring to the bottom line of the table, we see that the eduction passage before the piston is closed, and that behind it opened, thus destroying the whole moving power of the engine, when the piston is ·092 from the end of its stroke, the steam being cut off at $\frac{1}{4}$ from the end. Whereas if the steam is only cut off at $\frac{1}{24}$ from the end of the stroke, the moving power is not withdrawn till only ·011 of the stroke remains uncompleted. It will also be observed that increasing the lap on the exhausting side has the effect of retaining the action of the steam longer *behind* the piston, but it at the same time causes the eduction port *before* it to be closed sooner.

A very cursory examination of the action of the slide valve is sufficient to show that the lap on the steam side should always be greater than on the eduction side. If they were equal, the steam would be admitted on one side of the piston at the same time that it was allowed to escape from the other; but universal experience has shown that when this is the case a very considerable part of the power of the engine is destroyed by the resistance opposed to the piston, by the escaping steam not getting away to the condenser with sufficient rapidity. Hence we see the necessity of the lap on the eduction side being always less than the lap on the steam side; and the difference should be the greater the higher the velocity of the piston is intended to be, because the quicker the piston moves, the passage for the waste steam requires to be the larger, so as to admit of its getting away to the condenser with as great rapidity as possible. In locomotive or other engines, where it is not wished to expand the steam in the cylinder at all, the slide valve is sometimes made with very little lap on the steam side; and in these circumstances, in order to get a sufficient difference between the lap on the steam

and the eduction sides of the valve, it may be necessary not only to take away all the lap on the eduction side, but to take off still more, so as to cause both eduction passages to be, in some degree, open, when the valve is at the middle of its stroke. This, accordingly, is sometimes done in such circumstances as we have described; but, when there is a considerable amount of lap on the steam side, this plan of taking *more than all* the lap off the eduction side ought never to be resorted to, as it can serve no good purpose, and will materially increase an evil we have already explained: viz., the opening of the eduction port behind the piston before the stroke is nearly completed. In the case of locomotive or other engines moving rapidly, it is very conducive to efficiency to begin the eduction before the end of the stroke, as the piston moves slowly at that time; and a very small amount of travel in the piston at that point corresponds to a considerable additional time given for the accomplishment of the eduction. The tables apply equally to the common short-slide three-ported valves, and to the long D valves.

The extent to which expansion can be carried conveniently by means of lap upon the valve is about one-third of the stroke; that is, the valve may be made with so much lap that the steam will be cut off when one-third of the stroke has been performed, leaving the residue to be accomplished by the agency of the expanding steam; but if much more lap be put on than answers to this amount of expansion a distorted action of the valve will be produced, which will impair the efficiency of the engine. By the use of the link motion, however, much of this distorted action can be compensated. If a farther amount of expansion than this is wanted, where the link motion is not used, it may be attained by wire-drawing the steam, or by so contracting the steam passage that the pressure within the cylinder must decline when the speed of the piston is accelerated, as it is about the middle of the stroke. Thus, for example, if the valve be so made as to shut off the steam by the time two-thirds of the stroke have been performed, and the steam be at the same time throttled in the steam pipe, the full pressure of the steam within the cylinder cannot be maintained except near the beginning of the stroke,

where the piston travels slowly; for as the speed of the piston increases, the pressure necessarily subsides, until the piston approaches the other end of the cylinder, where the pressure would rise again but that the operation of the lap on the valve by this time has had the effect of closing the communication between the cylinder and steam pipe, so as to prevent more steam from entering. By throttling the steam, therefore, in the manner here indicated, the amount of expansion due to the lap may be doubled, so that an engine with lap enough upon the valve to cut off the steam at two-thirds of the stroke, may, by the aid of wire-drawing, be virtually rendered capable of cutting off the steam at one-third of the stroke.

The Link Motion.—The rules and proportions here given, are equally applicable, whether the valve is moved by a single eccentric, or by the arrangement called the *link motion*, and which has now been very generally introduced into steam engines. In the link motion there are two eccentrics, one of which is set so as to drive the engine in one direction, and the other is set so as to drive the engine in the opposite direction, and when the stud in communication with the valve is shifted to one end of the link, that stud partakes of the motion of the forward eccentric, whereas, when it is placed at the other end of the link, it partakes of the motion of the backing eccentric. A common length of the link is three times the stroke of the valve. Generally the stud is placed either at one end of the link or the other, not by moving the stud but by moving up or down the link; and it is better that this movement should be vertical, and be made by means of a screw, than that the movement should be produced by a lever travelling through an arc. The point of suspension should be near the middle of the link where its motion is the least. The link connects together the ends of the two eccentric rods, and is sometimes made straight, but generally curved, the curvature being an arc of such radius that the link may be raised up or down without sensibly altering the position of the stud with which the valve is connected. But the link should be convex or concave towards the valve, according as the eccentric rods are crossed or uncrossed when the throw

of the eccentrics are turned towards the link. In the case of new arrangements of engine, it is advisable to make a skeleton model in paper of the link and its connexions, so as to obtain full assurance that it works in the best way.

VELOCITY OF WATER IN RIVERS, CANALS, AND PIPES, ANSWERABLE TO ANY GIVEN DECLIVITY.

When a river runs in its bed with a uniform velocity, the gravitation of the water down the inclined plane of the bed, is just balanced by the friction. In the case of canals, culverts, and pipes, precisely the same action takes place. The head of water, therefore, which urges the flow through a pipe, may be divided into two parts, of which one part is expended in giving to the water its velocity, and the other part is expended in overcoming the friction. If water be let down an inclined shoot, its motion at the top will be slow, but will go on accelerating until the friction generated by the high velocity will just balance the gravitation down the plane, and after this point has been attained, the shoot may be made longer and longer without any increase in the velocity of the water taking place. In the case of a ball falling in the air or in water, the velocity of the descent will go on increasing until the resistance becomes so great as to balance the weight; and, in the case of a steam vessel propelled through the water, the speed will go on increasing until the resistance just balances the tractive force exerted by the engines, when the speed of the vessel will become uniform. In all these cases the resistance increases with the speed; and as the speed increases, the resistance increases also, until it becomes equal to the accelerating force.

The resistance which is occasioned by the friction of water increases more rapidly than the increase of the velocity. In other words, there will be more than twice the friction with twice the velocity. It is found by experiment that the friction of water increases nearly as the square of its velocity, so that there will be about four times the resistance with twice the speed. This law, however, is only approximately correct. The

friction does not increase quite so rapidly at high velocities as the square of the speed.

It is easy to determine the friction in lbs. per square foot of any given pipe or conduit, with any given velocity of the stream, when the slope or declivity of the surface of the water is known. For as the gravitation down the inclined plane of the conduit just balances the friction, the friction in the whole length of the conduit will be equal to the whole weight of the water in it, reduced in the same proportion as any other body descending an inclined plane. Thus, if the conduit be 2,000 feet long, and have 1 foot of fall in that length, the total friction will be equal to the total weight of the water divided by 2,000, and the friction per square foot will be equal to this 2000th part of the weight of the water divided by the number of square feet exposed to the water in the conduit. The friction will in all cases vary as the rubbing surface, or, what is the same thing, as the wetted perimeter As a cylindrical pipe has a less perimeter than any other form, it will occasion less resistance than any other form to water passing through it. In like manner, a canal or a ship with a semicircular cross section will have the minimum amount of friction.

The propelling power of flowing water being gravity, the amount of such power will vary with the magnitude of the stream; but the resisting power being friction, which varies with the amount of surface, or in any given length with the wetted perimeter, it will follow that the larger the area is relatively with the wetted perimeter, the less will be the resistance relatively with the propelling power, and the greater will be the velocity of the water with any given declivity. Now, as the circumference or perimeter of a pipe increases as the diameter, and the area as the square of the diameter, it is clear, that with any given head, water will run more swiftly through large pipes than through small; and in like manner with any given proportion of power to sectional area, large vessels will pass more swiftly than small vessels through the water. The sectional area of a pipe or canal divided by the wetted perimeter, is what is termed the *hydraulic mean depth*, and this depth is what would result if we suppose the perimeter to be bent out to a

straight line, and the sectional area to be spread evenly over it, so that each foot of the perimeter had its proper share of sectional area above it. The greater the hydraulic mean depth, the greater with any given declivity will be the velocity of the stream. With any given fall, therefore, deep and large rivers will run more swiftly than small and shallow ones. The hydraulic mean depth of a steam vessel will be the indicated power divided by the wetted perimeter of the cross section.

TO DETERMINE THE MEAN VELOCITY WITH WHICH WATER WILL FLOW THROUGH CANALS, ARTERIAL DRAINS, OR PIPES, RUNNING PARTLY OR WHOLLY FILLED.

RULE.—*Multiply the hydraulic mean depth in feet by twice the fall in feet per mile; take the square root of the product and multiply it by 55. The result is the mean velocity of the stream in feet per minute. This again multiplied by the sectional area in square feet gives the discharge in cubic feet per minute.*

Example.—What is the mean velocity of a river falling a foot in the mile, and of which the mean hydraulic depth is 8 feet?

Here $8 \times 2 = 16$, the square root of which is 4, and this multiplied by $55 = 220$, which will be the mean velocity of the stream in feet per minute.

In cylindrical pipes running full, the hydraulic mean depth is one-fourth of the diameter. For the hydraulic mean depth being the area divided by the wetted perimeter, it is $\frac{\cdot 7854 d^2}{3 \cdot 1416 d} = \frac{d}{4}$.*

* The surface, bottom, and mean velocities of rivers have fixed relations to one another. Thus, if the surface velocity in inches per second be denoted by V, the mean velocity will be $(V + 0{\cdot}5) - \sqrt{V}$ and the bottom velocity by $(V + 1) - 2\sqrt{V}$. With surface velocities therefore of 4, 16, 32, 64, and 100 inches per second, the corresponding mean velocities will be 2·5, 12·5, 26·8, 56·5, and 90·5 inches per second, and the corresponding bottom velocities will be 1, 9, 21·6, 49, and 81 inches per second.

The common rule for finding the number of cubic feet of water delivered each minute by a pipe of any given diameter is as follows:—*Divide* 4·72 *times the square root of the fifth power of the diameter of the pipe in inches by the square root of the quotient obtained by dividing the length of the pipe in feet by the head of water in feet.* Hawksley's rule for ascertaining the delivery in gallons per hour is as follows:—*Multiply* 15 *times the fifth power of the diameter of the pipe*

M. Prony has shown by a comparison of a large number of experiments that if H be the head in feet per mile required to balance the friction, V the velocity of the water through the pipe in feet per second, and D the diameter of the pipe in feet, then

$$H = \frac{2{\cdot}25V^2}{D}.$$

This equation is identical with that which has been used by Boulton and Watt in their practice for the last half century, and which is as follows:—

If l be the length of the main in miles, V the velocity of the water in the main in feet per second, D the diameter of the pipe in feet, and 2·25 a constant,

$$\text{then } \frac{2{\cdot}25lV^2}{D} = \text{feet of head due to friction.}$$

This equation put into words gives us the following Rule:—

TO DETERMINE THE HEAD OF WATER THAT WILL BALANCE THE FRICTION OF WATER RUNNING WITH ANY GIVEN VELOCITY THROUGH A PIPE OF A GIVEN LENGTH AND DIAMETER.

RULE.—*Multiply 2·25 times the length of the pipe in miles by the square of the velocity of the water in the pipe in feet per second, and divide the product by the diameter of the pipe in feet. The quotient is the head of water in feet that will balance the friction.*

The law indicated by this Rule is expressed numerically in the Tables on pp. 204, 205.

in inches by the head of water in feet, and divide the product by the length of the pipe in yards. Finally, extract the square root of the quotient, which gives the delivery in gallons per hour.

The annual rain-fall in England varies from 20 to 70 inches, the mean being 42 inches, and it is reckoned that about $\frac{8}{10}$ths of the rain-fall on any given area may be collected for storage. A cubic foot of water is about $6\frac{1}{4}$ gallons, and it is found in supplying towns with water that about on the average 16 gallons per head per day are required in ordinary towns, and 20 gallons per head per day in manufacturing towns, but the pipes should be large enough to convey twice this quantity. In the rainy districts of England collecting reservoirs should contain 120 days' supply, and in dry districts 200 days' supply. Service reservoirs are usually made to contain 8 days' supply. The mean daily evaporation in England is ·08 of an inch, and the loss from the overflow of storm water is reckoned to be about 10 per cent.

Explanation of the Tables.—The top horizontal row of figures represents either the diameter of a cylindrical pipe, or four times the area of any other shaped pipe divided by the circumference, or four times the area of the cross section of a canal, divided by the sum of all its sides, or bottom and sides, all being in inches.

The first vertical column indicates the slope of the pipe or canal, that is, the whole length of the pipe or canal, divided by the perpendicular fall.

Any number in any other column indicates the velocity, in inches per second, with which water would run through a pipe of such a diameter as the number at the head of such column expresses, having such a slope as that number in the first column expresses which is horizontally against such velocity.

Example 1.—With what velocity will water run through a pipe of 16 inches diameter, its length being 8,000 feet, and fall 16 feet? Here the slope manifestly is $8{,}000 \div 16 = 500$. Against 500 in the first column, and under 16, the diameter in the top row of figures, the number 29·8 is found, which is the velocity in inches per second.

Example 2.—With what velocity will water pass through a pipe of 21 inches diameter, having a slope of 900? 21 is not found in the head of the Table, in which case such a number must be found in the top row as will bear such proportion to 21 as some other two numbers in the top row bear to each other, and these latter numbers should be as near to 21 as they can be found.

In this case it will be seen that 18 is to 21 as 6 is to 7, or (for compliance with the indication just mentioned) rather as 12 to 14, or still better as 24 to 28. Then say as the velocity (against 900, the slope) under 24 is to 28 (28·7), so is the velocity under 18 (22·7) to that of 21 (viz. 24·7) the velocity in inches per second.

By the same process may the velocity for slopes be found or assigned, which are not to be found in the first column of the Table, proceeding with proportions found in the vertical column instead of the horizontal rows; the first vertical column

VELOCITY IN INCHES PER SECOND OF WATER FLOWING THROUGH PIPES WITH VARIOUS SLOPES AND DIAMETERS.

BY BOULTON, WATT & CO.

Slope or Length divided by Fall.	INTERNAL DIAMETERS OF THE PIPES IN INCHES.											
	1	2	3	4	5	6	7	8	9	10	12	14
10	63·3	96·0	121·	142·	161·	180·	193·	208·	221·	234·	255·	280·
20	41·8	63·4	80·	93·6	104·	118·	127·	137·	146·	154·	170·	185·
30	32·8	49·5	62·6	73·4	83·3	93·1	99·7	107·	114·	121·	133·	145·
40	27·5	41·5	52·5	61·5	69·8	78·0	83·7	90·1	95·8	103·	112·	121·
50	**23·9**	**36·3**	**45·7**	**53·7**	**60·9**	**68·0**	**73·0**	**78·7**	**83·5**	**88·4**	**97·5**	**106·**
60	21·6	32·7	41·2	48·3	54·8	61·2	65·7	70·8	75·3	79·8	87·8	95·4
70	19·6	29·7	37·5	44·0	49·8	55·7	59·7	64·4	68·5	72·5	80·0	86·6
80	18·1	27·4	34·7	40·7	46·1	51·5	55·3	59·5	63·3	67·1	74·0	80·2
90	16·9	25·6	32·4	37·9	43·0	48·0	51·5	55·5	59·0	62·5	69·0	74·8
100	**15·8**	**24·0**	**30·3**	**35·5**	**40·2**	**45·1**	**48·4**	**52·2**	**55·5**	**58·7**	**64·8**	**70·3**
200	10·5	16·0	20·2	23·7	26·8	30·0	32·2	34·7	36·9	39·0	43·1	46·7
300	8·43	12·7	16·1	18·6	21·4	23·8	25·6	27·7	29·4	30·6	34·3	37·7
400	7·11	10·8	13 6	15·9	18·1	20·2	21·6	23·3	24·8	26·3	29·0	31·4
500	**6·26**	**9·50**	**12·0**	**14·0**	**15·9**	**17·8**	**19·1**	**20·6**	**21·9**	**23·2**	**25·5**	**27·7**
600	5·64	8·57	10·8	12·7	14·3	16·1	17·2	18·6	19·7	20·9	23·0	25·0
700	5·17	7·85	9·90	11·6	13·2	14·7	15·8	17·0	18·1	19·2	21·1	22·9
800	4·81	7·30	9·21	10·8	12·2	13·7	14·7	15·8	16·8	17·8	19·6	21·3
900	4·50	6·83	8·62	10·1	11·4	12·8	13·7	14·8	15·7	16·6	18·3	19·9
1000	**4·25**	**6·45**	**8·15**	**9·54**	**10·8**	**12·1**	**12·9**	**14·0**	**14·8**	**15·7**	**17·3**	**18·8**
2000	2·88	4·37	5·52	6·48	7·33	8·2	8·77	9·48	10·1	10·6	11·7	12·7
3000	2·30	3·48	4·40	5·17	5·86	6·55	7·02	7·57	8·05	8·52	9·40	10·2
4000	1·96	2·97	3·75	4·40	4·98	5·57	5·97	6·44	6·84	7·25	8·00	8·66
5000	**1·73**	**2·62**	**3·31**	**3·88**	**4·40**	**4·92**	**5·28**	**5·69**	**6·04**	**6·40**	**7·06**	**7·66**
6000	1·57	2·38	3·00	3·52	3·99	4·45	4·79	5·15	5·44	5·80	6·40	6·95
7000	1·43	2·17	2·73	3·21	3·63	4·06	4·36	4·70	5·00	5·29	5·82	6·32
8000	1·32	2·01	2·53	2·97	3·46	3·76	4·03	4·35	4·62	4·90	5·40	5·85
9000	1·24	1·87	2·38	2·79	3·16	3·58	3·79	4·08	4·34	4·60	5·07	5·50
10000	**1·17**	**1·77**	**2·24**	**2·62**	**2·9**	**3·32**	**3·57**	**3·84**	**4·08**	**4·32**	**4·76**	**5·16**

being substituted in this case for the top row in the former case.

In all cases an addition must be made to the fall equal to that which would generate the existing velocity in a body falling freely by gravity. For instance, in the first case, to the fall of

VELOCITY IN INCHES PER SECOND OF WATER FLOWING THROUGH PIPES WITH VARIOUS SLOPES AND DIAMETERS.—(*Continued.*)

BY BOULTON, WATT & CO.

Slope or Length divided by Fall.	INTERNAL DIAMETERS OF THE PIPES IN INCHES.										
	16	18	20	24	28	32	36	40	60	80	100
10	301·	320·	338·	372·	403·	432·	459·	484·	597·	692·	775·
20	199·	211·	223·	245·	256·	285·	303·	320·	394·	456·	511·
30	155·	165·	175·	192·	208·	223·	238·	250·	309·	358·	400·
40	130·	138·	146·	161·	174·	187·	199·	210·	259·	300·	336·
50	**113·**	**121·**	**127·**	**140·**	**152·**	**163·**	**173·**	**183·**	**226·**	**261·**	**293·**
60	102·	109·	115·	127·	137·	147·	156·	165·	204·	236·	264·
70	93·2	99·0	104·	115·	124·	134·	142·	150·	185·	214·	240·
80	86·3	91·6	97·	106·	115·	123·	131·	139·	171·	198·	222·
90	80·5	85·5	90·4	99·4	107·	115·	122·	129·	159·	185·	207·
100	**75·5**	**80·2**	**85·0**	**93·5**	**102·**	**108·**	**115·**	**121·**	**150·**	**173·**	**194·**
200	50·2	53·3	56·4	62·0	67·2	72·1	76·7	80·8	99·5	115·	129·
300	39·9	42·5	45·0	49·5	53·6	57·5	61·1	64·4	79·5	92·0	105·
400	33·3	35·9	38·0	41·8	45·2	48·5	51·5	54·4	67·0	77·7	87·0
500	**29·8**	**31·6**	**33·4**	**36·8**	**39·8**	**42·8**	**45·5**	**47·9**	**59·0**	**68·4**	**76·7**
600	26·8	28·6	30·2	33·2	36·0	38·6	41·0	43·2	53·3	61·7	69·2
700	24·6	26·2	27·7	30·4	33·0	35·3	37·6	39·6	48·8	56·5	63·4
800	22·9	24·3	25·7	28·3	30·6	32·9	34·9	36·8	45·4	52·5	58·9
900	21·4	22·7	24·0	26·4	28·7	30·7	32·7	34·4	42·5	49·1	55·1
1000	**20·2**	**21·5**	**22·7**	**25·0**	**27·1**	**29·1**	**30·9**	**32·5**	**40·1**	**46·4**	**52·0**
2000	13·7	14·5	15·4	16·9	18·3	19·7	20·9	22·0	27·2	31·3	35·3
3000	10·9	11·6	12·3	13·5	14·6	15·7	16·7	17·6	21·7	25·2	28·2
4000	9·32	9·90	10·4	11·5	12·4	13·4	14·2	15·0	18·5	21·4	24·0
5000	**8·23**	**8·73**	**9·25**	**10·3**	**11·0**	**11·8**	**12·5**	**13·2**	**16·3**	**18·9**	**21·2**
6000	7·47	7·93	8·40	9·23	10·0	10·7	11·4	12·0	14·8	17·1	19·2
7000	6·80	7·22	7·65	8·40	9·10	9·76	10·3	10·9	13·5	15·6	17·5
8000	6·30	6·69	7·07	7·78	8·43	9·05	9·62	10·1	12·5	14·5	16·2
9000	5·91	6·28	6·64	7·30	7·92	8·50	9·02	9·52	11·7	13·05	15·2
10000	**5·55**	**5·91**	**6·25**	**6·87**	**7·45**	**7·98**	**8·50**	**8·93**	**11·0**	**12·7**	**14·3**

16 feet we must add the fall which would generate the velocity of 29·8 inches per second, namely, 1·15 inches, which will make the total fall 16 feet 1·15 inches that will be requisite to give such a velocity; but in such cases as this it is evident that the addition of this small fraction might have been disregarded.

In some cases Messrs. Boulton and Watt have employed the constant 1·82 instead of 2·25. Mr. Mylne's constant is 1·94; but some careful experiments made by him at the West Middlesex Waterworks, gave a constant as high as 2·62.

OTHER TOPICS OF THE THEORY OF STEAM-ENGINES.

It will not be necessary to extend these remarks by an investigation of the theory of the crank as an instrument for converting rectilinear into rotatory motion, since the idea, once widely prevalent, that there was a loss of power consequent upon its use, is now universally exploded. Neither will it be necessary to enter into any explanation of the structure of the numerous rotatory engines which have at different times been projected, since none of those engines are in common or beneficial operation. The proper dimensions of the cold water and feed pumps, the action of the fly-wheel in redressing irregularities of the motion of the engine, and other material points which might properly fall to be discussed under the head of the Theory of the Steam-Engine, and which have not already been treated of, will, for the sake of greater conciseness, be disposed of in the chapter on the Proportions of Steam-Engines, when these various topics must necessarily be considered. Nor is it deemed advisable here to recapitulate the rules for proportioning the various kinds of parallel motion, since parallel motions have now almost gone out of use, and since also any particular case of a parallel motion which has to be considered, can easily be resolved geometrically by drawing the parts on a convenient scale,—the principle of all parallel motions being that the versed sine of an arc, pointing in one direction, shall be compensated by an equal versed sine of an arc pointing in the opposite direction; and the effect of these opposite motions is to produce a straight line. In the case of the parallel motions sometimes employed in side-lever engines, and in which the attachment is made not to the cross-head but to the side-rod, it is only necessary to provide that the end of the bar connected to the side-rod shall move, not in a straight line, but in an arc, the versed sine of which is equal to the versed sine of the arc described by the point of at-

tachment on the side-rod. As the bottom of the side-rod is attached to the beam and the top to the cross-head, and as the bottom moves in an arc and the top in a straight line, it is clear that every intermediate point of the side-rod must describe an arc which will more and more approach to a straight line, or have a smaller and smaller versed sine, the nearer such point is to the top of the rod. By drawing down the side-rod at the end of the stroke, and also, at half stroke, the amount of deviation from the vertical at those positions, can easily be determined for any point in the length of the rod; and the point of attachment of the parallel bar has only to be such, and the length and travel of the radius crank has also to be such, that the end of the parallel bar attached to the side-rod shall describe an arc whose versed sine is equal to the deviation from the perpendicular, or, in other words, to the side-travel of that point of the side-rod at which the attachment is made. Since, then, the side-rod is guided at the bottom by the arc of the beam, and near the top by that less arc described by the end of the parallel bar, which answers to the supposition of the cross-head moving in a vertical line, the result is that the cross-head will be constrained to move in this vertical line; since only on that supposition can the two arcs already fixed be described.

The method of balancing the momentum of the moving parts of marine engines which I introduced in 1852 has now been very generally adopted; and the practice is found to be very useful in reducing the tremor and uneasy movements to which engines working at a high rate of speed are otherwise subject. Nearly all the engines now employed for driving the screw propeller are direct-acting engines, which necessarily work at a high rate of speed to give the requisite velocity of rotation to the screw shaft. The principle on which the balancing is effected is that of applying a weight to the crank or shaft, and when the piston and its connexions move in one direction the weight moves in the opposite with an equal momentum.

CHAPTER IV.

PROPORTIONS OF STEAM-ENGINES.

We now come to the question how we are to determine the proportions of steam-engines of every class.

The *nominal* power of a low pressure engine is determined by the diameter of the cylinder and length of the stroke, as follows:—

TO DETERMINE THE NOMINAL POWER OF A LOW PRESSURE ENGINE OF WATT'S CONSTRUCTION.

Rule.—*Multiply the square of the diameter of the cylinder in inches by the cube root of the stroke in feet, and divide the product by* 47. *The quotient is the nominal horse-power of the engine.*

Example 1.—What is the nominal power of a low pressure engine with a cylinder 64 inches diameter and 8-feet stroke?

Here $64 \times 64 = 4{,}096$, which multiplied by 2, the cube root of $8 = 8{,}192$ and $\div 47 = 174{\cdot}3$.

The nominal powers of engines of different sizes, both high pressure and low pressure, are given in the following tables:—

NOMINAL HORSE POWER OF HIGH PRESSURE ENGINES.

NOMINAL HORSE POWER OF LOW PRESSURE ENGINES.

Example 2.—What is the nominal power of a low pressure engine of 40 inches diameter of cylinder and 5-feet stroke.

Here $40 \times 40 = 1{,}600$, which multiplied by 1·71—which is the cube root of 5 very nearly—we get 2,736, which divided by 47 gives 58·21 as the nominal horse power.

The *actual* horse power of an engine is determinable by the application of an instrument to determine the amount of power it actually exerts. The mode of determining this will be explained hereafter. Meanwhile it may be repeated that an actual horse power is a dynamical unit capable of raising a load of 33,000 lbs. one foot high in each minute of time. The *nominal power of a high pressure engine* may be taken at three times that of a low pressure engine of the same size.

The assumed pressure in computing the nominal power of low pressure engines is 7 lbs. on each square inch of the piston, and the assumed pressure in computing the nominal power of high pressure engines is 21 lbs. on each square inch of the piston. The assumed speed of the piston varies with the length of stroke from 160 to 256 feet per minute, namely, for a 2 ft. stroke, 160 ft.; 2½ ft., 170; 3 ft., 180; 4 ft., 200; 5 ft., 215; 6 ft., 228; 7 ft., 245; and 8 ft., 256 feet per minute.

In point of fact, in all modern low pressure engines the unbalanced pressure of steam upon the piston is much more than 7 lbs., and in most modern high pressure engines the unbalanced pressure of steam upon the piston is much more than 21 lbs. The speed of the piston is also frequently much more than 256 feet per minute. In the case of screw engines the Admiralty employs a rule to determine the power, in which the old assumed pressure of 7 lbs. per square inch is retained, but in which the actual speed of piston is taken into account. This rule is as follows:—

ADMIRALTY RULE FOR DETERMINING THE NOMINAL POWER OF AN ENGINE.

RULE.—*Multiply the square of the diameter of the cylinder in inches by the speed of the piston in feet per minute, and divide by* 6,000. *The quotient is the nominal power.*

Example.—What is the power of an engine with a cylinder

of 42 inches diameter, and $3\frac{1}{2}$ feet stroke, and which makes 85 revolutions per minute?

Here $42 \times 42 = 1,764$. The length of a double stroke will be $3\frac{1}{2} \times 2 = 7$ feet, and as there are 85 revolutions or double strokes per minute, $85 \times 7 = 595$ will be the speed of the piston in feet per minute. Now $1,764 \times 595 = 1,049,580$, which, divided by $6,000 = 175$ horses power.

The area of the piston in circular inches, it will be recollected, is found by multiplying the diameter by itself. Thus a piston 50 inches diameter contains 50×50, or 2,500 circular inches. Now as every circular inch is ·7854 of a square inch, we must, in order to find the area of the piston in square inches, multiply the diameter by itself and by ·7854, which will give the area in square inches. Thus, $2,500 \times {\cdot}7854 = 1,963{\cdot}5$ square inches, which is the area in square inches of a piston 50 inches in diameter. The circumference of any circle is obtained by multiplying the diameter by 3·1416. Hence the length of a string or tape that will be required to encircle a piston 50 inches in diameter will be $50 \times 3{\cdot}1416 = 157{\cdot}08$ inches. The areas of pumps, pipes, safety-valves, and all other circular objects, is computed in the same way as the areas of circles or pistons. Some valves are annular valves, consisting not of a flat circular plate, but of a ring or annulus of a certain breadth. To compute the area of such a valve we must first compute the area of the outer circle, and then the area of the inner, and subtract the less from the greater, which will give the area of the annulus. So in like manner, in trunk engines, we must subtract the area of the trunk from the area of the piston.

GENERAL CONSIDERATIONS AND INSTRUCTIONS.

In proceeding to design an engine for any given purpose, the nominal power may either be fixed or the nominal power may be left indeterminate, and only the work be fixed which the engine has to perform. In the first case we have only to ascertain by the foregoing rules or tables what the dimensions of a cylinder are which correspond to the nominal power,

and we have then to make all the other parts of dimensions corresponding thereto, which we shall be enabled to do by the rules here laid down. Of course the engineer settles for himself some particular type of engine which he prefers to adopt as the one that is to govern his practice, *and any drawing of an engine of a given size or power is applicable to the construction of a similar engine of any other size or power by merely altering the scale of the drawing.* If, therefore, any engineer decides upon the class of land engine, paddle engine, or screw engine which he prefers to construct, and chooses to get a set of drawings of such engine on any given scale lithographed, such drawings will be applicable to all sizes and powers of that class of engine by altering the scale in the proportion rendered necessary by the enlarged or diminished diameter of the cylinder answerable to the required power. Thus, if we have a drawing of a marine engine of 32 inches diameter of cylinder and 4-feet stroke, made to the scale of $\frac{1}{8}$-inch to the foot, we may from such drawing construct a similar engine of 64 inches diameter and 8-feet stroke by merely altering the scale to one of $\frac{1}{4}$-inch to the foot, so that every part will in fact measure twice what it measured before. In order to make the same drawing applicable to any size of engine, whether large or small, we have only to divide the diameter of the cylinder into the number of parts that the cylinder is to have of inches, and then we may use the scale so formed for the scale of the drawing. Thus, if we wish the engine to have a cylinder of 30 inches diameter, we must divide the diameter of the cylinder as shown in the drawing into 30 equal parts, each of which will represent an inch, and of course any twelve of them will represent a foot. If we now measure any other part of the engine, such as the diameter of the air pump, diameter of crank shaft, or any other part by this scale, we shall find the proper dimensions of the part in question, If we wish to construct from the drawing an engine of 60 or 100 inches, and of corresponding stroke, we have only to divide the diameter of the cylinder into 60 or 100 equal parts, and use each of those parts as an inch of the scale, when the proper dimensions of all the parts will be at once obtained.

It will be needless to guard these remarks against the obvious exception that in case of very large and very small engines it will be proper to make such slight modifications in some of the details as will conduce to greater convenience in working or in construction. For instance, as the height and strength of a man are a given quantity, it will obviously not be proper in doubling the size of all the other parts to double the height of the starting handles, or even to double their strength. In the case of oscillating engines, again, with a crank in the intermediate shaft, it may be difficult to get a sound crank made in the case of very large engines, and some other expedient may have to be adopted. Again, in the case of very small engines, the flanges and bolts may require to be a little larger than the proportion derived from a drawing of large engines, and the valve chests of the feed pumps and other parts may be too small if made strictly to scale to get the hand into conveniently to clear them out. All such points however are matters of practical convenience, only to be determined by the thoughtfulness and experience of the engineer, and in nowise affect the main conclusion that a drawing of an engine of any one size will suffice for the construction of engines of other sizes by merely changing the scale. It will consequently save much trouble in drawing offices to have one certain type of engine of each kind lithographed in all its details, and then engines of all sizes may be made therefrom by adding the proper scale, and by marking upon the drawing the proper dimensions of each part in feet or inches—the measurements being taken from a table fixed once for all, either by computation or by careful measurement of the drawing with the different suitable scales. By thus systematising the work of the drawing office, labour may be saved and mistakes prevented.

It easy to understand the principle on which the main parts of an engine must be proportioned. We must in the first place have the requisite quantity of boiler surface to generate the steam, the requisite quantity of water sent into the boiler to keep up the proper supply, and the requisite quantity of cold water to condense the steam after it has given motion to the piston. In common boilers about 10 square feet of heating

surface will boil off a cubic foot of water in the hour, and this in the older class of engines was considered the equivalent of a horse power. At the atmospheric pressure, or with no load on the safety valve, a cubic inch of water makes about a cubic foot of steam; and at twice the atmospheric pressure, or with 15 lbs. per square inch on the safety valve, a cubic inch of water will make about half a cubic foot of steam. For every half cubic foot of such steam therefore abstracted from the boiler there must be a cubic inch of water forced into it. So if we take the latent heat of steam in round numbers at 1,000 degrees, and if the condensing water enters at 60°, and escapes at 100°, the condensing water has obviously received 40 degrees of heat, and it has received this from the steam having 1,000° of heat, and the 112° which the steam if condensed into boiling water would exceed the waste-water in temperature. It follows that in order to reduce the heat of the steam to 100° there must be 1,112° of heat extracted, and if the condensing water was only to be heated 1 degree, there would require to be 1,112 times the quantity of condensing water that there is water in the steam. Since, however, the water is to be heated 40°, there will only require to be one-fortieth of this, or about $\frac{1}{28}$th the quantity of injection water that there is water in the steam. These rough determinations will enable the principle to be understood on which such proportions are determined. The proportions of the condenser and of the air-pump were determined by Mr. Watt at one-eighth of the capacity of the cylinder. In more modern engines, and especially in marine engines where there are irregularities of motion, the air-pump is generally made a little larger than this proportion, and with advantage. The condenser is also generally made larger, and many engineers appear to consider that the larger the condenser is the better. Mr. Watt, however, found that when the condenser was made larger than one-eighth of the capacity of the engine the efficiency of the engine was diminished. The fly-wheel employed in land engines to control the irregularities of motion that would otherwise exist, is constructed on the principle that there shall be a revolving mass of such weight, and moving with such a velocity, as to

constitute an adequate reservoir of power to redress irregularities. It is found that in those cases where the most equable motion is required, it is proper to have as much power treasured up in the fly-wheel as is generated in 6 half-strokes, though in many cases the proportion is not more than half this. It is quite easy to tell what the weight and velocity of the fly-wheel must be to possess this power. When we know the area of the piston and the unbalanced pressure per sq. inch, we easily find the pressure urging it, and this pressure multiplied by the length of 6 half-strokes represents the amount of power which, in the most equable engines, the fly-wheel must possess. Thus, suppose that the pressure on the piston were a ton, and that the length of the cylinder were 5 feet, then in 6 half-strokes the space described by the piston would be 30 feet. The measure of the power therefore is 1 ton descending through 30 feet, and if there were any circumstance which limited the weight of the fly-wheel to 1 ton, then the velocity of the rim—or more correctly of the centre of gyration—must be equal to that which any heavy body would have at the end of the descent by falling from a height of 30 feet, and which velocity may easily be determined by the rule already given for ascertaining the velocity of falling bodies. If the weight of the fly-wheel can be 2 tons, then the velocity of the rim need only be equal to that of a body falling through 15 feet, and so in all other proportions, so that the weight and velocity can easily be so adjusted as to represent most conveniently the prescribed store of power.

With these preliminary remarks it will now be proper to proceed to recapitulate the rules for proportioning all the parts of steam engines illustrated by examples:—

STEAM PORTS.

The area of steam port commonly given in the best engines working at a moderate speed is about 1 square inch per nominal horse-power, or $\frac{1}{25}$th of the area of the cylinder, and the area of the steam pipe leading into the cylinder is less than this, or ·66 square inch per nominal horse power. Since however engines

are now worked at various rates of speed it will be proper to adopt a rule in which the speed of the piston is made an element of the computation. This is done in the rules which follow both for the steam port and branch steam pipe.

TO FIND THE PROPER AREA OF THE STEAM OR EDUCTION PORT OF THE CYLINDER.

Rule.—*Multiply the square of the diameter of the cylinder in inches by the speed of the piston in feet per minute and by the decimal* ·032, *and divide the product by* 140. *The quotient is the proper area of the cylinder port in square inches.*

Example.—What is the proper area of each cylinder port in an engine with 64-inch cylinder, and with the piston travelling 220 feet per minute?

Here $64 \times 64 = 4{,}096$, which multiplied by $220 = 901{,}120$, and this multiplied by $·032 = 28{,}835·8$, which divided by 140, gives 206 inches as the area of each cylinder port in square inches.

This is a somewhat larger proportion than is given in some excellent engines in practice. But inasmuch as the application of lap to the valve virtually contracts the area of the cylinder ports, and as the application of such lap is now a common practice, it is desirable that the area of the ports should be on the large side. In the engines of the 'Clyde,' 'Tweed,' 'Tay,' and 'Teviot,' by Messrs. Caird and Co., the diameter of the cylinder was $74\frac{3}{8}$ inches, and the length of the stroke $7\frac{1}{2}$ feet, so that the nominal power of each engine was about 234 horses. The cylinder ports were $33\frac{1}{4}$ inches long and $6\frac{3}{4}$ inches broad, so that the area of each port was 224·4 square inches, being somewhat less than the proportion of 1 square inch per nominal horse power, but somewhat more than the proportion of $\frac{1}{25}$th of the area of the cylinder. As the areas of circles are in the proportion of the square roots of their respective diameters, the area of a circle of one-fifth of the diameter of the piston will have one-twenty-fifth of the area of the piston. One-fifth of $74\frac{3}{8}$ths is 15 nearly, and the area of a circle 15 inches in diameter is 176·7 square inches, which is considerably less than the actual

area of the port. By the rule we have given the area of the ports of this engine would, at a speed of 220 feet per minute, be about 277 square inches, which is somewhat greater than the actual dimensions. At a speed of the piston of 440 feet per minute the area of the port would be double the foregoing.

STEAM PIPE.

In the engines already referred to, the internal diameter of each steam pipe leading to the cylinder is 13⅝ inches, which gives an area of 145·8 square inches. It is not desirable to make the steam pipe larger than is absolutely necessary, as an increased external surface causes increased loss of heat from radiation. The following rule will give the proper area of the steam pipe for all speeds of piston:—

TO FIND THE AREA OF THE STEAM PIPE LEADING TO EACH CYLINDER.

RULE.—*Multiply the square of the diameter of the cylinder in inches by the speed of the piston in feet per minute and by the decimal* ·02, *and divide the product by* 170. *The quotient is the proper area of the steam pipe leading to the cylinder in inches.*

Example.—What is the proper area of the branch steam pipe leading to each cylinder in an engine with a cylinder 74½ inches diameter, and with the piston moving at a speed of 220 feet per minute?

Here $74·5 \times 74·5 = 5{,}550·25$, which multiplied by $220 = 1{,}221{,}055$. and this multiplied by $·02 = 24{,}421·1$, which divided by $170 = 144$ square inches nearly. The diameter of a circle of 144 square inches area is a little over 13½ inches, so that 13½ inches would be the proper internal diameter of each branch steam pipe in such an engine. The main steam pipe employed in steamers usually transmits the steam for both the engines to the end of the engine-house, where it divides into two branches—one extending to each cylinder. The main steam pipe will require to have nearly, but not quite, double the area of each of the branch steam pipes. It would require to

have exactly double the area, only that the friction in a large pipe is relatively less than in a small; and as, moreover, the engines work at right angles, so that one piston is at the end of its stroke when the other is at the beginning, and therefore moving slowly, it will follow that when one engine is making the greatest demand for steam the other is making very little, so that the area of the main steam pipe will not require to be as large as if the two engines were making their greatest demand at the same time.

SAFETY VALVES.

It is easy to determine what the size of an orifice should be in a boiler to allow any volume of steam to escape through it in a given time. For if we take the pressure of the atmosphere at 15 lbs., and if the pressure of the steam in the boiler be 10 lbs. more than this, then the velocity with which the steam will flow out will be equal to that which a heavy body would acquire in falling from the top of a column of the denser fluid that is high enough to produce the greater pressure to the top of a column of the same fluid high enough to produce the less pressure, and this velocity can easily be ascertained by a reference to the law of falling bodies. In practice, however, the area of safety valves is made larger than what answers to this theoretical deduction, partly in consequence of the liability of the valves to stick round the rim, and because the rim or circumference becomes relatively less in the case of large valves. One approximate rule for safety valves is to allow one square inch of area for each inch in the diameter of the cylinder, so that an engine with a 64-inch cylinder would require a safety valve on the boiler of 64 square inches area, which answers to a diameter of about 9 inches. The rule should also have reference, however, to the velocity of the piston, and this condition is observed in the following rule:—

TO FIND THE PROPER DIAMETER OF A SAFETY VALVE THAT WILL LET OFF ALL THE STEAM FROM A LOW PRESSURE BOILER.

RULE.—*Multiply the square of the diameter of the cylinder in inches by the speed of the piston in feet per minute, and*

divide the product by 14,000. *The quotient is the proper area of the safety valve in square inches.*

Example.—What is the proper diameter of the safety valve of a boiler that supplies an engine with steam, having a 64-inch cylinder, and with the piston travelling 220 feet per minute?

Here $64 \times 64 = 4{,}096$, which multiplied by $220 = 901{,}120$, and this divided by $14{,}000 = 64{\cdot}3$, which is the proper area of the safety valve in square inches.

ANOTHER RULE FOR SAFETY VALVES.

Multiply the nominal horse power of the engine by ·375, *and to the product add* 16·875. *The sum is the proper area of the safety valve in square inches, when the boiler is low pressure.*

Example.—What is the proper diameter of the safety valve for a low pressnre engine the nominal power of which is 140 horses?

Here $140 \times {\cdot}375 = 52{\cdot}5$, adding to which the constant number 16·875, we get 69·375, which is the proper area of the safety valve in square inches for a low pressure engine.

A 60-inch cylinder and 6-feet stroke is equal to 140 nominal horses power, so that this rule gives somewhat more than a square inch of area in the valve for each inch of diameter in the cylinder in that particular size of engine.

The opening through the safety valve must be understood to be the *effective* opening clear of bridges or other obstacles, and the area to be computed is the area of the smallest diameter of the valve. Most safety valves are made with a chamfered edge, which edge constitutes the steam tight surface, and the effective area is what corresponds to the smaller diameter of the valve and not to the larger. All boilers should have an extra or additional safety valve of the same capacity as the other, which may act in case of accident to the first from getting jammed or otherwise. The dimensions of safety valve here computed is that adequate for letting off all the steam. But in some cases the whole steam is not supplied from one boiler, and a safety valve in such case must be put on each boiler, but of a less area,

in proportion to the smaller volume of steam it has to let off. If there are two boilers, the safety valve on each will be half the area of the foregoing; if three boilers, one-third of the area; if four boilers, one-fourth of the area; and so of all other proportions. The area of the waste steam pipe should be the same as that of the safety valve.

TO FIND THE PROPER DIAMETER OF THE FEED PIPE.

RULE.—*Multiply the nominal horse power of the engine as computed by the Admiralty rule by ·04, to the product add* 3; *extract the square root of the sum. The result is the diameter of the feed pipe in inches.*

Example 1.—What is the proper diameter of the feed pipe in inches of an engine whose nominal horse power is 140?

140 = nominal horse power of engine
·04 = constant multiplier

5·6
3 = constant to be added

8·6

and $\sqrt{8·6} = 2·93$ diameter of feed pipe in inches.

Example 2.—What is the proper diameter of the feed pipe in inches in the case of an engine whose nominal horse power is 385?

385 = nominal horse power of engine
·04 = constant multiplier

15·4
3 constant to be added

18·4

and $\sqrt{18·4} = 4·29$ diameter of feed pipe in inches.

TO FIND THE PROPER DIMENSIONS OF THE AIR PUMP AND CONDENSER.

In land engines the diameter of the air pump is made half that of the cylinder, and the length of stroke half that of the cylinder, so that the capacity is ⅛th that of the cylinder; and the condenser is made of the same capacity. But in marine engines the diameter of the air pump is made ·6 of the diameter of the cylinder, and the length of the stroke is made from ·57 to ·6 times the stroke of the cylinder, and the condenser is made at least as large. In some cases the air pump is now made double-acting, in which case its capacity need only be half as great as when made single-acting.

TO FIND THE PROPER AREA OF THE INJECTION PIPE.

RULE.—*Multiply the nominal horse power of the engine, as computed by the Admiralty rule, by* 0·69, *and to the product add* 2·81. *The sum is the proper area of the injection pipe in square inches.*

Example 1.—What is the proper area of the injection pipe in square inches of an engine whose nominal horse power is 140?

	140	= nominal horse power of engine
	·069	= constant multiplier
	9·66	
	2·81	= constant to be added
Answer	12·47	= area of injection pipe in square inches.

Example 2.—What is the proper area of the injection pipe in square inches of an engine whose nominal horse power is 385?

	385	= nominal horse power of engine
	·069	= constant multiplier
	26·56	
	2·81	= constant to be added
Answer	29·37	= area of injection pipe in square inches.

The area of the injection orifice is usually made about 1-250th part of the area of the piston, which, in an engine of 385 horse power, would be about 27·7 inches of area. For warm climates the area should be increased.

TO FIND THE PROPER AREA OF THE FOOT VALVE PASSAGE.

RULE. —*Multiply the nominal horse power of the engine by* 9, *divide the product by* 5, *add* 8 *to the quotient. The sum is the proper area of foot valve passage in square inches.*

Example 1.—What is the proper area of the foot valve passage in square inches of an engine whose nominal horse power is 140?

	140	= nominal horse power of engine
	9	= constant multiplier
constant divisor	5)1260	
	252	
	8	= constant to be added
Answer	260	= area of foot valve passage in square inches.

Example 2.—What is the area of foot valve passage in square inches of an engine whose nominal horse power is 385?

	385	= nominal horse power of engine
	9	= constant multiplier
constant divisor	5)3465	
	693	
	8	= constant to be added
Answer	701	= area of foot valve passage in square inches.

The discharge valve passage is made of the same size as the foot valve passage.

A common rule for the area of the foot and discharge valve passages is one-fourth of the area of the air pump, and the waste

water pipe is made one-fourth of the diameter of the cylinder, which gives a somewhat less area than that through the foot and discharge valve passages. Such rules, however, are only applicable to slow-going engines. In rapid-working engines, such as those employed for driving the screw propeller by direct action, and in which the air-pump is usually double acting, the area through the foot and discharge valves should be equal to the area of the air-pump, and the waste water pipe should also have the same area. In all cases, therefore, in which these or other rules dependent on the nominal power are applied to fast-going engines, the nominal power must be computed by the Admiralty rule, in which the speed of the piston is taken into account.

TO FIND THE PROPER DIAMETER OF THE WASTE WATER PIPE.

RULE.—*Multiply the square root of the nominal horse power of the engine by* 1·2. *The product is the diameter of the waste water pipe in inches.*

Example 1.—What is the diameter of the waste water pipe, in inches, of an engine whose nominal horse power is 140?

$$
\begin{array}{rl}
140 = & \text{nominal horse power of engine} \\
\text{and } \sqrt{140} = & 11.83 \\
& 1{\cdot}2 \ = \text{constant multiplier} \\
\hline
\text{Answer} & 14{\cdot}19 = \text{diameter of waste water pipe in inches.}
\end{array}
$$

Example 2.—What is the diameter of waste water pipe, in inches, of an engine whose nominal horse power is 385?

$$
\begin{array}{rl}
385 = & \text{nominal horse power of engine} \\
\text{and } \sqrt{385} = & 19{\cdot}62 \\
& 1{\cdot}2 \ = \text{constant multiplier} \\
\hline
\text{Answer} & 23{\cdot}54 = \text{diameter of waste water pipe in inches.}
\end{array}
$$

CAPACITY OF THE FEED PUMP.

The relative volumes of steam and water are at 15 lbs. on the square inch, or the atmospheric pressure, 1,669 to 1; at 30 lbs., or

15 lbs. on the square inch above the atmospheric pressure, 881 to 1; at 60 lbs., or 45 lbs. above the atmospheric pressure, 467 to 1; and at 120 lbs., or 105 lbs. above the atmospheric pressure, 249 to 1.

In every engine, taking into account the risks of leakage and priming in the boiler, the feed pump should be capable of discharging twice the quantity of water that is consumed in the generation of steam; and in marine boilers it is necessary to blow out as much of the supersalted water as the quantity that is raised into steam, in order to keep the boiler free from saline incrustations. But if this water is discharged by leakage or priming, the object of preventing salting is equally fulfilled. Pumps, especially if worked at a high rate of speed, do not fill themselves with water at each stroke, but sometimes only half fill themselves, and sometimes do not even do that. Then in steam vessels, one pump should be able to supply both engines with steam, and the pump is generally only single-acting, while the cylinder is double-acting. If, therefore, we wish to see what size of pump we ought to supply to an engine in which the terminal elasticity of steam in the cylinder is equal to the atmospheric pressure, we know that the quantity of water in the steam is just $\frac{1}{1669}$th of the volume of the steam; but as we require to double the supply to make up for waste, the volume of water supplied will on this ground be $\frac{2}{1669}$; and as the pump may only half fill itself every stroke, the capacity of the pump must on this ground be $\frac{4}{1669}$ of the volume of steam. But then the pump is only single-acting, while the cylinder is double-acting, on which account the capacity of the pump must be doubled, in order that it may in a half stroke discharge the water required to produce the steam consumed in a whole stroke. This would make the capacity of the pump $\frac{8}{1669}$, or $\frac{1}{208}$ of the capacity of the cylinder, and a less proportion than this is inadvisable in the case of marine engines. Even with this proportion, one feed pump would not supply all the boilers, as it ought to be able to do in case of accident happening to the other, unless it should happen that the pump draws itself full of water at each stroke instead of half full, as it will nearly do if the motion of the engine is slow and the passages leading into it large,

and if at the same time the valves are large and have not much lift. In the case of engines working at a high speed, $\frac{1}{250}$ of the capacity of the cylinder for the capacity of the feed pump is scarcely sufficient, especially if there be no air vessel on the suction side of the pump, which in such pumps should always be introduced. In the engines of the 'Clyde,' 'Tweed,' 'Tay,' and 'Teviot,' by Messrs. Caird, the feed pump is $\frac{1}{240}$th of the capacity of the cylinder. In steam vessels there is no doubt always the resource of the donkey engine to make up for any deficiency in the feed. But it is much better to have the main feed pumps of the engine made of sufficient size to compensate for all the usual accidents befalling the supply of feed water. Of course, the supply of feed water required will vary materially with the amount of expansion with which the steam is worked, and also with the amount of superheating; and in the old flue boilers with the chimney passing up through the steam chest, there was always a considerable degree of superheating. A rule applicable to all pressures of steam and to moderate rates of expansion is as follows:—

TO FIND THE PROPER CAPACITY OF THE FEED PUMP.

RULE.—*Multiply the capacity of the cylinder in cubic inches by the total pressure of the steam in the boiler on each square inch (or by the load on each square inch of the safety valve plus* 15 *lbs. on each square inch for the pressure of the atmosphere), and divide the product by* 4,000. *The quotient is the proper capacity of the feed pump in cubic inches when the pump is single-acting and the engine is double-acting.*

If the pump should be double-acting, one-half of the above capacity will suffice.

Example 1.—What is the proper volume of the working part of the plunger of an engine with a 74-inch cylinder and 7½-feet stroke, the steam in the boiler being 5 lbs. per square inch above the atmospheric pressure?

The area in square inches of a circle 74 inches diameter is 4,300, which, multiplied by 7½ feet or 90 inches, gives 387,000

cubic inches as the capacity of the cylinder. Now if the steam in the boiler be 5 lbs. per square inch above the atmosphere, it will have a total pressure of 5 + 15, or 20 lbs. per square inch. Multiplying, therefore, 387,000 by 20, we get 7,740,000, which, divided by 4,000, gives 1,935 as the proper capacity of the feed pump in cubic inches. If now the stroke of the pump be 51 inches, we divide 1,935 by 51, which gives us 38 inches nearly as the proper area of the feed pump plunger. This area corresponds to a diameter of 7 inches, which is a better proportion than that subsisting in the engines of the 'Clyde,' 'Tweed,' 'Tay,' and 'Teviot,' which, with a 74-inch cylinder, 7½ feet stroke, and 51 inches stroke of pump, had the feed pump plungers of only 6 inches diameter.

Example 2.—What is the proper volume of the working part of the plunger of a locomotive feed pump, having cylinders of 18 inches diameter and 2 feet stroke, working with a pressure of 85 lbs. pressure above the atmosphere?

The area of a circle 18 inches diameter is 254·5 square inches, which, multiplied by 24 inches, which is the length of the stroke, gives 6,108 cubic inches as the capacity of the cylinder. If the steam be 85 lbs. above the atmosphere, then the total pressure must be 100 lbs. per square inch, and $6{,}108 \times 100 = 610{,}800$, which, divided by 4,000, gives 152·7 as the capacity of the feed pump in cubic inches. This is a somewhat larger proportion of feed pump than is usually given in locomotive engines. In the locomotive 'Iron Duke' the diameter of the feed pump plunger is 2¼ inches and the stroke 24 inches. But 152·7 divided by 24 inches gives an area of 6·36 square inches, which answers to a diameter of plunger of 2⅞ inches. In locomotives, however, as in marine engines, the feed pumps are very generally made too small, so that the proportion given in the rule appears preferable to that commonly adopted.

COLD-WATER PUMP.

The proper dimensions of the cold-water pump can easily be determined by a reference to the number of cubic inches of water, at a given temperature, that are required to condense a

cubic inch in the form of steam. There is no need, however, of going through the details of the process, and the proper dimensions of the pump will be found by the following rule:—

TO DETERMINE THE PROPER DIMENSIONS OF THE COLD-WATER PUMP.

RULE.—*Multiply the square of the diameter of the cylinder in inches by the length of the stroke in feet, and divide the product by* 4,400. *The quotient is the proper capacity of the cold-water pump in cubic feet.*

Example 1.—What is the proper capacity of the cold-water pump in an engine, having a 60-inch cylinder and a $5\frac{1}{2}$-feet stroke?

Here $60 \times 60 = 3{,}600$, which multiplied by $5\frac{1}{2}$ is 19,800, and this divided by 4,400 is 4·5, which is the proper capacity of the cold-water pump in cubic feet.

Example 2.—What is the proper capacity of the cold-water pump in the case of an engine, with a 2-feet cylinder and 3-feet stroke?

Here $24 \times 24 = 576$, and this multiplied by $3 = 1{,}728$, which divided by $4{,}400 = {\cdot}39$ cubic feet, or multiplying ·39 by 1,728, we get the capacity in cubic inches, which is 673·92. This is a somewhat larger content than is sometimes given in practice. Maudslay's 16-horse land engine has a 24-inch cylinder and 3-feet stroke, and the cold-water pump has a diameter of $6\frac{1}{2}$ inches, and a stroke of 18 inches, which gives a capacity of 594 cubic inches, instead of 673, as specified above. The larger dimension is the one to be preferred.

FLY-WHEEL.

Boulton and Watt's rule for finding the sectional area of the fly-wheel rim is as follows:—

RULE.—*Multiply* 44,000 *times the length of the stroke in feet by the square of the diameter of the cylinder in inches, and divide the product by the square of the number of revolutions per minute, multiplied by the cube of the diameter of*

the fly-wheel in feet. The resulting number will be the proper sectional area of the fly-wheel rim in square inches.

Example.—What will be the proper sectional area of the fly-wheel rim in square inches in the case of an engine, with a cylinder 24 inches diameter and 5 feet stroke, the fly-wheel being 20 feet diameter.

Here 44,000 multiplied by 5, which is the length of the stroke in feet, is 220,000. The square of the diameter of the cylinder in inches is 576, and 220,000 × 576 = 126,720,000. The engine will make about 21 revolutions, the square of which is 441, and the cube of the diameter of the fly-wheel in feet is 8,000, which multiplied by 441 is 3,528,000. Finally 126,720,000 divided by 3,528,000 is 35·8, which is the proper area in square inches of the section of the fly-wheel rim.

In an engine constructed by Mr. Caird, with a 24-inch cylinder, 5-feet stroke, and 20-foot fly-wheel, the width of the rim was 10 inches, and the thickness 3¾ inches, giving a sectional area of 37·5 square inches, which is somewhat larger than Boulton and Watt's proportion.

Suppose that we take the sectional area in round numbers at 36 square inches, and the circumference of the fly-wheel or length of rim if opened out at 62 feet or 744 inches, then there will be 36 times 744, or 26,784 cubic inches of cast iron in the rim, or dividing by 1,728, we shall have 15·5 cubic feet of cast iron. But a cubic foot of cast iron weighs 444 lbs. Hence 15½ cubic feet will weigh 6,882 lbs., and this weight revolves with a speed of 21 times 62, or 1,303 feet per minute, or 21·7 feet per second, or 260·4 inches per second. To find the height in inches from which a body must have fallen, to acquire any given velocity in inches per second, we square the velocity in inches, and divide the square by 772·84, which gives the height in inches. Now the square of 260·4 is 67,808, which divided by 772·84 = 87 inches, or 7¼ feet, so that the energy treasured in the fly-wheel is equal to a weight of 6,882 lbs. falling through 7¼ feet, or to a weight of 49,984·5 lbs. falling through 1 foot. Now the area of the cylinder being in round numbers 452 square inches, the total pressure upon it, if we allow an effec-

tive pressure including steam and vacuum of 7 lbs. per square inch, as was the proportion allowed in Watt's engines, will be 3,164 lbs., and the length of stroke being 5 feet, we shall have 3,164 lbs. moved through 5 feet, or 5 times this, which is 15,820 lbs. moved through 1 foot in each half stroke of the engine. Dividing now 49,984·5 foot-pounds, the total power resident in the fly-wheel at its mean velocity, by 158·20 foot-pounds, which is the power developed in each half stroke of the engine, we get 3·1 as the resulting number, which shows that there is over three times the power resident in the fly-wheel that is developed in each half stroke of the engine. In cases where great equability of motion is required, this power of fly-wheel is not sufficient, and in some engines, the proportion is made six times the power developed in each half stroke, or, in other words, the fly-wheel is twice as heavy as that computed above.

GOVERNOR.

The altitude of the height of the cone in which the arms revolve, measuring from the plane of revolution to the centre of suspension, will be the same as that of a pendulum which makes the same number of double beats per minute that the governor makes of revolutions; or if the number of revolutions per minute be fixed, and we wish to obtain the proper height of cone, we divide the constant number 375·36 by twice the number of revolutions, which gives the square root of the height of the cone; and, consequently, the height itself is equal to the square of this number. These relations are exhibited in the following rules:—

TO DETERMINE THE SPEED AT WHICH A GOVERNOR MUST BE DRIVEN, WHEN THE HEIGHT OF THE CONE IS FIXED IN WHICH THE ARMS REVOLVE.

RULE.—*Divide the constant number* 375·36 *by twice the square root of the height of the cone in inches. The quotient is the proper number of revolutions per minute.*

Example.—A governor with arms 30½ inches long, measuring from the centre of suspension to the centre of the ball, revolves

in the mean position of the arms at an angle of about 30 degrees, with the vertical spindle forming a cone about 26½ inches high. At what number of revolutions per minute should this governor be driven?

Here the height of the cone being 26·5 inches, the square root of which is 5·14, and twice the square root 10·28, we divide 375·36 by 10·28, which gives us 36·5 as the proper number of revolutions at which the governor should be driven.

TO DETERMINE THE HEIGHT OF THE CONE IN WHICH THE ARMS MUST REVOLVE, WHEN THE VELOCITY OF ROTATION OF THE GOVERNOR IS DETERMINED.

RULE.—*Divide the constant number* 375·36 *by twice the number of revolutions which the governor makes per minute, and square the quotient, which will be the height in inches which the cone will assume.*

Example.—Suppose that a governor be driven with a speed of 36½ revolutions per minute, what will be the height of the cone in which the balls will necessarily revolve, measuring from the centre of suspension of the arms to the plane of revolution of the balls?

Here 36·5 × 2 = 73, and 375·36 divided by 73 = 5·14, and 5·14 squared is equal to 26·4196, or very nearly 26·5 inches, which will be the height of the cone.

When the arms revolve at an angle of 45 degrees with the spindle, or at right angles with one another, the centrifugal force is equal to the weight of the balls; and when the arms revolve at an angle of 30 degrees with the spindle, they form with the base of the cone an equilateral triangle.

STRENGTHS OF LOW-PRESSURE LAND ENGINES.

PISTON ROD.

The piston rod is made one-tenth of the diameter of the cylinder, except in locomotives, where it is made one-seventh

of the diameter. The piston rod is sometimes made of steel, or of iron converted into steel for a certain depth in. This enables it to acquire and maintain a better polish than if made of iron.

MAIN LINKS.

The main links are the parts which connect the piston rod with the beam. They are usually made half the length of the stroke, and their sectional area is 113th the area of the piston.

AIR-PUMP ROD.

The diameter of the air-pump rod is commonly made one-tenth of the diameter of the air-pump.

BACK LINKS.

The sectional area of the back links is made the same as that of the air-pump rod.

END STUDS OF THE BEAM.

The end studs of the beam are usually made the same diameter as the piston rod. Sometimes they are of cast-iron, but generally now of wrought. The gudgeons of water wheels are generally loaded with about 500 lbs. for every circular inch of their transverse section, which is nearly the proportion that obtains in the end studs of engine beams. But the main centre is usually loaded beyond this proportion.

MAIN CENTRE.

The strength of this part will be given in the strengths of marine engines. But when of cast-iron it is usually made about one-fifth of the diameter of the cylinder.

In a cylinder of 24 inches diameter this will be 4·8 inches, or say $4\frac{3}{4}$ inches; and this proportion of strength will be about nine times the breaking weight, if we suppose the main centre to be overhung as in marine engines. Thus, in a cylinder of 24 inches diameter, and, consequently, of 452 square inches area, the total load on the piston with 20 lbs. on each square inch is 9,040 lbs.

But as the strain at the main centre is doubled from the beam acting as a lever of 2 to 1, it follows that the strain at the main centre will be 18,080 lbs. The ultimate tensile strength of common cast-iron being 12,000 per square inch of section, and the tensile and shearing strength being about the same, $\frac{1}{9}$th of 12,000, or 1,333 lbs., will be the proper load to place on each square inch of section; and 18,080 divided by 1,333 will give the proper sectional area in square inches, which will be $13\frac{1}{2}$ square inches nearly. This area corresponds to a diameter of a little over $4\frac{1}{8}$ inches. But the strength is virtually doubled by the circumstance of the main centre of land engines being supported at both ends.

MAIN BEAM.

The rules in common use for proportioning the main beams of engines are the same as those which existed prior to Mr. Hodgkinson's researches on the strength of cast-iron girders, which showed that the main element of strength was the bottom flange. But as in the case of engine beams the strain is alternately up and down, the top and bottom flanges, or beads of the beam, require to be of equal strength. Cast-iron is a bad material for engine beams, unless the central part be made of open work of cast-iron, and the edge of the beam be encircled by a great elliptical or lozenge-formed hoop, as is done in some of the American engines. But if the beam be made wholly of cast-iron, a much larger proportion of the metal should be collected in the top and bottom flanges than is at present the ordinary practice.

The usual length of the main beam is three times the length of the stroke; the usual breadth is equal to the diameter of the cylinder, and the usual mean thickness is $\frac{1}{108}$th of the length. The rule is as follows:—

TO FIND THE PROPER DIMENSIONS OF THE MAIN BEAM OF A LAND ENGINE.

RULE.—*Divide the weight in lbs. acting at the centre by* 250 *and multiply the quotient by the distance between the extreme centres. To find the depth, the breadth being given: Divide the*

product by the breadth in inches, and extract the square root of the quotient, which is the depth.

The depth of the beam at the ends is usually made one-third of the depth at the middle.

It will be preferable, howevei, to investigate a rule on the basis of Mr. Hodgkinson's rule for proportioning cast-iron girders, which is as follows:

Multiply the sectional area of the bottom flange in inches by the depth of the beam in inches, and divide the product by the distance between the supports also in inches, and 514 *times the quotient will be the breaking weight in cwts.*

If the breaking weight be expressed in tons, the constant number 514 must be divided by 20, which gives the breaking weight as 25·7, or say 26 tons, whereas experiment has shown that if the flange were to be formed of malleable iron instead of cast, the breaking weight would not be less than 80 tons; or, in other words, that with the same sectional area of flange, the beam would be more than three times stronger.

It is a common practice in the case of girders to make the strength equal to three times the breaking weight when the load is stationary, and to six times the breaking weight when the load is movable. But these proportions are too small, and less than nine or ten times the breaking weight will not give a sufficient margin of strength in the case of engines where the motion is so incessant, and where heavy strains may be accidentally encountered from priming or otherwise. In the case of an engine, the weight answering to the breaking weight is the load on the piston; and if we suppose the fly-wheel to be jammed, and the piston to be acting with its full force to lift or sink the main centre, it is clear that the strain on the main centre, and, therefore, on the beam, will be equal to *twice* the strain upon the piston, since the beam acts under such circumstances as a lever of 2 to 1. The problem we have now to consider is how many times the working weight must be less than the breaking weight to give a sufficient margin of strength in any given beam or, in other words, what proportions must the beam have to possess adequate working strength.

To take a practical example from an engine in constant work. The engine with a cylinder of 24 inches diameter has a main beam 15 feet (or 180 inches) long; 30 inches deep in the middle; and with a sectional area of flange of 7 square inches. The breaking weight of such a beam in cwts. will be $7 \times 30 \times 514$ divided by $180 = 600$ cwt. nearly, and this multiplied by 112 lbs. $= 67,200$ lb., which is the breaking weight in pounds avoirdupois. The area of the cylinder in round numbers is 452 square inches; but as there is a leverage of 2 to 1, this is equivalent to an area of cylinder of 904 square inches set under the middle of the beam and pulling it downwards, the beam being supposed to be supported at both ends. Dividing now 67,200 by 904 we get the pressure per square inch on the piston that would break the beam, which is a little over 74 lbs. per square inch of the area of the piston, or 58 lbs. per circular inch. If we suppose the working pressure of steam on the piston to be 6·27 lbs. per circular inch, or 7·854 lbs. per square inch, then the working strength of the beam will be about $9\frac{1}{3}$ times its breaking strength, which would give an adequate margin for safety. But if we suppose the working pressure to be 12·54 lbs. per circular inch, or 15·718 lbs. per square inch, the working strength would in such case be only about $4\frac{3}{4}$ times the breaking strength, and the beam would be too weak.

The strength of a cast-iron beam of any given dimensions varies directly as the sectional area of the edge flange; or, if the sectional area of that flange be constant, the strength of the beam varies directly as the depth, and inversely as the length. If while the sectional area of the flange remains the same the depth of the beam is doubled without altering the length, then the strength is doubled. But if the length be also doubled, the strength remains the same as at first. As the length of an engine-beam is doubled when we double the length of the stroke, and as in any symmetrical increase of an engine when we double the length of the stroke we also double the diameter of the cylinder, to which the depth of the beam is generally made equal, large beams with the same area of flange, and made in the ordinary proportions, would be as strong as small beams, except that

the load increases as the square of the diameter of the cylinder, and consequently the area of the edge flange must increase in the same proportion. These considerations enable us to fix the following rule for the strength of main beams:—

TO FIND THE PROPER DIMENSIONS OF THE MAIN BEAM OF AN ENGINE.

RULE.—*Make the depth of the beam equal to the diameter of the cylinder, and the length of the beam equal to three times the length of the stroke. Then to find the area of the edge flange: Multiply the area of the cylinder in square inches by the total pressure of steam and vacuum on each square inch of the piston, and divide the product by* 650. *The quotient is the proper area of the flange of the beam in square inches.*

Example 1.—What is the proper sectional area of the flange of the main beam of an engine, with cylinder 24 inches diameter and 5-feet stroke, the pressure on the piston being 20 lbs. per square inch?

Here the area of the cylinder will be 452 inches, which multiplied by 20 gives 9,040, and dividing by 650 we get 13·9 square inches, which is the proper sectional area of the edge bead or flange of the beam.

Example 2.—What is the proper sectional area of the flange of the main beam of an engine with a cylinder 60 inches diameter, 12½ feet stroke, and with a pressure of steam on the piston of 20 lbs. per square inch?

The area of a cylinder 60 inches diameter is 2,824 square inches, and 2,824 multiplied by 20=56,480, which divided by 650=87 square inches nearly. Such a flange, therefore, if 14½ inches broad, would be 6 inches thick. The beam would be 5 feet deep at the middle, and 37½ feet long between the extreme centres.

ANOTHER RULE FOR FINDING THE SECTIONAL AREA OF EACH EDGE FLANGE OF THE MAIN BEAM.

RULE.—*Multiply the diameter of the cylinder in inches by one-third of the length of the stroke in inches, and by the total pressure on each square inch of the piston, and divide the product by 650. The quotient is the proper sectional area in square inches of each flange or bead on the edge of the beam.*

Example 1.—What is the proper sectional area of the flange on the edge of the main beam of an engine with a 24-inch cylinder, 20 lbs. total pressure on piston per square inch, and 5 feet stroke?

Here 24 × 20 (which is one-third of the stroke in inches) × 20 (the pressure of the steam and vacuum per square inch) = 9,600, which divided by 650=14·7 sq. in., which is the area required.

Example 2.—What is the proper sectional area of the flange on the edge of the main beam of an engine with a 60-inch cylinder, 12½-feet stroke, and with a pressure on the piston of 20 lbs. per square inch?

Here 60 × 50 (which is one-third of the stroke in inches) × 20 (the pressure of the steam per square inch) = 6,000, which divided by 650 gives 92 as the sectional area of the edge bead in square inches. Such a flange, if 15⅓ inches broad, would be 6 inches thick. These results it will be seen are very nearly the same as those obtained by the preceding rule; and one inference from these rules is that nearly all engine beams are at present made too weak. The purpose of the web of the beam is mainly to connect together the top and bottom flanges, so that there is no advantage in making it thicker than suffices to keep the beam in shape; with which end, too, stiffening feathers, both vertical and horizontal, should be introduced upon the sides of the beam. The first cast-iron beams were made like a long hollow box to imitate wooden beams, and this form would still be the best, unless an open or skeleton beam, encircled with a great wrought-iron hoop after the American fashion, be adopted.

CONNECTING-ROD.

The connecting-rods of land engines are now usually made of wrought-iron, and when so made, the proportions will be the same, or nearly so, as those given under the head of marine engines. When made of cast-iron the configuration is such that the transverse section at the middle assumes the form of a cross, this form being adopted to give greater lateral stiffness. The length of the rod is usually made the same as the length of the beam, namely, three times the length of the stroke, and the area of the cross section of the rod at the middle is commonly made $\frac{1}{20}$th of the area of the cylinder, and the sectional area at the ends $\frac{1}{35}$th of the area of the cylinder. Such a strength is needlessly great, and is quite out of proportion to the strength commonly given to the beam. Thus, in the case of an engine with a 24-inch cylinder, the area of the piston is 452 square inches; and if we take 20 lbs. per square inch as the load on the piston, then the total load on the piston will be 9,040 lbs. If the working load be made $\frac{1}{9}$th of the breaking load, as in the case of the beam, then the breaking load should be 81,360 lbs., and the strength of the connecting-rod should be such that it would just break with that load on the piston. Now the tensile strength of the weakest cast-iron is about 12,000 lbs. per square inch of section, while its crushing strength is about five times that amount. Dividing 81,361 lbs., the total tensile strength of the rod, by 12,000, the tensile strength of one square inch, we get about 7 square inches as the proper area of the smallest part of the connecting-rod when of cast-iron. But $\frac{1}{35}$th of 452 (which is the area of the cylinder in square inches) is 13 square inches, from all of which it follows that while the main beams of engines are commonly made too weak, the cast-iron connecting-rods are commonly made too strong. This, however, is partly done for the purpose of balancing the weight of the piston and its connections.

FLY-WHEEL SHAFT.

The fly-wheel shaft of land engines is usually made of cast-iron. The following is the rule on which such shafts are usually proportioned:—

TO FIND THE DIAMETER OF THE FLY-WHEEL SHAFT AT SMALLEST PART, WHEN IT IS OF CAST-IRON.

RULE.—*Multiply the square of the diameter of the cylinder in inches by the length of the crank in inches; extract the cube root of the product; finally multiply the result by* ·3025. *The product is the diameter of the fly-wheel shaft at the smallest part in inches.*

Example 1.—What is the proper diameter of the fly-wheel shaft, when of cast-iron, in the case of an engine with a diameter of cylinder of 64 inches and a stroke of 8 feet?

$$\begin{array}{rl} 64 & = \text{diameter of the cylinder in inches} \\ 64 & \\ \hline 4096 & = \text{square of the diameter} \\ 48 & = \text{length of crank in inches} \\ \hline 196608 & \end{array}$$

$58·15 = \sqrt[3]{196608}$ and $58·15 \times ·3025 = 17·59$, which is the proper diameter of the fly-wheel shaft at the smallest part.

Example 2.—What is the proper diameter, at the smallest part, of the cast-iron fly-wheel shaft of an engine, with a diameter of cylinder of 40 inches, and 5 feet stroke?

$$\begin{array}{rl} 40 & = \text{diameter of cylinder in inches} \\ 40 & \\ \hline 1600 & = \text{square of diameter of cylinder} \\ 30 & = \text{length of crank in inches} \\ \hline 48000 & \end{array}$$

$36·30 = \sqrt[3]{48000}$ and $36·30 \times ·3025 = 10·98$, which is the proper diameter of the shaft in inches.

MR. WATT'S RULE FOR THE NECKS OF HIS CRANK SHAFTS.

RULE.—*Multiply the area of the piston in square inches by the pressure on each square inch (and which Mr. Watt took at* 12 *lbs.), and by the length of the crank in feet. Divide the product by* 81·4, *and extract the cube root of the quotient, which is the proper diameter of the shaft in inches.*

Example 1.—What is the proper diameter of the fly-wheel shaft in an engine, with a cylinder 64 inches diameter and 8 feet stroke, the pressure on the piston being taken at 12 lbs. per square inch?

The area of a cylinder 64 inches diameter is 3,217 square inches, which multiplied by 12 = 38,604, and this multiplied by 4, which is the length of the crank in feet, is 154,416. This divided by 31·4 = 4,917·7, the cube root of which is 17·01 inches.

Example 2.—What is the right diameter, according to Mr. Watt's rule, of the fly-wheel shaft of an engine, with a 24-inch cylinder, 5 feet stroke, and with a pressure of 12 lbs. on each square inch of the piston?

The area of the cylinder is 452 square inches, which multiplied by 12 = 5,424, and this multiplied by 2½, which is the length of the crank in feet = 13,560, which divided by 31·4 = 431, the cube root of which is 7½ inches, which is the proper diameter of the shaft. In Mr. Caird's engine the diameter is 8 inches.

TO FIND THE PROPER THICKNESS OF THE LARGE EYE OF THE CRANK FOR FLY-WHEEL SHAFT, WHEN OF CAST-IRON.

RULE.—*Multiply the square of the length of the crank in inches by* 1·561, *and then multiply the square of the diameter of the cylinder in inches by* ·1235; *multiply the sum of these products by the square of the diameter of the cylinder in inches; divide this product by* 666·283; *divide this quotient by the length of the crank in inches; finally extract the cube root of the quotient. The result is the proper thickness of the large eye of crank for fly-wheel shaft in inches, when of cast-iron.*

Example 1.—Required the proper thickness of the large eye of crank for fly-wheel shaft, when of cast-iron, of an engine whose length of stroke is 8 feet, and diameter of cylinder 64 inches.

$$
\begin{array}{r l}
48 & = \text{length of crank in inches} \\
48 & \\
\hline
2304 & = \text{square of length of crank in inches} \\
1{\cdot}561 & = \text{constant multiplier} \\
\hline
3596{\cdot}5 &
\end{array}
$$

$$
\begin{array}{r l}
64 & = \text{diameter of cylinder in inches} \\
64 & \\
\hline
4096 & = \left\{ \begin{array}{l} \text{square of diameter of cylinder} \\ \text{in inches} \end{array} \right. \\
{\cdot}1235 & = \text{constant multiplier} \\
\hline
505{\cdot}8 & \\
3596{\cdot}5 & \\
\hline
4102{\cdot}3 & = \text{sum of products} \\
4096 & = \left\{ \begin{array}{l} \text{square of the diameter of the cy-} \\ \text{linder in inches} \end{array} \right.
\end{array}
$$

$$
\left.\begin{array}{l}\text{Constant}\\ \text{divisor}\end{array}\right\} = 666{\cdot}283)16803020{\cdot}8
$$

$$
\text{Length of crank} = 48)25219{\cdot}045
$$

$$
525{\cdot}397
$$

and $\sqrt[3]{525{\cdot}397} = 8{\cdot}07$ which is the proper thickness of the large eye of the crank in inches, when of cast-iron.

Example 2.—Required the proper thickness of the large eye of the crank for fly-wheel shaft, when of cast-iron, of an engine, whose length of stroke is 5 feet, and diameter of cylinder 40 inches.

$$
\begin{array}{r l}
30 & = \text{length of crank in inches} \\
30 & \\
\hline
900 & = \text{square of length of crank in inches} \\
1{\cdot}561 & = \text{constant multiplier} \\
\hline
1404{\cdot}9 &
\end{array}
$$

40 = diameter of cylinder in inches
40
1600
·1235 = constant multiplier
197·6
1404·9
1602·5 = sum of products
1600 = square of diameter of cylinder

Constant divisor = 666·283)2564000·0

Length of crank = 30 inches 3848·2
128·3

and $\sqrt[3]{128{\cdot}3} = 5{\cdot}04$ inches is the proper thickness in this engine of the large eye of the crank, when of cast-iron.

TO FIND THE PROPER BREADTH OF THE WEB OF THE CRANK AT THE CENTRE OF THE FLY-WHEEL SHAFT, WHEN OF CAST-IRON, SUPPOSING THE BREADTH TO BE CONTINUED TO THE CENTRE OF THE SHAFT.

RULE.—*Multiply the square of the length of the crank in inches by* 1·561, *and then multiply the square of the diameter of the cylinder in inches by* ·1235; *multiply the square root of the sum of these products by the square of the diameter of the cylinder in inches; divide the product by* 23·04, *and finally extract the cube root of the quotient. The final result is the breadth of the crank at the centre of the fly-wheel shaft, when the crank is of cast-iron.*

Example 1.—What is the proper breadth of the web of the crank at the centre of fly-wheel shaft, when of cast-iron, in the case of an engine, with a diameter of cylinder of 64 inches, and length of stroke 8 feet?

48 = length of crank in inches
48
2304 = square of length of crank
1·561 = constant multiplier
3596·5

64 = diameter of cylinder in inches
64

4096 = square of diameter of cylinder
·1235 = constant multiplier

505·8
3596·5

4102·3 = sum of products

$\sqrt{4102·3} = 64·05$ nearly
4096 = square of diameter of cylinder in inches.

23·04)262348·80(11395·34
2304

3214
2304

9108
6912

21968
20736

12320
11520

8000
6912

10880
9216

1664

$\sqrt[3]{11395·34} = 22·5$ inches, which is the proper breadth of the web of the crank, when of cast-iron, supposing the breadth to be continued to the centre of the fly-wheel shaft.

Example 2.—What is the proper breadth of the web of a cast-iron crank at the centre of the fly-wheel shaft (supposing it to be so far extended), in the case of an engine with 40 inches diameter of cylinder and 5 feet stroke?

$$
\begin{array}{r l}
30 & = \text{length of crank in inches} \\
30 & \\
\hline
900 & = \text{square of length of crank in inches} \\
1{\cdot}561 & = \text{constant multiplier} \\
\hline
1404{\cdot}9 & \\
\\
40 & = \text{diameter of cylinder in inches} \\
40 & \\
\hline
1600 & = \text{square of diameter of cylinder} \\
{\cdot}1235 & = \text{constant multiplier} \\
\hline
197{\cdot}6 & \\
1602{\cdot}5 & = \text{sum of products} \\
\sqrt{1602{\cdot}5} & = 40{\cdot}3 \text{ nearly} \\
\end{array}
$$

$$
\begin{array}{r}
1600 \\
\hline
23{\cdot}04)64480{\cdot}0 \\
\hline
2798{\cdot}6 \text{ nearly}
\end{array}
$$

$\sqrt[3]{2798{\cdot}6} = 14{\cdot}09$, which is the proper breadth in inches of a cast iron crank in an engine of this size, supposing the breadth to be continued to the fly-wheel shaft.

TO FIND THE PROPER THICKNESS OF THE WEB OF A CAST-IRON CRANK AT THE CENTRE OF THE FLY-WHEEL SHAFT.

RULE.—*Multiply the square of the length of the crank in inches by* 1·561, *and then multiply the square of the diameter of the cylinder in inches by* ·1235; *multiply the square root of the sum of these products by the square of the diameter of the cylinder in inches; divide the product by* 1·32; *finally extract the cube root of the quotient. The result is the proper thickness of the web of a cast-iron crank in inches at the centre of the fly-wheel shaft, supposing the thickness to be extended to that point.*

Example 1.—Required the proper thickness of the web of a cast-iron crank at the centre of the fly-wheel shaft (supposing it

to be so far extended), in the case of an engine with 64 inches diameter of cylinder, and 8 feet stroke.

$$
\begin{array}{rl}
48 & = \text{length of crank in inches} \\
48 & \\
\hline
2304 & = \text{square of the length of crank} \\
1{\cdot}561 & = \text{constant multiplier} \\
\hline
3596{\cdot}5 &
\end{array}
$$

$$
\begin{array}{rl}
64 & = \text{diameter of cylinder in inches} \\
64 & \\
\hline
4096 & = \text{square of diameter of cylinder} \\
{\cdot}1235 & = \text{constant multiplier} \\
\hline
505{\cdot}8 & \\
3596{\cdot}5 & \\
\hline
4102{\cdot}3 & = \text{sum of products}
\end{array}
$$

$$\text{and } \sqrt{4102{\cdot}3} = 64{\cdot}05 \text{ nearly}$$

$$4096 = \text{square of diameter of cylinder}$$

$$\left.\begin{array}{l}\text{Constant}\\ \text{divisor}\end{array}\right\} = 184{\cdot}32)262348{\cdot}5$$

$$1422{\cdot}33$$

$$\text{and } \sqrt[3]{1423{\cdot}33} = 11{\cdot}25$$

Example 2.—What is the proper thickness of the web of a cast-iron crank at centre of fly-wheel shaft (supposing it to be so far extended), in the case of an engine with 40 inches diameter of cylinder, and 5 feet stroke?

$$
\begin{array}{rl}
30 & = \text{length of crank in inches} \\
30 & \\
\hline
900 & = \left\{\text{square of length of crank in inches}\right. \\
1{\cdot}561 & = \text{constant multiplier} \\
\hline
1404{\cdot}9 &
\end{array}
$$

$$
\begin{array}{rl}
40 & = \text{diameter of cylinder in inches} \\
40 & \\
\hline
1600 & = \text{square of diameter of cylinder} \\
\cdot 1235 & = \text{constant multiplier} \\
\hline
197\cdot 6 & \\
1404\cdot 9 & \\
\hline
1602\cdot 5 & \\
\sqrt{1602\cdot 5} = 40\cdot 3 & \text{nearly} \\
1600 & \\
\hline
\left.\begin{array}{l}\text{Constant}\\ \text{divisor}\end{array}\right\} = 184\cdot 32)64480\cdot 0 & \\
\hline
349\cdot 8 &
\end{array}
$$

and $\sqrt[3]{349\cdot 8} = 7\cdot 04$, which is the proper thickness in inches of the web of a cast-iron crank for this engine, measuring at the centre of the fly-wheel shaft.

CRANK PIN.

The crank pins of land engines having cast-iron cranks, are generally made of cast-iron, and are in diameter about one-sixth of the diameter of the cylinder.

MILL GEARING.

Boulton and Watt, by whom the present system of iron gearing was introduced, proportioned their wheels on the following consideration:—'That a bar of cast-iron 1 inch square and 12 inches long, bears 600 lbs. before it breaks; 1 inch long will bear 7,200 lbs., and $\frac{1}{15}$th of this = 480 lbs., is the load which should be put on the wheel,' for each square inch in section of the tooth. Boulton and Watt's rule for the strength of geared wheels is consequently as follows:—If H = the actual horses power which the wheel has to transmit; d, the diameter of the wheel in feet, and r, the revolutions of the wheel per minute; then $\frac{H \times 306}{d \times r}$ = the strength, and the strength divided by the breadth in inches $= p^2$, or the square of the pitch in inches.

Hence $\mathrm{H} = \frac{p^2 \times b \times d \times r}{306}$ and $p = \sqrt{\frac{\mathrm{H} \times 306}{b \times d \times r}}$, which equations put into words are as follows:

TO FIND THE NUMBER OF ACTUAL HORSES POWER WHICH A GIVEN WHEEL WILL TRANSMIT, ACCORDING TO BOULTON AND WATT'S PRACTICE.

RULE.—*Multiply the square of the pitch in inches by the breadth of the wheel in inches, by its diameter in feet, and by the number of revolutions it makes per minute, and divide the product by the constant number* 306. *The quotient is the number of actual horses' power which the wheel will safely transmit, according to Boulton and Watt's practice.*

TO FIND THE PROPER PITCH OF A WHEEL IN INCHES TO TRANSMIT A GIVEN POWER, ACCORDING TO BOULTON AND WATT'S PRACTICE.

RULE.—*Multiply the breadth of the teeth in inches by the diameter of the wheel in feet, and by the number of revolutions it makes per minute, and reserve the product as a divisor. Next multiply the number of actual horses' power which the wheel has to transmit by the constant number* 306, *and divide the product by the divisor found as above. Finally, extract the square root of the quotient, which is the proper pitch of the wheel in inches, according to Boulton and Watt's practice.*

Instead, however, of reckoning the strain in horses' power, it is preferable to reckon it as a pressure or weight applied to the acting tooth of the driving wheel. If t = the thickness of the tooth in inches, w = the pressure upon it in lbs., and c a constant multiplier, which for cast-iron is ·025, for brass, ·035, and for hard wood, ·038, then $t = c\sqrt{w}$, by which formula we can easily find the proper thickness of the tooth, and twice the thickness of the tooth with the proper allowance for clearance, gives the pitch. This formula put into words is as follows:—

TO FIND THE PROPER THICKNESS OF TOOTH OF A CAST-IRON WHEEL TO TRANSMIT WITH SAFETY ANY GIVEN PRESSURE.

RULE.—*Multiply the square root of the pressure in pounds acting at the pitch line by the constant number* ·025. *The product is the proper thickness of the tooth in inches.*

Example 1.—What is the proper thickness of the teeth of a cast-iron wheel moved by a pressure of 233·33 lbs. at the pitch circle?

Here $\sqrt{233 \cdot 33} = 15 \cdot 27$, and this multiplied by $\cdot 025 = \cdot 381$, which is the proper thickness of the teeth in inches.

Example 2.—What is the proper thickness of the teeth of a cast-iron wheel which is moved round by a pressure of 46,666·6 lbs. at the pitch circle?

It will be easiest to solve this question by means of logarithms. As the index of the logarithm is always one less than the number of places above unity filled by the number of which the logarithm has to be found; and as there are five such places in the number 46,666·6, it follows that the index of the logarithm will be 4, and the rest of the logarithm will be found by looking for the nearest number to 46,666·6 in the tables, and which number will be 4,666, the logarithm answering to which is 668945. The residue 6·6, however, has not yet been taken into account, and to include it we must multiply the number found opposite to the logarithm in the column marked D, commonly introduced in logarithmic tables (and which is a column of common differences), by the number we have not yet reckoned, namely, 6·6; and cut off a number of figures from the product equal to those in the multiplier, adding the residue to the logarithm, which will thereupon become the correct logarithm of the whole quantity. The common difference in this case is 93, which multiplied by 6·6 gives 613·8, and cutting off the 3·8 we add the 61 to the logarithm already found, which then becomes 4·669006. Dividing this by 2, we get 2·334503, which will be the logarithm of the number that is the square root of 46,666·6. As the index of the logarithm is 2, there will be three places above unity in the number, and looking now in the logarithm tables for the number answer-

ing to the logarithm nearest 334503, we get the number 216, the logarithm of which is 334454. The number 216 is consequently the square root of 46,666·6 very nearly, as to extract the square root by logarithms, we have only to divide the logarithm of the number by 2, and the number answering to the new logarithm thus found will be the square root of the original number. Now 216 multiplied by ·025 = 5·400, which consequently is the thickness in inches of each of the teeth of this wheel.

GENERAL RULES REGARDING GEARING.

The pitch should be in all cases as fine as is consistent with the required strength. When the velocity of the motion exceeds $3\frac{1}{2}$ feet per second, the larger of the two wheels should be fitted with wooden teeth, the thickness of which should be a little greater than that of the iron teeth. The breadth of the teeth in the direction of the axis varies very much in practice. But where the velocity does not exceed 5 feet per second, a breadth of tooth in the line of the axis equal to four times the thickness of the tooth will suffice. This is nearly the same thing as a breadth equal to twice the pitch. Where the velocity at the pitch circle is greater than 5 feet per second, the breadth of the teeth should be 5 times the thickness of tooth, the surfaces being kept well greased. But if the teeth be constantly wet, the breadth should be 6 times the thickness of tooth at all velocities. The best length of the teeth is $\frac{5}{8}$ths of the pitch, and the length should not exceed $\frac{3}{4}$ths of the pitch, and the effective breadth of the teeth should not be reckoned as exceeding twice the length; any additional breadth being good for wear, but not for strength. In the Soho practice the length of the teeth is made $\frac{4}{12}$ths of the pitch outside, and $\frac{5}{12}$ths of the pitch inside of the pitch circle, the whole length being $\frac{9}{12}$ths or $\frac{3}{4}$ths of the pitch. The London practice is to divide the pitch into 12 parts, and to adjust the length of the tooth by allowing $\frac{3}{12}$ths without, and $\frac{4}{12}$ths within the pitch circle, the entire length of tooth being $\frac{7}{12}$ths of the pitch. The projection of the teeth beyond the pitch circle will be $\frac{1}{4}$th of the pitch, and the surface in contact between

the teeth of the two wheels will be half the pitch. About ⅛th of the pitch should be left unoccupied at the bottom of the teeth for clearance.

With regard to the least number of teeth that is admissible in the smaller of two wheels working together, 12 to 18 teeth will answer well enough in crane work, where a pinion is employed to give motion to a wheel at a low rate of speed. But for quick motions, a pinion driven by a wheel should never have less than from 30 to 40 teeth.

The best form of teeth is the epicycloidal, and in general the proper curve is obtained by rolling a circle of wood carrying a pencil on another circle of wood answering to the pitch circle, the point of the tooth being described by the rolling circle traversing the *outside* of the pitch line, and the root by traversing the *inside* of the pitch line. The diameter of the rolling circle should be 2·22 times the pitch. Some teeth are not epicycloidal, but the roots are radii of the pitch circle, and the points are described with compasses from the pitch centre of the next tooth.

In the following table will be found the thickness and pitch of teeth answering to different amounts of load or pressure at the pitch circles. But it may here be remarked that such large pitches as 12 and 13 inches are practically not used. In cases where such large pressures are to be transmitted as answer to pitches over 5 inches or thereabout, it is usual to distribute the load by placing two or more parallel wheels upon the same shaft, working into corresponding pinions; and it is also usual to set the teeth of each wheel a little in advance of the teeth of the wheel next it, so as to divide the pitch, and thus render the action of the teeth smoother and more continuous.

PROPORTIONS OF THE TEETH OF CAST-IRON WHEELS.

Pressure in lbs. at the pitch circle.	Pitch of teeth in inches, allowing one-tenth for clearance.	Thickness of teeth in inches.	Pressure in lbs. at the pitch circle.	Pitch of teeth in inches, allowing one-tenth for clearance.	Thickness of teeth in inches.
233·33	·798	·38	11666·65	5·6705	2·7002
349·95	·981	·467	13999·98	6·2118	2·9580
466·66	1·134	·540	16333·31	6·7099	3·1952
583·32	1·268	·604	18666·64	7·1728	3·4156
699·99	1·388	·661	20999·97	7·6079	3·6228
816·65	1·5	·716	23333·3	8·0194	3·8188
933·32	1·604	·763	25666·63	8·4109	4·0052
1049·98	1·7	·809	27999·96	8·7848	4·1832
1166·65	1·793	·854	30333·29	9·1470	4·3557
1283·31	1·88	·895	32666·62	9·4887	4·5184
1399·98	1·964	·935	34999·95	9·8218	4·6770
1516·64	2·044	·973	37333·28	10·1439	4·8304
1633·31	2·121	1·04	39666·61	10·4560	4·9790
1749·97	2·196	1·045	41999·94	10·7592	5·1234
1866·64	2·268	1·08	44333·27	11·0540	5·2638
1983·3	2·338	1·113	46666·6	11·3412	5·4006
2099·97	2·405	1·145	49999·93	11·7381	5·5896
2216·63	2·471	1·177	52333·26	12·0103	5·7192
2333·3	2·538	1·208	54666·59	12·2749	5·8452
2449·96	2·598	1·237	56999·92	12·5341	5·9686
2566·63	2·659	1·266	59333·25	12·7883	6·0897
2683·29	2·720	1·295	60666·58	12·9310	6·1576
2799·96	2·777	1·322	62999·91	13·1773	6·2749
4666·66	3·586	1·7078	65333·24	13·3893	6·3759
6999·99	4·3924	2·0916	67666·57	13·6566	6·5031
9333·32	5·0719	2·4152	69999·99	13·8901	6·6143

It will be useful to illustrate the application of these rules to the case of heavy gearing by one or two practical examples.

In a steamer with engines by Messrs. Penn and Son there are two cylinders of 82½ inches diameter and 6 feet stroke, giving motion to a toothed wheel 14 feet diameter consisting of four similar wheels bolted together, the teeth being 12 inches broad and 5·86 inches pitch. The area of a cylinder 82½ inches being 5,346 square inches, there will be a total pressure on the piston—if we reckon the mean average pressure upon each square inch at 25 lbs.—of 133,650 lbs. But as there are two pistons, the total pressure on the two pistons will be 267,300 lbs. Now the diameter of the geared wheel being 14 feet, its circumference will be 44 feet, and as at each movement of the pistons up and down through the length of the stroke, or through a distance of 12 feet, the wheel makes one revolution, or moves through 44 feet, the pressure at the circumference of the wheel will be less

than that on the pistons in the proportion in which 44 exceeds 12, so that by multiplying 267,300 by 12 and dividing the product by 44 we get the equivalent or balancing pressure at the circumference of the wheel, and which is 69,673 lbs. As, however, this load is distributed among four wheels, there will only be one-fourth of 69,673, or 17,418 lbs. to be borne by each of them. According to the rule we have given, therefore, the square root of 17,418 multiplied by ·025 will be the proper thickness of each tooth in inches. Now $\sqrt{17,418} = 132$, and $132 \times \cdot025 = 3\cdot3$, which by our rule is the proper thickness of the tooth in inches, and twice this, or 6·6, with one-tenth or ·3 for clearance, will be the pitch $= 6\cdot9$, whereas the actual pitch is 1 inch less than this. If the multiplier be made ·02, instead of ·025, the value obtained will agree more nearly with this example, as $132 \times \cdot02 = 2\cdot64$, which will be the thickness of tooth, and $2\cdot64 \times 2 = 5\cdot28$, to which adding $\frac{1}{10}$th of the thickness of the tooth for clearance, or ·264, we get 5·544 inches as the pitch. If we take the pressure at 20 lbs. per square inch on the pistons instead of 25 lbs., then the total pressure on the two pistons will be 213,840 lbs., which reduced to the equivalent pressure at the periphery of the wheel will be 58,320 lbs. The fourth of this is 14,580, the logarithm of which is 4·163758, the half of which is 2·081879, the natural number answering to which is 120·7, which multiplied by $\cdot025 = 3\cdot1175$, which is the proper thickness of the tooth in inches for this amount of strain. It will be seen, therefore, that the strength which our rule gives is somewhat greater than that of this example.

Let us now take an example by a different maker, and we select the geared engines of the steamer 'City of Glasgow,' constructed by Messrs. Tod and Macgregor. There were two cylinders in this vessel, each 66 inches diameter and 5 feet stroke, and the motion was communicated from the crank shaft to the screw shaft by means of four parallel wheels, 7 feet diameter, 8 inches broad, and 4 inches pitch. The area of a cylinder 66 inches diameter is 3,421 square inches, and the area of two such cylinders will, consequently, be 6,842 square inches. If we take the pressure urging the pistons at 20 lbs. per square inch, the total pressure on the pistons will be 136,840, which reduced to the

pressure at the periphery of the wheel—which moves 2·2 times faster than the pistons—will be 62,200 lbs.; and as the pressure is divided among four wheels there will be one-fourth of 62,200, or 15,550 lbs. on each. The logarithm of this number is 4·191430, the half of which is 2·095715, the natural number answering to which is 124·7, and 124·7 multiplied by ·025 = 3·1175, which is half as much again as the actual strength given in these wheels.

We may take still another example, and shall select the case of the 'Fire Queen,' a screw yatch constructed by Messrs. Robert Napier and Sons. In this vessel there are two cylinders, each of 36 inches diameter and 36 inches stroke, and the motion is communicated from the crank shaft to the screw shaft through the medium of three parallel wheels 8½ feet diameter placed on the end of the crank shaft. The pitch of the teeth is 3·55 inches and two of the wheels are 4 inches broad, and one of them 6 inches broad. The two narrow wheels may be reckoned as equivalent to one broad one, so we may consider the strain to be divided between two wheels. The area of each cylinder is 1,018 square inches, and if we reckon two cylinders of this area, with a pressure of 20 lbs. per square inch, urging the piston of each, the total pressure urging the pistons will be 40,720 lbs. The double stroke of the piston is 6 feet, and the circumference of the wheel is 26·7 feet; and as the wheel revolves once while the pistons are making a double stroke, the relative velocities will be 6 and 26·7, and the relative pressures 26·7 and 6. Multiplying, therefore, 40,720 by 6 and dividing by 26·7, we get 9,150 lbs. as the pressure at the circumference of the wheel; and as this load is to be divided between two wheels, there will be a load of 4,575 lbs. upon each. The logarithm of 4,575 is 3·660391, the half of which is 1·830195, the natural number answering to which is 67·64, which multiplied by ·025 gives 1·691 as the proper thickness of tooth in this wheel. Twice 1·691 is 3·382, to which if we add $\frac{1}{10}$th of the thickness of the tooth, or ·169 for clearance, we get 3·55 as the proper pitch of this wheel, and this is the very pitch which is really given. In this case, therefore, the rule and the example perfectly correspond. The rule gives

sufficient strength to represent the *mean* thickness of wooden and iron teeth—the wooden teeth being a little thicker, and the iron teeth a little thinner than the amount which the rule prescribes.

MARINE ENGINES.

The rules which I have given in my "Catechism of the Steam-Engine" for fixing the proper proportions of the parts of marine engines, take into account the pressure of the steam with which the engine works. But in order that the proportions thus arrived at may be more easily comparable with the proportions subsisting in the engines of different constructors, in which the pressure is assumed as tolerably uniform, it will be more convenient so to frame the rules that a uniform pressure of 25 lbs. per square inch of the area of the piston shall be supposed to be at all times existing. In cases where it is desired to ascertain the dimensions proper for a greater pressure than 25 lbs., it will be easy to arrive at the right result by taking an imaginary cylinder of as much greater area than the real cylinder as the real pressure exceeds the assumed pressure of 25 lbs., and then by computing the strengths and other proportions as if for this imaginary cylinder, they will be those proper for the real cylinder. Thus if it be desired to ascertain the strengths proper for an engine with a cylinder of 30 inches diameter, and with a pressure on the piston of 100 lbs. on the square inch, the end will be attained if we determine the strengths proper for an engine of 60 inches diameter, and with 25 lbs. pressure on the square inch; for the area of the larger cylinder being four times greater than that of the smaller, the same total force will be exerted with one-fourth of the pressure. So, in like manner, if it be wished to ascertain the strengths proper for an engine with a cylinder 30 inches diameter, and with a pressure on the piston of 50 lbs. per square inch, we shall find them by determining the proportions suitable for an engine with an area of piston twice greater than the area of a piston 30 inches diameter, and which area will be that answering to a diameter of 42½ inches.

By this mode of procedure a table of proportions adapted to the ordinary pressures will be made available for determining the proportions suitable for all pressures, as we have only to fix upon an assumed cylinder which shall have as much more area as the intended pressure has an excess of pressure over 25 lbs. per square inch, and the proportions proper for this assumed cylinder will be those proper for the real cylinder with the pressure intended. In this way the strengths fixed for marine engines may also be made applicable to locomotives and to high and low pressure engines of every kind. In the following rules, therefore, it will be understood the strengths and other proportions are those proper to an assumed pressure on the piston, including steam and vacuum, of 25 lbs. per square inch, and the computations are for side lever engines, but for the most part are applicable to all kinds of engines.

CROSSHEAD.

TO FIND THE PROPER THICKNESS OF THE WEB OF THE CROSSHEAD AT THE MIDDLE.

RULE.—*Multiply the diameter of the cylinder in inches by ·072.*

Example 1.—Let 40 inches be the diameter of the cylinder.

Then 40 inches × ·072 = 2·880 inches, which is the proper thickness of the web of the crosshead at the middle in this engine.

Example 2.—Let 64 inches be the diameter of cylinder.

Then 64 inches × ·072 = 4·608 inches, which is the proper thickness of the web of the crosshead at the middle in this engine.

TO FIND THE PROPER THICKNESS OF THE WEB OF THE CROSSHEAD AT THE JOURNAL.

RULE.—*Multiply the diameter of the cylinder in inches by ·061.*

Example 1.—Let 40 inches be the diameter of the cylinder.

Then 40 inches × ·061 = 2·440 inches, which is the proper

thickness of the web of the crosshead at the journal in this engine.

Example 2.—Let 64 inches be the diameter of the cylinder.

Then 64 inches × ·061 = 3·904 inches, which is the proper thickness of the web of the crosshead at the journal in this engine.

TO FIND THE PROPER DEPTH OF THE WEB OF THE CROSSHEAD AT THE MIDDLE.

RULE.—*Multiply the diameter of the cylinder in inches by* ·268

Example 1.—Let 40 inches be the diameter of the cylinder.

Then 40 inches × ·268 = 10·720 inches, which is the proper depth of the web of the crosshead at the middle in this engine.

Example 2.—Let 64 inches be the diameter of the cylinder.

Then 64 inches × ·268 = 17·152 inches, which is the proper depth of the web of the crosshead at the middle in this engine.

TO FIND THE PROPER DEPTH OF THE WEB OF THE CROSSHEAD AT JOURNALS.

RULE.—*Multiply the diameter of the cylinder in inches by* ·101.

Example 1.—Let 40 inches be the diameter of cylinder.

Then 40 inches × ·101 = 4·040 inches, which is the proper depth of the web of the crosshead at journals in this engine.

Example 2.—Let 64 inches be the diameter of the cylinder.

Then 64 inches × ·101 = 6·464 inches, which is the proper depth of the web of the crosshead at journals in this engine.

TO FIND THE PROPER DIAMETER OF THE JOURNALS OF THE CROSSHEAD.

RULE.—*Multiply the diameter of the cylinder in inches by* ·086.

Example 1.—Let 40 inches be the diameter of the cylinder.

Then 40 inches × ·086 = 3·440 inches, which is the proper diameter of the journals of the crosshead in this engine.

Example 2.—Let 64 inches be the diameter of cylinder.

Then 64 inches × ·086 = 5·504 inches, which is the proper diameter of the journal of crosshead in this engine.

TO FIND THE PROPER LENGTH OF THE JOURNALS OF THE CROSSHEAD.

The length of the journals of the crossheads should be equal to about 1¼ times their diameter, but on the whole it appears to be advisable to make the journals of the crosshead as long as they can be conveniently got.

TO FIND THE PROPER THICKNESS OF THE EYE OF THE CROSSHEAD.

RULE.—*Multiply the diameter of the cylinder in inches by* ·041.

Example 1.—Let 40 inches be the diameter of the cylinder.

Then 40 inches × ·041 = 1·640 inches, which is the proper thickness of the eye of the crosshead in this engine.

Example 2.—Let 64 inches be the diameter of the cylinder.

Then 64 inches × ·041 = 2·624 inches, which is the proper thickness of the eye of the crosshead in this engine.

TO FIND THE PROPER DEPTH OF THE EYE OF THE CROSSHEAD.

RULE.—*Multiply the diameter of the cylinder in inches by* ·286.

Example 1.—Let 40 inches be the diameter of the cylinder.

Then 40 inches × ·286 = 11·440 inches, which is the proper depth of the eye of the crosshead in this engine.

Example 2.—Let 64 inches be the diameter of the cylinder.

Then 64 inches × ·286 = 18·304 inches, which is the proper depth of the eye of the crosshead in this engine.

TO FIND THE PROPER DEPTH OF GIBS AND CUTTER PASSING THROUGH THE CROSSHEAD.

RULE.—*Multiply the diameter of the cylinder in inches by* ·105.

Example 1.—Let 40 inches be the diameter of the cylinder.

Then 40 inches × ·105 = 4·200 inches, which is the proper depth of the gibs and cutter passing through the crosshead in this engine.

Example 2.—Let 64 inches be the diameter of the cylinder.

Then 64 inches × ·105 = 6·720 inches, which is the proper depth of the gibs and cutter passing through the crosshead in this engine.

TO FIND THE PROPER THICKNESS OF THE GIBS AND CUTTER PASSING THROUGH THE CROSSHEAD.

RULE.—*Multiply the diameter of the cylinder in inches by* ·021.

Example 1.—Let 40 inches be the diameter of the cylinder.

Then 40 inches × ·021 = ·840 inches, which is the proper thickness of the gibs and cutter passing through the crosshead in this engine.

Example 2.—Let 64 inches be the diameter of the cylinder.

Then 64 inches × ·021 = 1·344 inches, which is the proper thickness of the gibs and cutter passing through the crosshead in this engine.

SIDE RODS.

TO FIND THE PROPER DIAMETER OF THE CYLINDER SIDE RODS AT THE ENDS.

RULE.—*Multiply the diameter of the cylinder in inches by* ·065.

Example 1.—Let 40 inches be the diameter of the cylinder.

Then 40 inches × ·065 = 2·600 inches, which is the proper diameter of cylinder side rods at ends in this engine.

Example 2.—Let 64 inches be the diameter of the cylinder.

Then 64 inches × ·065 = 4·160 inches, which is the proper diameter of the cylinder side rods at ends in this engine.

The diameter of the side rods at the middle should be about

one-fourth more than the diameter at the ends. Thus a side rod 5 inches diameter at the ends will be $6\frac{1}{4}$ inches diameter at the middle.

The area of the horizontal section of iron through the middle of eye of side rod is usually about one-half greater than the sectional area of the side rod at ends.

TO FIND THE PROPER BREADTH OF THE BUTT OF THE SIDE ROD IN INCHES.

RULE.—*Multiply the diameter of the cylinder in inches by* ·077.

Example 1.—Let 40 inches be the diameter of the cylinder.

Then 40 inches × ·077 = 3·080 inches, which is the proper breadth of butt of side rod in this engine.

Example 2.—Let 64 inches be the diameter of the cylinder.

Then 64 inches × ·077 = 4·928 inches, which is the proper breadth of butt in this engine.

TO FIND THE PROPER THICKNESS OF THE BUTT OF THE SIDE RODS.

RULE.—*Multiply the diameter of the cylinder in inches by* ·061.

Example 1.—Let 40 inches be the diameter of the cylinder.

Then 40 inches × ·061 = 2·440 inches, which is the proper thickness of the butt of the side rod in this engine.

Example 2.—Let 64 inches be the diameter of the cylinder.

Then 64 inches × ·061 = 3·904 inches, which is the proper thickness of the butt of the side rod in this engine.

TO FIND THE PROPER MEAN THICKNESS OF THE STRAP OF THE SIDE ROD AT THE CUTTER.

RULE.—*Multiply the diameter of the cylinder in inches by* ·032.

Example 1.—Let 40 inches be the diameter of the cylinder.

Then 40 inches × ·032 = 1·280 inches, which is the proper mean thickness of the strap of side rod at the cutter in this engine.

Example 2.—Let 64 inches be the diameter of the cylinder.

Then 64 inches × ·032 = 2·048 inches, which is the proper mean thickness of the strap of side rod at the cutter in this engine.

TO FIND THE PROPER MEAN THICKNESS OF THE STRAP OF SIDE ROD BELOW THE CUTTER.

RULE.—*Multiply the diameter of the cylinder in inches by* ·023.

Example 1.—Let 40 inches be the diameter of the cylinder.

Then 40 inches × ·023 = ·92 inches, which is the proper mean thickness of the strap of the side rod below the cutter in this engine.

Example 2.—Let 64 inches be the diameter of the cylinder.

Then 64 inches × ·023 = 1·472 inches, which is the proper mean thickness of the strap of the side rod below the cutter in this engine.

TO FIND THE PROPER DEPTH OF THE GIBS AND CUTTER OF SIDE ROD.

RULE.—*Multiply the diameter of the cylinder in inches by* ·08.

Example 1.—Let 40 inches be the diameter of the cylinder.

Then 40 inches × ·08 = 3·20 inches, which is the proper depth of gibs and cutter of side rod in this engine.

Example 2.—Let 64 inches be the diameter of the cylinder.

Then 64 inches × ·08 = 5·12 inches, which is the proper depth of gibs and cutter of side rod in this engine.

TO FIND THE PROPER THICKNESS OF GIBS AND CUTTER OF SIDE ROD.

RULE.—*Multiply the diameter of the cylinder in inches by* ·016.

Example 1.—Let 40 inches be the diameter of the cylinder.

Then 40 inches × ·016 = ·64 inches, which is the proper thickness of gibs and cutter of side rod in this engine.

Example 2.—Let 64 inches equal the diameter of cylinder.

Then 64 inches × ·016 = 1·02 inches, which is the proper thickness of gibs and cutter of side rod in this engine.

PISTON ROD.

TO FIND THE PROPER DIAMETER OF THE PISTON ROD.

RULE.—*Divide the diameter of the cylinder in inches by* 10.

Example 1.—Let 40 inches be the diameter of the cylinder.

Then 40 inches ÷ 10 = 4·0 inches, which is the proper diameter of piston rod in this engine.

Example 2.—Let 64 inches be the diameter of the cylinder.

Then 64 inches ÷ 10 = 6·4 inches, which is the proper diameter of piston rod in this engine.

TO FIND THE PROPER LENGTH OF THE PART OF THE PISTON ROD IN THE PISTON.

RULE.—*Divide the diameter of the cylinder in inches by* 5.

Example 1.—Let 40 inches be the diameter of the cylinder.

Then 40 inches ÷ 5 = 8·0 inches, which is the proper length of the part of the piston rod in the piston in this engine.

Example 2.—Let 64 inches be the diameter of the cylinder.

Then 64 inches ÷ 5 = 12·8 inches, which is the proper length of the part of the piston rod in the piston in this engine.

TO FIND THE MAJOR DIAMETER OF THE PART OF THE PISTON ROD IN THE PISTON.

RULE.—*Multiply the diameter of the cylinder in inches by* ·14.

Example 1.—Let 40 inches equal the diameter of cylinder.

Then 40 inches × ·14 = 5·60 inches, which is the proper major diameter of the part of the piston rod in piston in this engine.

Example 2.—Let 64 inches be the diameter of the cylinder.

Then 64 inches × ·14 = 8·96 inches, which is the proper major diameter of the part of the piston rod in piston in this engine.

TO FIND THE MINOR DIAMETER OF THE PART OF THE PISTON ROD IN THE PISTON.

RULE.—*Multiply the diameter of the cylinder in inches by* ·115.

Example 1.—Let 40 inches be the diameter of the cylinder.

Then 40 inches × ·115 = 4·600 inches, which is the proper minor diameter of the part of the piston rod in piston in this engine.

Example 2.—Let 64 inches be the diameter of the cylinder.

Then 64 inches × ·115 = 7·360 inches, which is the proper minor diameter of the part of the piston rod in piston in this engine.

TO FIND THE MAJOR DIAMETER OF THE PART OF THE PISTON ROD IN THE CROSSHEAD.

RULE.—*Multiply the diameter of the cylinder in inches by* ·095.

Example 1.—Let 40 inches be the diameter of the cylinder.

Then 40 inches × ·095 = 3·800 inches, which is the proper major diameter of the part of the piston rod in the crosshead in this engine.

Example 2.—Let 64 inches be the diameter of the cylinder.

Then 64 inches × ·095 = 6·080 inches, which is the proper major diameter of the part of the piston rod in the crosshead in this engine.

TO FIND THE MINOR DIAMETER OF THE PART OF THE PISTON ROD IN CROSSHEAD.

RULE.—*Multiply the diameter of the cylinder in inches by* ·09.

Example 1.—Let 40 inches be the diameter of the cylinder.

Then 40 inches × ·09 = 3·60 inches, which is the proper minor diameter of the part of the piston rod in crosshead in this engine.

Example 2.—Let 64 inches be the diameter of the cylinder.

Then 64 inches × ·09 = 5·76 inches, which is the proper minor diameter of the part of the piston rod in crosshead in this engine.

TO FIND THE PROPER DEPTH OF THE CUTTER THROUGH PISTON.

RULE.—*Multiply the diameter of the cylinder in inches by* ·085.

Example 1.—Let 40 inches be the diameter of the cylinder.

Then 40 inches × ·085 = 3·400 inches, which is the propei depth of the cutter through the piston in this engine.

Example 2.—Let 64 inches be the diameter of the cylinder.

Then 64 inches × ·085 = 5·440 inches, which is the proper depth of the cutter through the piston ir this engine.

TO FIND THE PROPER THICKNESS OF THE CUTTER THROUGH PISTON.

RULE.—*Multiply the diameter of the cylinder in inches by* ·035.

Example 1.—Let 40 inches by the diameter of the cylinder.

Then 40 inches × ·035 = 1·400 inches, which is the proper thickness of cutter through the piston in this engine.

Example 2.—Let 64 inches be the diameter of the cylinder.

Then 64 inches × ·035 = 2·240 inches, which is the proper thickness of cutter through piston in this engine.

CONNECTING-ROD.

TO FIND THE PROPER DIAMETER OF THE CONNECTING-ROD AT THE ENDS.

RULE.—*Multiply the diameter of the cylinder in inches by* ·095.

Example 1.—Let 40 inches be the diameter of the cylinder.

Then 40 inches × ·095 = 3·800 inches, which is the proper diameter of the connecting-rod at the ends in this engine.

Example 2.—Let 64 inches be the diameter of the cylinder.

Then 64 inches × ·095 = 6·080 inches, which is the proper diameter of the connecting-rod at the ends in this engine.

The diameter of the connecting-rod at the middle will vary with the length, but is usually one-fifth more than the diameter at the ends. Thus a connecting-rod 7·7 inches diameter at the ends will be 9·25 inches diameter at the middle.

TO FIND THE MAJOR DIAMETER OF THE PART OF CONNECTING-ROD IN THE CROSSTAIL.

RULE.—*Multiply the diameter of the cylinder in inches by* ·098.

Example 1.—Let 40 inches be the diameter of the cylinder.

Then 40 inches × ·098 = 3·920 inches, which is the proper major diameter of the part of the connecting-rod in the cross tail in this engine.

Example 2.—Let 64 inches be the diameter of the cylinder.

Then 64 inches × ·098 = 6·272 inches, which is the proper major diameter of the part of connecting-rod entering the cross tail in this engine.

TO FIND THE PROPER MINOR DIAMETER OF THE PART OF CONNECTING-ROD ENTERING THE CROSSTAIL.

RULE.—*Multiply the diameter of the cylinder in inches by* ·09.

Example 1.—Let 40 inches be the diameter of the cylinder.

Then 40 inches × ·09 = 3·60 inches, which is the proper minor diameter of the part of the connecting-rod in the cross-tail in this engine.

Example 2.—Let 64 inches be the diameter of the cylinder.

Then 64 inches × ·09 = 5·76 inches, which is the proper minor diameter of the part of the connecting rod in the cross-tail in this engine.

TO FIND THE PROPER BREADTH OF BUTT OF THE CONNECTING-ROD.

RULE.—*Multiply the diameter of the cylinder in inches by* ·156.

Example 1.—Let 40 inches be the diameter of the cylinder.

Then 40 inches × ·156 = 6·240 inches, which is the proper breadth of the butt of connecting-rod in this engine.

Example 2.—Let 64 inches be the diameter of the cylinder.

Then 64 inches × ·156 = 9·984 inches, which is the proper breadth of the butt of the connecting-rod in this engine.

TO FIND THE PROPER THICKNESS OF THE BUTT OF THE CONNECTING-ROD.

RULE.—*Divide the diameter of the cylinder in inches by* 8.

Example 1.—Let 40 inches be the diameter of the cylinder.

Then 40 inches ÷ 8 = 5·00 inches, which is the proper thickness of the butt of the connecting-rod in this engine.

Example 2.—Let 64 inches be the diameter of the cylinder.

Then 64 inches ÷ 8 = 8·00 inches, which is the proper thickness of the butt of the connecting-rod in this engine.

TO FIND THE PROPER MEAN THICKNESS OF THE STRAP OF CONNECTING-ROD AT THE CUTTER.

RULE.—*Multiply the diameter of the cylinder in inches by* ·043.

Example 1.—Let 40 inches be the diameter of the cylinder.

Then 40 inches × ·043 = 1·720 inches, which is the proper mean thickness of the connecting-rod strap at the cutter in this engine.

Example 2.—Let 64 inches be the diameter of the cylinder.

Then 64 inches × ·043 = 2·752 inches, which is the proper mean thickness of the connecting-rod strap at the cutter in this engine.

TO FIND THE PROPER MEAN THICKNESS OF THE CONNECTING-ROD STRAP ABOVE CUTTER.

RULE.—*Multiply the diameter of the cylinder in inches by* ·032.

Example 1.—Let 40 inches be the diameter of the cylinder.

Then 40 inches × ·032 = 1·280 inches, which is the proper mean thickness of the connecting-rod strap above the cutter in this engine.

Example 2.—Let 64 inches be the diameter of the cylinder.

Then 64 inches × ·032 = 2·048 inches, which is the proper mean thickness of the connecting-rod strap above the cutter in this engine.

TO FIND THE PROPER DISTANCE OF CUTTER FROM END OF STRAP OF CONNECTING-ROD.

RULE.—*Multiply the diameter of the cylinder in inches by* ·048.

Example 1.—Let 40 inches be the diameter of the cylinder.

Then 40 inches × ·048 = 1·920 inches, which is the proper distance of the cutter from the end of the strap of the connecting-rod in this engine.

Example 2.—Let 64 inches be the diameter of the cylinder.

Then 64 inches × ·048 = 3·072 inches, which is the proper distance of the cutter from the end of the strap of the connecting-rod in this engine.

TO FIND THE PROPER DEPTH OF THE GIBS AND CUTTER PASSING THROUGH THE CROSSTAIL.

RULE.—*Multiply the diameter of the cylinder in inches by* ·105.

Example 1.—Let 40 inches be the diameter of the cylinder.

Then 40 inches × ·105 = 4·20 inches, which is the proper depth of the gibs and cutter passing through the crosstail in this engine.

Example 2.—Let 64 inches be the diameter of the cylinder.

Then 64 inches × ·105 = 6·720 inches, which is the proper depth of the gibs and cutter passing through the crosstail in this engine.

The thickness of the cutters passing through the crosstail will be the same as the thickness of those passing through the crosshead.

TO FIND THE PROPER DEPTH OF THE GIBS AND CUTTER THROUGH THE BUTT OF THE CONNECTING-ROD.

RULE.—*Multiply the diameter of the cylinder in inches by* ·11.

Example 1.—Let 40 inches be the diameter of the cylinder.

Then 40 inches × ·11 = 4·40 inches, which is the proper depth of the gibs and cutter passing through the butt of the connecting-rod in this engine.

Example 2.—Let 64 inches be the diameter of the cylinder.

Then 64 inches × ·11 = 7·04 inches, which is the proper depth of the gibs and cutter passing through the butt of the connecting-rod in this engine.

TO FIND THE THICKNESS OF THE GIBS AND CUTTER THROUGH THE BUTT OF THE CONNECTING-ROD.

RULE.—*Multiply the diameter of the cylinder in inches by* ·029.

Example 1.—Let 40 inches be the diameter of the cylinder.

Then 40 inches × ·029 = 1·160 inches, which is the proper

thickness of the gibs and cutter passing through the butt of the connecting-rod in this engine.

Example 2.—Let 64 inches be the diameter of the cylinder.

Then 64 inches $\times \cdot 029 = 1 \cdot 856$ inches, which is the proper thickness of the gibs and cutter passing through the butt of the connecting-rod in this engine.

CROSSTAIL.

The crosstail is made in all respects the same as the crosshead, except that the end journals, where the crosstail butts fit on, are made so that the length is only equal to the diameter of the journal, instead of being about 1¼ times, as in the crosshead. But as the crosstail butts do not work on these journals or gudgeons, but are keyed fast upon them, the shorter length is preferable. The butts of the crosstail have the eyes nearly twice the diameter of the journals, or more accurately 1·8 times, and the butts for the reception of the straps for connecting to the side lever are made of the same dimensions as the butts of the side rods.

SIDE LEVER AND STUDS OR CENTRES.

The side lever is usually made of cast-iron. But it should be in all cases encircled by a strong wrought-iron hoop, thinned at the edge so that it may be riveted or bolted all along to a flange cast on the beam for this purpose, and forming an extension of the usual edge bead. The proportions given in the rules are those of the common cast-iron side levers as usually constructed. But the strength will be increased three times if wrought-iron be substituted for cast in the top and bottom flanges or edge beads.

TO FIND THE PROPER DEPTH OF THE SIDE LEVER ACROSS THE CENTRE.

RULE.—*Multiply the length of the side lever in feet by ·7423; extract the cube root of the product and reserve the root for a multiplier. Then square the diameter of the cylinder in*

inches; extract the cube root of the square. The product of the last result, and the reserved multiplier, is the depth of the side lever in inches across the centre.

Example 1.—What is the proper depth across the centre of the side lever in the case of an engine with a diameter of cylinder of 64 inches and length of side lever of 20 feet?

Here 20 = length of side lever in feet
·7433 length of multiplier

14·848 and $\sqrt[3]{14.846} = 2·458$ nearly

Also 64 = diameter of cylinder
64

4096 and $\sqrt[3]{4096} = 16$

Hence depth at centre = 16 × 2·458 = 39·30 inches, or between 39½ and 39 inches.

Example 2.—What is the proper depth across the centre of the side lever in the case of an engine with a diameter of cylinder of 40 inches, and length of side lever of 15 feet.

Here 15 = length of side lever
·7423

11·1345 and $\sqrt[3]{11·1345} = 2·232$

Also 40 = diameter of cylinder
40

1600 and $\sqrt[3]{1600} = 11·69$ which × 2·232 = 26·09,
or a little over 26 inches.

The depth of the side lever at the ends is determined by the depth of the eyes round the end studs. The thickness of the side lever is usually made about $\frac{1}{80}$th of its length, and the breadth of the edge bead is usually made about $\frac{1}{15}$ of the length of the lever between the end centres.

TO FIND THE PROPER DIAMETER OF THE MAIN CENTRE JOURNAL.

RULE.—*Multiply the diameter of the cylinder in inches by* ·183.

Example 1.—Let 40 inches be the diameter of the cylinder.

Then 40 inches × ·183 = 7·32 inches, which is the proper diameter of the main centre journal in this engine.

Example 2.—Let 64 inches be the diameter of the cylinder.

Then 64 inches × ·183 = 11·712 inches, which is the proper diameter of the main centre journal in this engine.

TO FIND THE LENGTH OF THE MAIN CENTRE JOURNAL.

RULE.—*Multiply the diameter of the cylinder in inches by* ·275.

Example 1.—Let 40 inches be the diameter of the cylinder.

Then 40 inches × ·275 = 11·00 inches, which is the proper length of the main centre journal in this engine.

Example 2.—Let 64 inches be the diameter of the cylinder.

Then 64 inches × ·275 = 17·60 inches, which is the proper length of the main centre journal in this engine.

TO FIND THE DIAMETER OF THE END STUDS OF THE SIDE LEVER.

RULE.—*Multiply the diameter of the cylinder in inches by* ·07.

Example 1.—Let 40 inches be the diameter of the cylinder.

Then 40 inches × ·07 = 2·80 inches, which is the proper diameter of the end studs of the side lever in this engine.

Example 2.—Let 64 inches be the diameter of the cylinder.

Then 64 inches × ·07 = 4·48 inches, which is the proper diameter of the end studs of the side lever in this engine.

TO FIND THE PROPER LENGTH OF THE END STUDS OF THE SIDE LEVER.

RULE.—*Multiply the diameter of the cylinder in inches by* ·076.

Example 1.—Let 40 inches be the diameter of the cylinder.

Then 40 inches × ·076 = 3·04 inches, which is the proper length of the end of studs of the side lever in this engine.

Example 2.—Let 64 inches be the diameter of the cylinder.

Then 64 inches × ·076 = 4·86 inches, which is the proper length of the end studs of the side lever in this engine.

TO FIND THE PROPER DIAMETER OF THE AIR-PUMP STUDS IN SIDE LEVER.

RULE.—*Multiply the diameter of the cylinder in inches by* ·045.

Example 1.—Let 40 inches be the diameter of the cylinder.

Then 40 inches × ·045 = 1·80 inches, which is the proper diameter of the stud in the side lever for working the air-pump of this engine.

Example 2.—Let 64 inches be the diameter of the cylinder.

Then 64 inches × ·045 = 2·88 inches, which is the proper diameter of the air-pump studs in the side levers of this engine.

TO FIND THE PROPER LENGTH OF THE AIR-PUMP STUDS SET IN THE SIDE LEVER.

RULE.—*Multiply the diameter of the cylinder in inches by* ·049.

Example 1.—Let 40 inches be the diameter of the cylinder.

Then 40 inches × ·049 = 1·96 inches, which is the proper length of the air-pump studs in this engine.

Example 2.—Let 64 inches be the diameter of the cylinder.

Then 64 inches × ·049 = 3·136 inches, which is the proper length of the air-pump studs in this engine.

TO FIND THE PROPER DEPTH OF THE EYE ROUND THE END STUDS OF SIDE LEVER.

RULE.—*Multiply the diameter of the cylinder in inches by* ·074.

Example 1.—Let 40 inches be the diameter of the cylinder.

Then 40 inches × ·074 = 2·96 inches, which is the proper depth of the eye round the end studs of the side lever in this engine.

Example 2.—Let 64 inches be the diameter of the cylinder.

Then 64 inches × ·074 = 4·736 inches, which is the proper depth of the eye round the end studs of the side lever in this engine.

It is clear that the diameter of the end stud added to twice the depth of the metal running round it will be equal to the depth of the side lever at the end

Hence 2·1 + twice 2·96 = 8·72 will be the depth in inches of the side lever at the ends in the engine with the 40-inch cylinder, and 4·48 + twice 4·736 = 13·95 will be the depth in inches of the side lever at the ends in the engine with the 64-inch cylinder.

TO FIND THE THICKNESS OF THE EYE ROUND THE END STUDS OF SIDE LEVER.

RULE.—*Multiply the diameter of the cylinder in inches by* ·052.

Example 1.—Let 40 inches be the diameter of the cylinder.

Then 40 inches × ·052 = 2·08 inches, which is the proper thickness of eye of side lever round the end studs in this engine.

Example 2.—Let 64 inches be the diameter of the cylinder.

Then 64 inches × ·052 = 3·328 inches, which is the proper thickness of eye of side lever round the end studs in this engine.

THE CRANK.

TO FIND THE PROPER DIAMETER OF THE CRANK-PIN JOURNALS.

RULE.—*Multiply the diameter of the cylinder in inches by* ·142.

Example 1.—Let 40 inches be the diameter of the cylinder.

Then 40 inches × ·142 = 5·680 inches, which is the proper diameter of the crank-pin journal in this engine.

Example 2.—Let 64 inches be the diameter of the cylinder.

Then 64 inches × ·142 = 9·080 inches, which is the proper diameter of the crank-pin journal in this engine.

TO FIND THE PROPER LENGTH OF THE CRANK-PIN JOURNAL.

RULE.—*Multiply the diameter of the cylinder in inches by* ·16.

Example 1.—Let 40 inches be the diameter of the cylinder.

Then 40 inches × ·16 = 6·40 inches, which is the proper length of the crank-pin journal in this engine.

Example 2.—Let 64 inches be the diameter of the cylinder.

Then 64 inches × ·16 = 10·24 inches, which is the proper length of the crank-pin journal in this engine.

TO FIND THE PROPER THICKNESS OF THE SMALL EYE OF CRANK.

RULE.—*Multiply the diameter of the cylinder in inches by* ·063.

Example 1.—Let 40 inches be the diameter of the cylinder.

Then 40 inches × ·063 = 2·52 inches, which is the proper thickness of the small eye of the crank in this engine.

Example 2.—Let 64 inches be the diameter of the cylinder.

Then 64 inches × ·063 = 4·032 inches, which is the proper thickness of the small eye of the crank in this engine.

TO FIND THE PROPER BREADTH OF THE SMALL EYE OF THE CRANK.

RULE.—*Multiply the diameter of the cylinder in inches by* ·187.

Example 1.—Let 40 inches be the diameter of the cylinder.

Then 40 inches × ·187 = 7·48 inches, which is the proper breadth of the small eye of the crank in this engine.

Example 2.—Let 64 inches be the diameter of the cylinder.

Then 64 inches × ·187 = 11·968 inches, which is the proper breadth of the small eye of the crank in this engine.

TO FIND THE PROPER THICKNESS OF THE WEB OF CRANK, SUPPOSING IT TO BE CONTINUED TO CENTRE OF CRANK PIN.

RULE.—*Multiply the diameter of the cylinder in inches by* ·11.

Example 1.—Let 40 inches be the diameter of the cylinder.

Then 40 inches × ·11 = 4·40 inches, which is the proper thickness of the web of crank in this engine, supposing it to be continued so far as centre of pin.

Example 2.—Let 64 inches be the diameter of the cylinder.

Then 64 inches × ·11 = 7·04 inches, which is the proper thickness of the web of the crank in this engine, supposing that the thickness were to be continued to the centre of the crank-pin and to be there measured.

TO FIND THE PROPER THICKNESS OF THE WEB OF THE CRANK, SUPPOSING THE THICKNESS TO BE CONTINUED TO THE CENTRE OF THE PADDLE-SHAFT.

RULE.—*Multiply the square of the length of the crank in inches by* 1·561, *and then multiply the square of the diameter of cylinder in inches by* ·1235. *Multiply the square root of the sum of these products by the square of the diameter of the cylinder in inches; divide this quotient by* 360; *finally extract the cube root of the quotient. The result is the thickness of the web of the crank at paddle shaft centre in inches.*

Example 1.—What is the proper thickness of the web of crank

at the centre of the paddle-shaft, supposing the thickness to be continued thither and there measured, in the case of an engine with a diameter of cylinder of 64 inches and stroke of 8 feet.

48 = length of crank in inches
48
2304
1·561 constant multiplier
3596·5
505·8 product of 64^2 and ·1235
4102·3

64 = diameter of cylinder
64
4096
·1235
505·8

and $\sqrt{4102{\cdot}3} = 64{\cdot}05$ nearly
4096 = square of diameter
360)262348·5
728·75

And $\sqrt[3]{728} = 9$ nearly, which is the proper thickness in inches of the crank of this engine measured at the centre of the paddle shaft.

Example 2.—What is the proper thickness of the web of crank at paddle-shaft centre in the case of an engine with a cylinder 40 inches in diameter and stroke of 5 feet?

30 = length of crank in inches
30
900 = square of length of crank
1·561 = constant multiplier
1404·9

12*

$$
\begin{array}{r l}
40 & = \text{diameter of cylinder in inches} \\
40 & \\
\hline
1600 & = \text{square of diameter of cylinder} \\
\cdot 235 & = \text{constant multiplier} \\
\hline
197 \cdot 9 & \\
1404 \cdot 9 & \\
\hline
1602 \cdot 8 & \text{and } \sqrt{1602 \cdot 8} = 40 \cdot 03
\end{array}
$$

$$
\begin{array}{r}
1600 \\
\hline
360)64048 \\
\hline
177 \cdot 9
\end{array}
$$

And $\sqrt[3]{177 \cdot 9} = 5 \cdot 62$, which is the proper thickness in inches of the web of the crank, supposing the web to be continued to the centre of the paddle shaft.

TO FIND THE PROPER BREADTH OF THE WEB OF THE CRANK AT PIN-CENTRE, SUPPOSING IT TO BE CONTINUED TO THE CENTRE OF THE CRANK-PIN.

RULE.—*Multiply the diameter of the cylinder by* ·16. *The product is the proper breadth of the web of the crank, supposing the web to be continued to the plane of the centre of the crank-pin.*

Example 1.—Let the diameter of the cylinder be 40 inches.

Then 40 inches × ·16 = 6·4, which is the proper breadth in inches of the web of the crank in the plane of the centre of the crank-pin in this engine.

Example 2.—Let the diameter of the cylinder be 64 inches.

Then 64 inches × ·16 = 10·24 inches, which is the proper breadth of the web of the crank at the crank-pin end in this engine.

TO FIND THE PROPER BREADTH OF THE CRANK AT PADDLE-CENTRE.

RULE.—*Multiply the square of the length of crank in inches by* 1·561, *and then multiply the square of the diameter of cylinder in inches by* ·1235; *multiply the square root of the sum of these products by the square of the diameter of the cyl-*

inder in inches; divide the product by 45. Finally, extract the cube root of the quotient.

Example 1.—What is the proper breadth of the crank at paddle-centre in the case of an engine with a diameter of cylinder of 64 inches and stroke of 8 feet?

$$
\begin{array}{rl}
48 & \text{length of crank in inches} \\
48 & \\
\hline
2304 & \\
1\cdot561 & \text{constant multiplier} \\
\hline
3596\cdot5 & \\
505\cdot8 & \\
\hline
4102\cdot3 & \\
\hline\hline
64 & \text{diameter of cylinder} \\
64 & \\
\hline
4086 & \\
\cdot1235 & \text{constant multiplier} \\
\hline
505\cdot8 & \\
\hline\hline
\end{array}
$$

and $\sqrt{4102\cdot3} = 64\cdot05$ nearly

$$
\begin{array}{r}
4096 \\
\hline
45)262348\cdot5 \\
\hline
\end{array}
$$

$5829\cdot97$ and $\sqrt[3]{5829} = 18$ nearly, which is the proper breadth in inches of the web of the crank at the shaft-centre in this engine.

Example 2.—What is the proper breadth of crank at paddle-centre in the case of an engine with a diameter of cylinder of 40 inches and stroke of 5 feet?

$$
\begin{array}{rl}
30 = & \text{length of crank in inches} \\
30 & \\
\hline
900 = & \text{square of length of crank} \\
1\cdot561 & \\
\hline
1404\cdot9 & \\
\hline\hline
\end{array}
$$

$$
\begin{array}{r l}
40 & = \text{diameter of cylinder} \\
40 & \\
\hline
1600 & = \text{square of diameter of cylinder} \\
\cdot 1235 & \\
\hline
197{\cdot}6 & \\
1404{\cdot}9 & \\
\hline
1602{\cdot}5 &
\end{array}
$$

and $\sqrt{1602{\cdot}5} = 40{\cdot}03$

$$
\begin{array}{r}
1600 \\
\hline
45)64048 \\
\hline
1466{\cdot}7
\end{array}
$$

and $\sqrt[3]{1466{\cdot}7} = 11{\cdot}24$ nearly.

The purpose of taking the breadth and thickness of the web of the crank at the shaft and pin-centres is to obtain fixed points for measurement. For, although the web of the crank does not extend either to the centre of the shaft or to the centre of the pin, it can easily be drawn in as if extending to those points, and the breadth and thickness being then laid down at those points the proper amount of taper in the web of the crank will be obtained.

TO FIND THE PROPER THICKNESS OF THE LARGE EYE OF THE CRANK.

RULE.—*Multiply the square of the length of the crank in inches by* 1·561, *then multiply the square of the diameter of the cylinder in inches by* ·1235; *multiply the sum of these products by the square of the cylinder in inches; divide the quotient by the length of the crank in inches; afterwards divide the product by* 1828·28. *Finally, extract the cube root of the quotient. The result is the proper thickness in inches of the large eye of crank.*

Example 1.—What is the proper thickness of large eye of the the crank in the case of an engine with a diameter of cylinder of 64 inches and stroke of 8 feet?

$$
\begin{array}{r l}
48 & = \text{length of crank in inches} \\
48 & \\
\hline
2304 & = \text{square of length of crank} \\
1{\cdot}561 & = \text{constant multiplier} \\
\hline
3596{\cdot}5 & \\
505{\cdot}8 & = \text{product of } 64^2 \text{ and } 1235 \\
\hline
4102{\cdot}3 & \\
\hline\hline
\end{array}
$$

$$
\begin{array}{r l}
64 & = \text{diameter of cylinder} \\
64 & \\
\hline
4096 & \\
{\cdot}1235 & = \text{constant multiplier} \\
\hline
505{\cdot}8 & \\
\hline\hline
\end{array}
$$

$$
\begin{array}{r l}
4102{\cdot}3 & \\
4096 & = \text{square of diameter} \\
\hline
48)16803020{\cdot}8 & \\
\hline
1828{\cdot}28)350062{\cdot}94 & \\
\hline
191{\cdot}47 & \\
\end{array}
$$

and $\sqrt[3]{191{\cdot}47} = 5{\cdot}77$ nearly, which is the proper thickness of the large eye of the crank in inches.

Example 2.—What is the proper thickness of the large eye of crank in the case of an engine with a diameter of cylinder of 40 inches and with a stroke of 5 feet?

$$
\begin{array}{r l}
30 & = \text{length of crank in inches} \\
30 & \\
\hline
900 & = \text{square of length of crank} \\
1{\cdot}561 & \text{constant multiplier} \\
\hline
1404{\cdot}9 & \\
\hline\hline
\end{array}
$$

$$
\begin{array}{r l}
40 & = \text{diameter of cylinder} \\
40 & \\
\hline
1600 & \\
\cdot 1235 & \text{constant multiplier} \\
\hline
197\cdot 6 & \\
1404\cdot 9 & \text{add} \\
\hline
1602\cdot 5 & \\
1600 & = \text{square of diameter} \\
\hline
1828\cdot 28)2564000 & \\
\hline
30)1402\cdot 41 & \\
\hline
46\cdot 74 &
\end{array}
$$

and $\sqrt[3]{46\cdot 74} = 3\cdot 60$, which is the proper thickness in inches of the large eye of the crank in this engine.

TO FIND THE PROPER DIAMETER OF THE PADDLE-SHAFT JOURNAL.

RULE.—*Multiply the square of the diameter of the cylinder in inches by the length of crank in inches; extract the cube root of the quotient. Finally, multiply the result by ·242. The final product is the diameter of the paddle-shaft journal in inches.*

Example 1.—What is the proper diameter of the paddle-shaft journal in the case of an engine with a diameter of cylinder of 64 inches and stroke of 8 feet?

$$
\begin{array}{r l}
64 & = \text{diameter of cylinder in inches} \\
64 & \\
\hline
4096 & \text{square of diameter of cylinder} \\
48 & = \text{length of crank in inches} \\
\hline
196608 &
\end{array}
$$

and $\sqrt[3]{196608} = 58\cdot 148$, and $58\cdot 148 \times \cdot 242 = 14\cdot 07$ inches.

Example 2.—What is the proper diameter of the paddle-shaft journal in the case of an engine with a diameter of cylinder of 40 inches and a stroke of 5 feet?

40 = diameter of cylinder

1600 = square of diameter of cylinder
30 = length of crank in inches

48000 and $\sqrt[3]{48000} = 36{\cdot}30$
and $36{\cdot}30 \times {\cdot}242 = 8{\cdot}79$ inches.

TO FIND THE PROPER LENGTH OF THE PADDLE-SHAFT JOURNAL.

RULE.—*Multiply the square of the diameter of the cylinder in inches by the length of the crank in inches; extract the cube root of quotient; multiply the result by ·303. The product is the length of the paddle-shaft journal in inches.*

Example 1.—What is the proper length of the paddle-shaft journal in the case of an engine with a diameter of cylinder 64 inches and stroke 8 feet?

64 = diameter of cylinder
64

4096 = square of diameter of cylinder
48 = length of crank in inches

196608 and $\sqrt[3]{196608} = 58{\cdot}148$
Length of journal $= 58{\cdot}148 \times {\cdot}303 = 17{\cdot}60$ inches.

Example 2.—What is the proper length of the paddle-shaft journal in the case of an engine with a diameter of cylinder of 40 feet and stroke of 5 feet?

40
40

1600
30

48000 and $\sqrt{48000} = 36{\cdot}30 \times {\cdot}303 = 10{\cdot}99$.

It will be seen from these examples that the length of the paddle-shaft journals is $1\frac{1}{4}$ times the diameter. The paddle-shafts, cranks, and all the other working parts of marine

engines are made of wrought-iron, except the side levers, which are of cast-iron, and the air-pump rod, which is of copper or brass.

THE AIR-PUMP.

TO FIND THE PROPER DIAMETER OF AIR-PUMP.

RULE.—*Multiply the diameter of the cylinder in inches by* ·6.

Example 1.—Let 40 inches be the diameter of the cylinder.

Then 40 inches × ·6 = 24·0 inches, which is the proper diameter of the air-pump in this engine.

Example 2.—Let 64 inches be the diameter of the cylinder.

Then 64 inches × ·6 = 38·4 inches, which is the proper diameter of the air-pump in this engine.

AIR-PUMP ROD.

TO FIND THE PROPER DIAMETER IN INCHES OF THE AIR-PUMP ROD WHEN OF COPPER.

RULE.—*Multiply the diameter of the cylinder in inches by* ·067.

Example 1.—Let the diameter of the cylinder be 40 inches.

Then 40 × ·067 = 2·68 inches, which is the proper diameter of the air-pump rod when of copper in this engine.

Example 2.—Let the diameter of the cylinder be 64 inches.

Then 64 × ·067 = 4·28 inches, which is the proper diameter of the air-pump rod when of copper in this engine.

TO FIND THE PROPER DEPTH OF GIBS AND CUTTER THROUGH THE AIR-PUMP CROSSHEAD IN INCHES.

RULE.—*Multiply the diameter of the cylinder in inches by* ·063.

Example 1.—Let the diameter of the cylinder be 40 inches.

Then 40 × ·063 = 2·52 inches, which is the proper depth of gibs and cutter through the air-pump crosshead in this engine.

Example 2.—Let the diameter of the cylinder be 64 inches.

Then 64 × ·063 = 4·03 inches, which is the proper depth of gibs and cutter through the air-pump crosshead in this engine.

TO FIND THE PROPER THICKNESS OF GIBS AND CUTTER THROUGH AIR-PUMP CROSSHEAD IN INCHES.

RULE.—*Multiply the diameter of the cylinder in inches by* ·013.

Example 1.—Let the diameter of the cylinder be 40 inches.

Then 40 × ·013 = ·52 inches, which is the proper thickness of gibs and cutter through the air-pump crosshead in this engine.

Example 2.—Let the diameter of the cylinder be 64 inches.

Then 64 × ·013 = ·83 inches, which is the proper thickness of gibs and cutter through the air-pump crosshead in this engine.

TO FIND THE PROPER DEPTH IN INCHES OF THE CUTTER THROUGH THE AIR-PUMP BUCKET.

RULE.—*Multiply the diameter of the cylinder in inches by* ·051.

Example 1.—Let the diameter of the cylinder be 40 inches.

Then 40 × ·051 = 2·04 inches, which is the proper depth of the cutter through the air-pump bucket in this engine.

Example 2.—Let the diameter of the cylinder be 64 inches.

Then 64 × ·051 = 3·26 inches, which is the proper depth of the cutter through the air-pump bucket in this engine.

TO FIND THE PROPER THICKNESS OF THE CUTTER THROUGH THE AIR-PUMP BUCKET IN INCHES.

RULE.—*Multiply the diameter of the cylinder in inches by* ·021.

Example 1.—Let the diameter of the cylinder be 40 inches.

Then 40 × ·021 = ·84 inches, which is the proper thickness of the cutter through the air-pump bucket in this engine.

Example 2.—Let the diameter of the cylinder be 64 inches.

Then 64 × ·021 = 1·34 inches, which is the proper thickness of the cutter through the air-pnmp bucket in this engine.

The cutter through the air-pump bucket should be always made of brass or copper, but the gibs and cutter through the air-pump crosshead will be of iron. The air-pump bucket should always be of brass, and it is advisable to insert the rod into the crosshead and also into the bucket with a good deal of taper, so as to facilitate its detachment should the bucket require to be taken out. It is usual to form the part of the rod projecting

through the crosshead into a screw, and to screw a nut upon it. This also is a common practice at the top of the piston rod and at the bottom of the connecting-rod.

AIR-PUMP CROSSHEAD.

TO FIND THE PROPER DEPTH OF THE EYE OF THE AIR-PUMP CROSSHEAD.

RULE.—*Multiply the diameter of the cylinder in inches by* ·171.

Example 1.—Let 40 inches be the diameter of the cylinder.

Then 40 inches × ·171 = 6·84 inches, which is the proper depth of eye of air-pump crosshead in this engine.

Example 2.—Let 64 inches be the diameter of the cylinder.

Then 64 inches × ·171 = 10·944 inches which is the proper depth of the eye of air-pump crosshead in this engine.

TO FIND THE PROPER DEPTH OF THE AIR-PUMP CROSSHEAD AT THE MIDDLE OF THE WEB.

RULE.—*Multiply the diameter of the cylinder in inches by* ·161.

Example 1.—Let 40 inches be the diameter of the cylinder.

Then 40 inches × ·161 = 6·44 inches, which is the proper depth at the middle of the web of the air-pump crosshead in this engine.

Example 2.—Let 64 inches be the diameter of the cylinder.

Then 64 inches × ·161 = 10·30 inches, which is the proper depth at the middle of the web of the air-pump crosshead in this engine.

TO FIND THE PROPER DEPTH OF THE WEB OF THE AIR-PUMP CROSSHEAD AT JOURNALS.

RULE.—*Mutiply the diameter of the cylinder in inches by* ·061.

Example 1.—Let 40 inches be the diameter of the cylinder.

Then 40 inches × ·061 = 2·44 inches, which is the proper depth of the web of the air-pump crosshead at the journals in this engine.

Example 2.—Let 64 inches be the diameter of the cylinder.

Then 64 inches × ·061 = 3·90 inches, which is the proper depth of the web of the air-pump crosshead at the journals in this engine.

TO FIND THE PROPER THICKNESS OF THE EYE OF THE AIR-PUMP CROSSHEAD.

RULE.—*Multiply the diameter of the cylinder in inches by* ·025.

Example 1.—Let 40 inches be the diameter of the cylinder.

Then 40 inches × ·025 = 1·00 inches, which is the proper thickness of the eye of the air-pump crosshead in this engine.

Example 2.—Let 64 inches be the diameter of the cylinder.

Then 64 inches × ·025 = 1·600 inches, which is the proper thickness of the eye of the air-pump crosshead in this engine.

TO FIND THE PROPER THICKNESS OF THE WEB OF THE AIR-PUMP CROSSHEAD AT THE MIDDLE.

RULE.—*Multiply the diameter of the cylinder in inches by* ·043.

Example 1.—Let 40 inches be the diameter of the cylinder.

Then 40 inches × ·043 = 1·72 inches, which is the proper thickness of the web of the air-pump crosshead at the middle in this engine.

Example 2.—Let 64 inches be the diameter of the cylinder.

Then 64 inches × ·043 = 2·75 inches, which is the proper thickness of the web of the air-pump crosshead at the middle in this engine.

TO FIND THE PROPER THICKNESS OF THE WEB OF THE AIR-PUMP CROSSHEAD AT THE JOURNALS.

RULE.—*Multiply the diameter of the cylinder in inches by* ·037.

Example 1.—Let 40 inches be the diameter of the cylinder.

Then 40 inches × ·037 = 1·48 inches, which is the proper thickness of the web of the air-pump crosshead at the journals in this engine.

Example 2.—Let 64 inches be the diameter of the cylinder.

Then 64 inches × ·037 = 2·36 inches, which is the proper thickness of the web of the air-pump crosshead at the journals in this engine.

TO FIND THE PROPER DIAMETER OF THE JOURNALS OF THE AIR-PUMP CROSSHEAD.

RULE.—*Multiply the diameter of the cylinder in inches by* ·051.

Example 1.—Let 40 inches be the diameter of the cylinder.

Then 40 inches × ·051 = 2·04 inches, which is the proper diameter of the journals of the air-pump crosshead in this engine.

Example 2.—Let 64 inches be the diameter of the cylinder.

Then 64 inches × ·051 = 3·26 inches, which is the proper diameter of the journals of the air-pump crosshead in this engine.

TO FIND THE PROPER LENGTH OF THE JOURNALS OF THE AIR-PUMP CROSSHEAD.

RULE.—*Multiply the diameter of the cylinder in inches by* ·058.

Example 1.—Let 40 inches be the diameter of the cylinder.

Then 40 inches × ·058 = 2·32 inches, which is the proper length of the end journals for the air-pump crosshead in this engine.

Example 2.—Let 64 inches be the diameter of the cylinder.

Then 64 inches × ·058 = 3·71 inches, which is the proper length of the end journals for the air-pump crosshead in this engine.

AIR-PUMP SIDE RODS.

TO FIND THE PROPER DIAMETER OF AIR-PUMP SIDE ROD AT THE ENDS.

RULE.—*Multiply the diameter of the cylinder in inches by* ·039.

Example 1.—Let 40 inches be the diameter of the cylinder.

Then 40 inches × ·039 = 1·56 inches, which is the proper diameter of air-pump side rod at ends in this engine.

Example 2.—Let 64 inches be the diameter of the cylinder.

Then 64 inches × ·039 = 2·49 inches, which is the proper diameter of air-pump side rod at ends in this engine.

TO FIND THE BREADTH OF BUTT FOR AIR-PUMP SIDE RODS.

RULE.—*Multiply the diameter of the cylinder in inches by* ·046

Example 1.—Let 40 inches be the diameter of the cylinder.

Then 40 inches × ·046 = 1·84 inches, which is the proper breadth of butt for air-pump side rod in this engine.

Example 2.—Let 64 inches be the diameter of the cylinder.

Then 64 inches × ·046 = 2·94 inches, which is the proper breadth of butt of air-pump side rod in this engine.

TO FIND THE PROPER THICKNESS OF BUTT FOR AIR-PUMP SIDE ROD.

RULE.—*Multiply the diameter of the cylinder in inches by* ·037.

Example 1.—Let 40 inches be the diameter of the cylinder.

Then 40 inches × ·037 = 1·48 inches, which is the proper thickness of butt for air-pump side rod in this engine.

Example 2.—Let 64 inches be the diameter of the cylinder.

Then 64 inches × ·037 = 2·36 inches, which is the proper thickness of butt for air-pump side rod in this engine.

TO FIND THE MEAN THICKNESS OF STRAP AT CUTTER OF AIR-PUMP SIDE ROD.

RULE.—*Multiply the diameter of the cylinder in inches by* ·019.

Example 1.—Let 40 inches be the diameter of the cylinder.

Then 40 inches × ·019 = ·76 inches, which is the proper mean thickness of the strap at cutter of air-pump side rod in this engine.

Example 2.—Let 64 inches be the diameter of the cylinder.

Then 64 inches × ·019 = 1·21 inches, which is the proper mean thickness of the strap at cutter of air-pump side rod in this engine.

TO FIND THE PROPER MEAN THICKNESS OF THE STRAP BELOW CUTTER OF AIR-PUMP SIDE ROD.

RULE.—*Multiply the diameter of the cylinder in inches by* ·014.

Example 1.—Let 40 inches be the diameter of the cylinder.

Then 40 inches × ·014 = ·56 inches, which is the proper mean thickness of the strap below cutter in the air-pump side rod of this engine.

Example 2.—Let 64 inches be the diameter of the cylinder.

Then 64 inches × ·014 = ·89 inches, which is the proper mean thickness of strap below cutter in the air-pump side rod of this engine.

TO FIND THE PROPER DEPTH OF THE GIBS AND CUTTER FOR AIR-PUMP SIDE ROD.

RULE.—*Multiply the diameter of the cylinder in inches by* ·048.

Example 1.—Let 40 inches be the diameter of the cylinder.

Then 40 inches × ·048 = 1·92 inches, which is the proper depth of gibs and cutter for air-pump side rod in this engine.

Example 2.—Let 64 inches be the diameter of the cylinder.

Then 64 inches × ·048 = 3·07 inches, which is the proper depth of gibs and cutter for the air-pump side rod in this engine.

TO FIND THE PROPER THICKNESS OF THE GIBS AND CUTTER FOR THE AIR-PUMP SIDE ROD.

RULE.—*Divide the diameter of the cylinder in inches by* 100.

Example 1.—Let 40 inches be the diameter of the cylinder.

Then 40 inches ÷ 100 = ·40 inches, which is the proper thickness of the gibs and cutter for the air-pump side rod in this engine.

Example 2.—Let 64 inches be the diameter of the cylinder.

Then 64 inches ÷ 100 = ·64 inches, which is the proper thickness of the gibs and cutter of the air-pump side rod in this engine.

It will be satisfactory to compare the dimensions of the parts of engines with the actual dimensions obtaining in some engines of good proportions which have for some time been in successful operation; and I select for the purpose of this comparison the side-lever engines constructed by Messrs. Caird & Co., for the West India Mail steamers 'Clyde,' 'Tweed,' 'Tay,' and 'Tevoit.' The dimensions of the main parts given by the rules, and the actual dimensions, are exhibited in the following table, touching which it is sufficient to remark that where there is any appreciable divergence between the two, the dimensions given by the rules appear to be the preferable ones:—

COMPARISON OF DIMENSIONS GIVEN BY THE FOREGOING RULES WITH THE ACTUAL DIMENSIONS OF THE MAIN PARTS OF THE SIDE LEVER ENGINES OF THE STEAMERS 'CLYDE,' 'TWEED,' 'TAY,' AND 'TEVIOT,' OF 450 HORSES POWER, CONSTRUCTED BY MESSRS. CAIRD & CO.

Diameter of Paddle-Shaft Journal.	Dimensions by Rules.	Actual Dimensions.
Diameter of paddle-shaft journal	15·15	15·25
Exterior diameter of large eye of crank	27·84	27·875
†Diameter of crank pin journal	10·49	9·5
Exterior diameter of small eye of crank	19·8177	20·625
†Length of small eye of crank	18·875	13·25
Thickness of web of crank at paddle centre	9·8	10·5
" " at crank pin centre	8·14	9·75
Breadth of crank at crank pin centre	12·21	15·0
Diameter of piston rod	7·4	7·75
Diameter of connecting rod at ends	7·03	7·6
† " " at middle	9·98	9·25
" side rod at ends	4·77	5·0
" " at middle	6·6	6·875
" eye of crosshead (outside)	13·5	14·5
Depth of eye of crosshead (outside)	21·133	21·25
Diameter of crosshead journal	6·349	6·375
Thickness of web of crosshead at centre	5·3	5·5
†Depth of web of crosshead at centre	19·85	19·5
Thickness of web of crosshead at journal	4·514	4·375
Depth of web of crosshead at journal	7·511	9·75
†Diameter of main centre journal	0·0367 × $P^{\frac{1}{2}} \times D =$ 13·579	11·5

The rules give generally smaller numbers than Messrs. Caird's practice. The difference is greatest in 'Breadth of crank at crank-pin centre,' and in 'Exterior diameter of eye of crosshead,' and 'Depth of web of crosshead of journals.'

In five cases above, marked thus †, the rules give greater strength than the example selected of Messrs. Caird's engine, especially in 'Diameter of main centre,' where Messrs. Caird's proportions are quite too small.

I have already explained that from any one drawing, all sizes of engines of that particular form may be constructed by merely altering the scale; and all the dimensions of ships and engines, and, in fact, every quantity whatever which increases or diminishes in a given ratio, or according to a uniform law, may be expressed graphically by a curve, which will have its corresponding equation, though sometimes that equation will be too complicated to be numerically expressible. Mr. Watt, in his early

practice, laid down most of the dimensions of his engines to curves, and, indeed, was in the habit of using that mode of investigation and expression in all his researches. A table of the dimensions of the parts of engines may easily be laid down in the form of a curve; and the benefit of that practice is, that if we have a certain number of points in the curve, we can easily find all the intermediate ones by merely measuring with a pair of compasses and a scale of equal parts. Thus, for example, we may lay down the table of the diameter of crank-shaft, given in page 294, to a curve as follows:—First draw a straight horizontal line, which divide into equal parts by any convenient scale, beginning, as in the table, with 20, and ending with 100. If now we erect vertical lines or ordinates at every division of the horizontal line, and if, with any given length of stroke, say 2 feet, we know the diameter of shaft proper for some of the diameters of cylinder—say for a 20-inch cylinder, 4·08 inches; for a 24-inch cylinder, 4·66 inches; for a 40-inch cylinder, 6·55 inches; and for an 80-inch cylinder, 10·29 inches—we can easily determine the diameters of shaft proper for all the intermediate diameters of cylinders, by marking off with the same scale, or any other, the vertical heights corresponding to all the diameters we know; and a curve traced through these points will intersect all the other ordinates, and give the diameters proper for the whole series. By thus setting down the known quantities in order to deduce the unknown, we shall at the same time see whether the quantities we set down follow a regular law of increase or not; for if they do not, instead of all the points marked off falling into a regular curve, some of them will be above the curve and some of them beneath it, thus showing that the quantities given do not form portions of a homogeneous system. If the quantity increases in arithmetical progression, the curve will become a straight angular line. Thus in the case of the diameter of the piston rod, as the increase follows the same law as the increase of the diameter of the cylinder, the law of increase will be expressed by a right-angled triangle, the diameters of the cylinder being represented by the divisions on the base, and the diameters of piston rod by the corresponding vertical ordinates. If to the

curve of diameter of crank shaft for each diameter of cylinder with any given length of stroke, we add below the base another curve pointing downwards, representing the increase of the diameter of the shaft due to every increase of the length of the stroke, the diameter of the cylinder remaining the same, the total height of the conjoint ordinates will show the diameter of the shaft for each successive diameter of cylinder and length of stroke. One of the curves will be convex and the other concave, and the convexity of the one will be equal to the concavity of the other, so that the ordinates will be the same as those of a triangle. Hence, if we double the diameter of the cylinder, and also double the length of the stroke, we shall double the diameter of the shaft; if we treble the diameter of the cylinder, and also treble the length of the stroke, we shall treble the diameter of the shaft, and so on in all other proportions. By referring to the table in page 294, we shall see that these relations are there preserved. Thus a 20-inch cylinder and a 2-feet stroke has a shaft of 4·08 inches in diameter; a 40-inch cylinder and a 4-feet stroke, a shaft of 8·16 inches diameter; a 60-inch cylinder and a 6-feet stroke, a shaft 12·25 inches diameter, and so on. If this were not so, an engine drawn on any one scale would not be applicable to any other of a different size; whereas we know that any one drawing will do for all sizes of engines by merely changing the scale.

It is very convenient in making drawings of engines to adopt some uniform size for the drawing-boards and drawings, and to adhere to them on all occasions. The best arranged drawing-office I have met with is that of Boulton and Watt, which was originally settled in its present form by Mr. Watt himself, who brought the same good sense and habits of methodical arrangement to this problem that he did to every other. The basis of Boulton and Watt's sizes of drawings is the dimensions of a sheet of double elephant drawing paper; and all their drawings are either of that size, of half that size, or of a quarter that size, leaving a proper width for margin. The drawing-boards are all made with a frame fitting around them, so that it is not necessary to glue the paper round the edges; but the damped sheet

TABLE OF THE DIMENSIONS OF THE PRINCIPAL

MARINE ENGINES OF

NAMES OF PARTS.	NOMINAL			
	10 H. P.	15 H. P.	20 H. P.	25 H. P.
Diameter of	in.	in.	in.	in.
Cylinder	20	24	27	29½
Piston rod	2	2⅜	2½	3
Air-pump	12	15	17	17½
Air-pump rod	1¼	1½	2	2⅛
Injection cock	1¼	1½	1⅝	1¾
Hot-water pump	2½	2½	3	3¼
Feed-pipe	1½	1½	2	2⅛
Steam-pipe	4	5	5½	6
Waste-water pipe	5	6	7	7½
Beam gudgeon	3½	4½	5	5½
Pins in beam ends	2	2⅜	2½	3
Air-pump pins in beam	1¼	1⅝	1¾	1⅞
Crank-pin	2½	3	3½	3¾
Main shaft	4¼	5½	6¼	6½
Paddle-wheels, in feet	9	11	11	12
Weight-shaft bearings	2	2¼	2½	2½
Stroke of				
Piston	24	30	30	33
Air-pump bucket	12	15	15	16½
Feed-pump plunger	6	7½	7½	8
Cylinder crosshead				
Depth of boss	6	7½	8	9
Diameter of boss	4	4½	5	5½
Depth of middle	5	5½	6¼	7
Thickness	1½	1⅝	1¾	2
Air-pump crosshead				
Depth of boss	4½	5	5½	6½
Diameter of boss	2¾	3⅜	3½	4
Depth of middle	3½	4	4½	5
Thickness	1	1⅛	1¼	1⅜
Columns				
Diameter at top	4	4¾	5½	5½
Diameter at bottom	4½	5½	6¼	6⅝
Centre to centre of				
Air-pump, side-rods transversely	29½	34¼	37½	39½
Beams " "	33	39	42½	45
Frames " "	21	23	25½	26
Engines " "	66	72	76	80
Length of steam port	7½	8½	10	11
Breadth of steam port	1½	1¾	2	2¼
Port valve passage				
Depth	2	2	2½	2½
Width	13	14	15½	17
Beam				
Breadth at middle	14	18	19	21
Breadth at ends	5		6¼	7½
Thickness	1	1½	1⅓	1⅜

PARTS OF MESSRS. MAUDSLAY, SONS, AND FIELD'S

DIFFERENT POWERS.

POWER OF ENGINE.									
30 H. P.	40 H. P.							110 H. P.	120 H. P.
in.	in.	in.	in.	in.	in.	in.	in.	in.	in.
33	36½	40	43	46	48	50	52½	55½	57
3½	[illegible]	4	4½	4½	4½	4½	5	5½	5½
18½	21	23	24	26	27½	28	30	31½	[illegible]
2½	2½	2½	2½	3	3½	3½	3½	4	4½
2	2½	2½	2½	3	3½	3½	3½	3½	3½
3½	4	4½	4½	5	5½	6	6½	7	7½
2½	2½	2½	2½	3	3½	3½	3½	3½	4
6½	7	7½	8½	9½	10	10½	11	11½	12
8	9	9½	10	10½	11½	12½	13	13½	14
5½	6	6½	7	7½	8	8½	9	9½	9½
3½	3½	4	4½	4½	4½	5	5½	5½	5½
2	2½	2½	2½	2⅞	2⅞	3	3½	3½	3½
4	4½	5	5½	6	6½	7	7½	7½	8
7	7½	8½	9½	10	10½	10½	11½	12	12½
13	13	15	17	17	19	19	21	21	23
2½	2½	2½	2½	3	3½	3½	3½	2½	3½
33	36	42	[illegible]	52	56	60	63	66	72
16	13	21	24	26	28	30	31½	33	36
[illegible]	9	10½	12	13	14	15	16	16½	18
9½	10½	12	13	14	14½	15	16	17	17½
6	6½	7½	8	8½	9	9½	10	11	12
7½	8½	9½	10½	11½	11½	12½	13	13½	14
2½	2½	2½	3	3½	3½	3½	3½	4	4½
6½	8	9	10	10½	10½	11	11½	12	12½
4½	4½	5½	5½	5½	6	6½	6½	7½	7½
6½	6½	7	7½	8	8½	8½	9	9½	9½
1½	1½	2	2½	2½	2½	2½	2½	2½	2½
6	7	8	8½	9½	9½	9½	10	10½	10½
6½	7½	9	9½	10½	10½	11	11½	11½	12
42½	47½	53	55½	60½	63	67	68½	[illegible]	72
48	54	60	63	69	69	72	78	80	88
27	30	34	34	40	40	42	44	45	46
84	88	96	100	108	108	112	126	126	130
11½	13	15	13½	13½	19	19	20	20	21
2½	2½	3	3	4	4	[illegible]	4½	4½	4½
3½	3½	4	4½	5	5½	5½	6	6½	7
13	20	24	26	28	28	29	31	31	32
23	25	28	29	33	34	35	36	38	39
8	8½	10	10½	12	12½	12½	12	16	15½
1½	1½	1½	2	2½	2½	2½	[illegible]	2½	2½

TABLE OF THE DIMENSIONS OF THE PRINCIPAL
ENGINES OF DIF-

Power in nominal Horses.	Diameter of Cylinder in inches.	Diameter of air-pump in inches.	Stroke.		Diameter of paddle-wheel.	Diameter of main shaft.	Depth of boss of crosshead.	Diameter of piston rod.	Diameter of air-pump rod, copper.	Diameter of air-pump rod, iron.	Diameter of steam pipe.
			ft.	in.	ft.	in.	in.	in.	in.	in.	in.
10	20	12	2	0	9	4	7	2	1¼	1	4
15	24	15	2	6	11	4¾	8⅜	2⅜	1⅜	1¼	5
20	27	17	2	6	11	5¾	10	3	1½	1⅜	5¾
30	31¾	18⅜	3	0	13	6¾	10¾	3¼	1⅞	1⅝	6⅜
40	36⅛	20	3	0	13	7⅜	11⅛	3⅛	2¼	1⅞	7
50	39⅛	22	3	6	15	8¼	12¼	3¾	2⅛	2	7¾
60	43	24	4	0	17	9	13	4	2½	2¼	8⅜
70	46	25⅜	4	3	17	8⅜	13¾	4¼	3	2⅜	9¼
80	48	27	4	6	19	10	14⅜	4⅜	3¼	2¾	10
90	50	28	4	9	19	10⅜	15¼	4¾	3⅜	3	10⅜
100	52⅜		5	0	21	11	16	5	3½	3⅛	11
110	55		5	0	21	11⅜	16¾	5¼	4	3¼	11⅜
120	57⅜		5	6	23	12⅜	17⅜	5⅜	4¼	3⅜	12

is laid upon the board, which it somewhat overlaps, and the frame then comes down and turns over the edges of the paper upon the sides of the board, and the frame being then fixed so that its face is flush with the paper, the paper by being thus bound all round the edges is properly stretched when dry. In Mr. Watt's time the drawings were made with copying ink, and an impression was taken from them by passing them through a roller press, so as to retain the original in the office, while a duplicate of it was sent out with the work; and the copying press was invented by Mr. Watt for this purpose. The whole of the drawings pertaining to each particular engine are placed in a small paper portfolio by themselves; and these small portfolios are numbered and arranged in drawers, with a catalogue to tell the particular engine delineated in the drawings of each portfolio. In this way I have found that the drawings illustrative of any engine, though it may have been made in the last

PARTS OF MESSRS. SEAWARD AND CO.'S MARINE
FERENT POWERS.

Number of lbs. plate for steam-pipe.	Diameter of waste-water pipe.	Diameter of feed-pipe.	Length of beam.		Bearing of crosshead (diameter).	Main-beam gudgeon (diameter).	[illegible] red bearing (diameter).	Weight shaft, length of bearing.	Weight shaft, diameter of bearing.		
	in.	in.	ft.	in.	in.	in.	in.	in.	in.	in.	in.
26	5	1⅛	6	0	2	3	1⅞	2⅛	2	2⅜	2⅝
28	6	1½	7	0	2¼	4	1⅛	2¼	2¼	3	3¼
30	7	2	8	0	2½	5	1½	3	2½	3¼	3¾
35	8	2¼	8	6	2¾	5½	2	3⅛	2½	4	4¼
38	9	2½	10	0	3	6	2¼	3½	2¾	4¼	4¾
40	9½	2¾	10	6	3¼	6½	2½	3¾	2¾	5	5½
44	10	3	11	6	3½	7	2¾	[illegible]	2¾	5½	6
50	10	3¼	12	6	3¾	7½	3	4¼	3	6	6½
55	11	3½	13	0	4	8	3¼	4½	[illegible]	6½	7
58	12½	3¾	13	6	4¼	8½	3½	4¾	3¼	7	7½
62	13	4	15	0	4½	9	3¾	5	3½	7¼	7¾
66	13½	4¼	16	0	4¾	9½	3¾	5½	3¾	7½	[illegible]
70	14	4½	17	6	5	9¾	3⅞	5¾	3¾	8	8¼

century, could be produced to me in a few minutes; and the system is altogether more perfect and more convenient than any other with which I am acquainted. The portfolios are not large, which would make them inconvenient, but are of such size that a double elephant sheet has to be folded to go into one of them; but most of the drawings are on small sheets of paper, which is a much more convenient practice than that of drawing the details upon large sheets.

It will be interesting to compare with the results given in the foregoing rules the actual sizes of some side lever engines of approved construction. Accordingly I have recapitulated, in the tables introduced above, the principal dimensions of the marine engines of Messrs. Maudslay and Messrs. Seaward. These tables are so clear, that they do not require further explanation, and the same remark is applicable to the tables which follow.

DIAMETER OF WROUGHT-IRON CRANK-SHAFT JOURNAL.

LENGTH OF WROUGHT-IRON CRANK-SHAFT JOURNAL.

Diameter of Cylinder in inches.	LENGTH OF STROKE IN FEET.											
	2	2½	3	3½	4	4½	·5	5½	6	7	8	9
20	5·10	5·49	5·84	6·15	6·43	6·69	6·93	7·16	7·36	7·75	8·11	8·43
21	5·37	5·69	6·03	6·31	6·62	6·87	7·14	7·36	7·59	7·99	8·36	8·69
22	5·53	5·85	6·21	6·50	6·82	7·08	7·35	7·59	7·82	8·23	8·61	8·96
23	5·68	6·02	6·39	6·69	7·02	7·29	7·57	7·81	8·05	8·47	8·86	9·23
24	5·84	6·19	6·57	6·88	7·22	7·50	7·78	8·04	8·28	8·71	9·11	9·50
25	5·99	6·36	6·75	7·07	7·42	7·70	8·00	8·26	8·51	8·95	9·36	9·76
26	6·15	6·53	6·93	7·26	7·62	7·91	8·21	8·48	8·74	9·19	9·61	10·02
27	6·30	6·70	7·11	7·45	7·82	8·11	8·43	8·70	8·97	9·43	9·86	10·28
28	6·46	6·87	7·29	7·64	8·02	8·32	8·64	8·92	9·20	9·67	10·11	10·54
29	6·61	7·04	7·47	7·83	8·22	8·53	8·86	9·14	9·43	9·91	10·36	10·80
30	6·77	7·21	7·65	8·02	8·42	8·74	9·07	9·36	9·65	10·15	10·61	11·04
31	6·92	7·37	7·82	8·19	8·60	8·93	9·27	9·56	9·86	10·37	10·84	11·28
32	7·06	7·52	7·98	8·36	8·78	9·11	9·47	9·76	10·06	10·58	11·07	11·52
33	7·20	7·67	8·14	8·53	8·96	9·30	9·56	9·96	10·27	10·79	11·30	11·76
34	7·34	7·82	8·30	8·70	9·14	9·48	9·75	10·15	10·47	11·00	11·53	11·99
35	7·48	7·97	8·46	8·87	9·31	9·67	9·94	10·35	10·68	11·22	11·76	12·22
36	7·62	8·12	8·63	9·04	9·49	9·85	10·13	10·55	10·88	11·43	11·98	12·45
37	7·76	8·27	8·79	9·21	9·67	10·04	10·32	10·74	11·09	11·64	12·20	12·68
38	7·90	8·42	8·95	9·38	9·85	10·22	10·51	10·94	11·29	11·86	12·42	12·91
39	8·05	8·57	9·11	9·55	10·03	10·41	10·80	11·14	11·49	12·07	12·64	13·14
40	8·19	8·72	9·27	9·72	10·20	10·58	10·99	11·34	11·69	12·29	12·86	13·37
41	8·32	8·86	9·42	9·88	10·37	10·75	11·17	11·53	11·88	12·49	13·07	13·59
42	8·44	9·00	9·57	10·03	10·54	10·92	11·34	11·71	12·06	12·69	13·28	13·81
43	8·56	9·13	9·72	10·18	10·70	11·09	11·52	11·89	12·25	12·88	13·49	14·03
44	8·68	9·27	9·87	10·37	10·86	11·26	11·69	12·07	12·43	13·08	13·70	14·25
45	8·80	9·40	10·02	10·49	11·02	11·43	11·87	12·25	12·62	13·28	13·91	14·46
46	8·92	9·54	10·17	10·64	11·18	11·59	12·04	12·48	12·80	13·47	14·12	14·67
48	9·16	9·81	10·47	10·94	11·50	11·93	12·39	12·79	13·17	13·87	14·52	15·09
50	9·40	10·08	10·76	11·26	11·82	12·27	12·75	13·15	13·55	14·27	14·92	15·51
52	9·65	10·34	11·04	11·56	12·18	12·59	13·09	13·50	13·90	14·64	15·32	15·92
54	9·89	10·60	11·32	11·86	12·44	12·91	13·43	13·85	14·25	15·02	15·71	16·32
56	10·13	10·86	11·60	12·16	12·75	13·23	13·77	14·19	14·60	15·39	16·09	16·72
58	10·37	11·13	11·88	12·46	13·06	13·55	14·11	14·53	14·95	15·76	16·47	17·12
60	10·61	11·37	12·15	12·75	13·37	13·87	14·44	14·86	15·31	16·11	16·85	17·52
62	10·84	11·62	12·41	13·00	13·61	14·15	14·76	15·18	15·64	16·47	17·23	17·90
64	11·07	11·86	12·67	13·25	13·85	14·42	15·06	15·50	15·97	16·83	17·59	18·28
66	11·30	12·11	12·93	13·50	14·09	14·70	15·36	15·82	16·30	17·18	17·95	18·66
68	11·53	12·35	13·19	13·75	14·33	14·97	15·66	16·14	16·63	17·52	18·31	19·04
70	11·76	12·60	13·46	14·01	14·55	15·25	15·96	16·46	16·96	17·86	18·67	19·42
72	11·98	12·84	13·71	14·29	14·88	15·56	16·26	16·77	17·28	18·20	19·03	19·78
74	12·20	13·07	13·96	14·58	15·21	15·87	16·56	17·08	17·60	18·54	19·39	20·14
76	12·42	13·30	14·20	14·87	15·54	16·18	16·86	17·39	17·92	18·87	19·73	20·50
78	12·64	13·53	14·44	15·15	15·87	16·49	17·16	17·70	18·24	19·19	20·07	20·86
80	12·86	13·76	14·70	15·44	16·20	16·81	17·45	18·00	18·55	19·52	20·41	21·22
82	13·07	13·99	14·95	15·70	16·46	17·08	17·73	18·29	18·86	19·84	20·75	21·58
84	13·28	14·22	15·19	15·95	16·72	17·35	18·03	18·58	19·16	20·16	21·09	21·94
86	13·49	14·45	15·43	16·20	16·98	17·62	18·31	18·87	19·46	20·48	21·43	22·28
88	13·70	14·68	15·67	16·45	17·24	17·89	18·59	19·16	19·75	20·80	21·75	22·62
90	13·91	14·91	15·92	16·76	17·50	18·15	18·87	19·47	20·06	21·11	22·07	22·96
92	14·11	15·14	16·15	16·95	17·76	18·42	19·14	19·75	20·36	21·42	22·40	23·30
94	14·31	15·36	16·38	17·19	18·02	18·69	19·40	20·01	20·65	21·73	22·73	23·64
96	14·51	15·59	16·61	17·43	18·28	18·96	19·67	20·38	20·94	22·03	23·05	23·98
98	14·71	15·82	16·84	17·67	18·54	19·23	19·95	20·57	21·23	22·33	23·37	24·32
100	14·91	16·08	17·07	17·92	18·80	19·52	20·25	20·83	21·52	22·65	23·69	24·64

BREADTH OF WEB OF CRANK, SUPPOSING IT TO BE CONTINUED TO PADDLE-SHAFT CENTRE.

THICKNESS OF WEB OF CRANK, SUPPOSING IT TO BE CONTINUED TO PADDLE-SHAFT CENTRE.

Diameter of Cylinder in inches.	LENGTH OF STROKE IN FEET.											
	2	2½	3	3½	4	4½	5	5½	6	7	8	9
20	2·64	2·72	2·81	3·02	3·23	3·36	3·49	3·59	3·70	3·89	4·06	4·22
21	2·75	2·84	2·93	3·14	3·34	3·47	3·61	3·71	3·82	4·02	4·19	4·36
22	2·86	2·96	3·05	3·25	3·45	3·58	3·73	3·83	3·94	4·14	4·32	4·50
23	2·97	3·08	3·17	3·36	3·56	3·69	3·85	3·95	4·06	4·26	4·45	4·64
24	3·08	3·19	3·29	3·47	3·67	3·80	3·97	4·07	4·18	4·38	4·58	4·77
25	3·19	3·30	3·41	3·58	3·78	3·91	4·09	4·19	4·30	4·50	4·71	4·90
26	3·30	3·41	3·52	3·69	3·89	4·02	4·20	4·30	4·42	4·62	4·84	5·03
27	3·41	3·52	3·63	3·80	4·00	4·13	4·31	4·41	4·53	4·74	4·97	5·16
28	3·52	3·63	3·74	3·91	4·10	4·24	4·42	4·52	4·64	4·86	5·10	5·29
29	3·63	3·74	3·85	4·02	4·20	4·35	4·53	4·63	4·75	4·98	5·22	5·42
30	3·75	3·85	3·96	4·13	4·30	4·46	4·62	4·74	4·87	5·11	5·34	5·55
31	3·83	3·94	4·06	4·23	4·39	4·56	4·72	4·85	4·98	5·23	5·46	5·67
32	3·91	4·03	4·16	4·33	4·49	4·66	4·82	4·96	5·09	5·34	5·57	5·79
33	3·99	4·12	4·26	4·43	4·59	4·76	5·92	5·07	5·20	5·45	5·69	5·91
34	4·07	4·21	4·36	4·53	4·69	4·86	5·02	5·18	5·31	5 57	5·80	6·03
35	4·15	4·30	4·45	4·62	4·79	4·96	5·12	5·28	5·42	5·68	5·92	6·15
36	4·22	4·38	4·54	4·71	4·89	5·06	5·22	5·38	5·53	5·79	6·03	6·27
37	4·29	4·46	4·63	4·80	4·98	5·16	5·32	5·48	5·64	5·90	6·15	6·39
38	4·36	4·54	4·72	4·89	5·08	5·26	5·42	5·58	5·74	6·01	6·26	6·51
39	4·43	4·62	4·81	4·98	5·18	5·36	5·52	5·68	5·84	6·12	6·38	6·62
40	4·50	4·70	4·90	5·09	5·28	5·45	5·62	5·78	5·94	6·23	6·49	6·74
41	4·60	4·80	5·00	5·19	5·38	5·54	5·70	5·86	6·03	6·34	6·61	6·86
42	4·70	4·90	5·10	5·29	5·48	5·63	5·78	5·94	6·11	6·45	6·73	6·97
43	4·80	5·00	5·20	5·39	5·58	5·72	5·86	6·02	6·20	6·56	6·85	7·08
44	4·89	5·09	5·30	5·49	5·68	5·81	5·94	6·10	6·28	6·67	6·97	7·19
45	4·98	5.18	5·39	5·58	5·78	5·90	6·02	6·18	6·37	6·77	7·09	7·30
46	5·07	5·27	5·48	5·67	5·87	5·98	6·10	6·26	6·45	6·87	7·21	7·41
48	5·25	5·45	5·66	5·85	6·05	6·14	6·26	6·42	6·62	7·07	7·45	7·63
50	5·43	5·63	5·84	6·03	6·23	6·30	6·38	6·58	6·79	7·27	7·67	7·85
52	5·61	5·81	6·03	6·23	6·43	6·52	6·62	6·81	7·03	7·47	7·87	8·07
54	5·78	5·98	6·21	6·42	6·63	6·74	6·86	7·04	7·27	7·67	8·05	8·29
56	5·94	6·14	6·39	6·60	6·82	6·96	7·10	7·27	7·50	7·87	8·23	8·51
58	6·10	6·30	6·57	6·78	7·00	7·16	7·33	7·50	7·72	8·07	8·41	8·71
60	6·26	6·50	6·75	6·96	7·18	7·36	7·55	7·78	7·92	8·27	8·59	8·91
62	6·48	6·71	6·95	7·16	7·36	7·56	7·75	7·98	8·11	8·47	8·79	9·11
64	6·70	6·91	7·15	7·34	7·54	7·74	7·94	8·13	8·30	8·65	8·97	9·31
66	6·92	7·10	7·33	7·52	7·72	7·92	8·12	8·31	8·49	8·84	9·17	9·52
68	7·12	7·30	7 51	7·70	7·90	8·10	8·30	8·49	8·68	9·04	9·37	9·72
70	7·32	7·50	7·69	7·88	8·08	8·28	8·48	8·67	8·87	9·24	9·59	9·92
72	7·51	7·69	7·87	8·07	8·28	8·46	8·66	8·85	9·07	9·48	9·83	10·12
74	7·70	7·88	8·06	8·26	8·47	8·64	8·82	9·03	9·26	9·71	10·06	10·32
76	7·89	8·07	8·25	8·46	8·67	8·82	8·98	9·21	9·44	9·93	10·28	10·53
78	8·08	8·26	8·44	8·65	8·87	9·00	9·14	9·39	9·62	10·16	10·51	10·73
80	8·27	8·44	8·62	8·84	9·06	9·18	9·30	9·55	9·80	10·37	10·75	10·94
82	8·46	8·63	8·81	9·02	9·24	9·37	9·41	9·75	9·99	10·58	10·91	11·13
84	8·65	8·82	9·00	9·20	9·42	9·56	9·72	9·94	10·13	10·69	11·07	11·31
86	8·85	9·01	9·18	9·38	9·60	9·74	9·92	10·13	10.36	10·84	11.23	11·50
88	9·04	9·20	9·36	9·56	9·75	9·93	10·13	10·33	10·54	10·98	11·37	11·69
90	9·23	9·39	9·55	9·74	9·93	10·13	10·33	10·53	10·78	11·13	11·51	11·89
92	9·43	9·58	9·74	9·93	10·11	10·32	10·51	10·71	10·92	11·32	11·69	12·03
94	9·63	9·78	9·93	10·12	10·30	10·50	10·70	10·90	11·11	11·50	11·89	12·27
96	9·82	9·97	10·12	10·30	10·49	10·69	10·89	11·09	11·29	11·68	12·08	12·46
98	10·01	10·16	10·31	10·49	10·68	10·88	11·08	11·27	11·48	11·87	12·27	12·64
100	10·20	10·35	10·50	10·68	10·86	11·06	11·26	11·46	11·66	12·06	12·45	12·84

THICKNESS OF LARGE EYE OF CRANK.

Diameter of Cylinder in inches.	LENGTH OF STROKE IN FEET.											
	2	2½	3	3½	4	4½	5	5½	6	7	8	9
20	1·71	1·80	1·89	1·97	2·05	2·12	2·19	2·25	2·32	2·44	2·55	2·64
21	1·77	1·87	1·95	2·04	2·13	2·20	2·27	2·32	2·40	2·52	2·64	2·75
22	1·83	1·93	2·01	2·11	2·20	2·28	2·35	2·39	2·48	2·60	2·72	2·86
23	1·89	1·99	2·07	2·18	2·28	2·36	2·43	2·46	2·56	2·68	2·80	2·97
24	1·95	2·06	2·14	2·25	2·35	2·44	2·51	2·54	2·64	2·76	2·88	3·08
25	2·01	2·12	2·21	2·32	2·43	2·52	2·59	2·62	2·72	2·84	2·96	3·18
26	2·07	2·19	2·28	2·39	2·50	2·59	2·66	2·70	2·80	2·92	3·04	3·28
27	2·13	2·25	2·35	2·46	2·58	2·66	2·73	2·78	2·87	2·99	3·12	3·38
28	2·19	2·31	2·42	2·53	2·65	2·73	2·80	2·86	2·94	3·06	3·20	3·48
29	2·25	2·37	2·49	2·60	2·73	2·80	2·87	2·94	3·01	3·13	3·28	3·58
30	2·30	2·43	2·56	2·68	2·80	2·87	2·94	3·01	3·08	3·18	3·36	3·68
31	2·36	2·50	2·63	2·74	2·87	2·94	3·00	3·07	3·15	3·26	3·43	3·73
32	2·42	2·56	2·69	2·80	2·94	3·01	3·06	3·13	3·22	3·34	3·50	3·78
33	2·49	2·62	2·75	2·86	3·00	3·08	3·12	3·20	3·29	3·41	3·57	3·83
34	2·55	2·69	2·81	2·92	3·06	3·15	3·18	3·27	3·36	3·48	3·64	3·88
35	2·61	2·75	2·87	2·98	3·12	3·22	3·25	3·34	3·43	3·55	3·71	3·93
36	2·67	2·81	2·93	3·04	3·18	3·29	3·32	3·41	3·50	3·62	3·78	3·98
37	2·74	2·87	2·99	3·10	3·24	3·35	3·39	3·48	3·57	3·69	3·85	4·03
38	2·81	2·94	3·05	3·16	3·30	3·41	3·46	3·55	3·64	3·77	3·91	4·07
39	2·87	3·00	3·11	3·22	3·36	3·47	3·53	3·62	3·71	3·84	3·97	4·11
40	2·93	3·05	3·17	3·29	3·42	3·51	3·60	3·69	3·78	3·90	4·03	4·15
41	3·03	3·13	3·24	3·37	3·49	3·58	3·67	3·75	3·85	3·98	4·11	4·24
42	3·13	3·22	3·31	3·45	3·56	3·65	3·74	3·81	3·92	4·06	4·19	4·32
43	3·23	3·30	3·39	3·53	3·63	3·72	3·81	3·87	3·99	4·14	4·27	4·40
44	3·33	3·39	3·47	3·60	3·69	3·79	3·88	3·94	4·06	4·22	4·35	4·48
45	3·43	3·47	3·55	3·67	3·75	3·86	3·95	4·01	4·13	4·30	4·43	4·56
46	3·53	3·56	3·63	3·74	3·81	3·93	4·02	4·08	4·19	4·38	4·51	4·64
48	3·73	3·73	3·79	3·88	3·93	4·06	4·15	4·22	4·32	4·52	4·66	4·80
50	3·93	3·92	3·95	4·02	4·09	4·18	4·27	4·36	4·46	4·64	4·81	4·96
52	4·10	4·05	4·09	4·18	4·24	4·31	4·41	4·48	4·60	4·78	4·95	5·10
54	4·28	4·19	4·24	4·33	4·38	4·45	4·55	4·61	4·74	4·92	5·09	5·24
56	4·45	4·33	4·40	4·47	4·52	4·59	4·69	4·75	4·88	5·06	5·23	5·38
58	4·62	4·46	4·56	4·61	4·66	4·73	4·83	4·89	5·00	5·19	5·37	5·52
60	4·79	4·60	4·72	4·76	4·80	4·87	4·95	5·03	5·12	5·21	5·49	5·66
62	4·98	4·80	4·88	4·90	4·96	5·02	5·09	5·17	5·26	5·45	5·63	5·80
64	5·16	5·00	5·04	5·03	5·10	5·16	5·23	5·31	5·40	5·59	5·77	5·94
66	5·34	5·20	5·20	5·15	5·24	5·30	5·37	5·45	5·54	5·73	5·90	6·08
68	5·52	5·40	5·35	5·27	5·38	5·44	5·51	5·59	5·68	5·87	6·04	6·22
70	5·70	5·60	5·49	5·40	5·52	5·58	5·64	5·72	5·80	5·98	6·16	6·35
72	5·90	5·78	5·67	5·56	5·68	5·74	5·80	5·86	5·94	6·12	6·30	6·49
74	6·09	5·96	5·84	5·72	5·84	5·89	5·95	6·00	6·08	6·26	6·44	6·63
76	6·27	6·14	6·00	5·88	5·98	6·03	6·09	6·14	6·22	6·40	6·58	6·75
78	6·46	6·32	6·16	6·02	6·12	6·17	6·23	6·28	6·36	6·53	6·71	6·87
80	6·66	6·49	6·32	6·16	6·26	6·31	6·37	6·43	6·50	6·66	6·84	7·02
82	6·86	6·69	6·50	6·35	6·42	6·46	6·51	6·57	6·64	6·80	6·98	7·16
84	7·06	6·88	6·68	6·54	6·58	6·61	6·65	6·71	6·78	6·94	7·12	7·30
86	7·27	7·06	6·84	6·73	6·74	6·76	6·79	6·85	6·90	7·08	7·26	7·44
88	7·47	7·24	7·00	6·92	6·90	6·91	6·93	6.99	7·02	7·21	7·39	7·57
90	7·67	7·42	7·18	7·11	7·05	7·06	7·08	7·11	7·14	7·34	7·51	7·69
92	7·88	7·62	7·37	7·29	7·21	7·22	7·23	7·27	7·30	7·48	7·65	7·83
94	8·09	7·81	7·55	7·47	7·37	7·38	7·38	7·43	7·45	7·62	7·79	7·97
96	8·31	7·99	7·73	7·65	7·53	7·54	7·53	7·59	7·60	7·76	7·93	8·11
98	8·52	8·17	7·91	7·81	7·69	7·70	7·68	7·73	7·76	7·90	8·06	8·24
100	8·72	8·40	8·09	7·97	7·86	7·84	7·83	7·87	7·91	8·04	8·19	8·36

Diameter of Cylinder in inches.	Diamet'r of Air-pump in inches	DIMENSIONS OF THE SEVERAL PARTS OF PISTON ROD IN INCHES.										Diameter of Air-pump Piston Rod, when copper, in inches.
		Diam'r of Piston Rod.	Length of Part in Piston.	Major Diameter of Part in Crosshead.	Minor Diameter of Part in Crosshead.	Maj'r Diameter of Part in Piston.	Min'r Diameter of Part in Piston.	D'pth of Gibs & Cutter thro' Crosshead.	Thickn's of Gibs & Cutt'r thro Crosshead.	Depth of Cutter thro' Piston.	Thickness of Cutter thro' Piston.	
20	12·0	2·0	4·0	1·90	1·80	2·80	2·30	2·11	·42	1·70	·70	1·34
21	12·6	2·1	4·2	1·99	1·89	2·94	2·41	2·21	·44	1·78	·73	1·40
22	13·2	2·2	4·4	2·09	1·98	3·08	2·53	2·32	·46	1·87	·77	1·47
23	13·8	2·3	4·6	2·18	2·07	3·22	2·64	2·42	·48	1·95	·80	1·53
24	14·4	2·4	4·8	2·28	2·16	3·36	2·76	2·53	·50	2·04	·84	1·60
25	15·0	2·5	5·0	2·37	2·25	3·50	2·87	2·63	·52	2·12	·87	1·67
26	15·6	2·6	5·2	2·47	2·34	3·64	2·99	2·74	·54	2·21	·90	1·73
27	16·2	2·7	5·4	2·56	2·43	3·78	3·11	2·84	·57	2·29	·94	1·80
28	16·8	2·8	5·6	2·66	2·52	3·92	3·22	2·95	·59	2·38	·97	1·87
29	17·4	2·9	5·8	2·75	2·61	4·06	3·34	3·05	·61	2·46	1·00	1·94
30	18·0	3·0	6·0	2·85	2·70	4·20	3·45	3·16	·63	2·55	1·04	2·00
31	18·6	3·1	6·2	2·94	2·79	4·34	3·57	3·26	·65	2·63	1·07	2·07
32	19·2	3·2	6·4	3·04	2·88	4·48	3·68	3·37	·67	2·72	1·10	2·14
33	19·8	3·3	6·6	3·13	2·97	4·62	3·80	3·47	·69	2·80	1·14	2·21
34	20·4	3·4	6·8	3·23	3·06	4·76	3·91	3·57	·71	2·89	1·19	2·27
35	21·0	3·5	7·0	3·32	3·15	4·90	4·02	3·67	·73	2·97	1·22	2·33
36	21·6	3·6	7·2	3·42	3·24	5·04	4·14	3·78	·75	3·06	1·26	2·40
37	22·2	3·7	7·4	3·51	3·33	5·18	4·25	3·88	·78	3·14	1·29	2·47
38	22·8	3·8	7·6	3·61	3·42	5·32	4·36	3·99	·80	3·23	1·33	2·54
39	23·4	3·9	7·8	3·70	3·51	5·46	4·48	4·09	·82	3·31	1·36	2·60
40	24·0	4·0	8·0	3·80	3·60	5·60	4·59	4·20	·84	3·40	1·40	2·67
41	24·6	4·1	8·2	3·89	3·69	5·74	4·70	4·30	·86	3·48	1·43	2·74
42	25·2	4·2	8·4	3·99	3·78	5·88	4·82	4·41	·89	3·57	1·47	2·81
43	25·8	4·3	8·6	4·08	3·87	6·02	4·93	4·51	·91	3·65	1·50	2·87
44	26·4	4·4	8·8	4·18	3·96	6·16	5·05	4·62	·93	3·74	1·54	2·93
45	27·0	4·5	9·0	4·27	4·05	6·30	5·17	4·72	·95	3·82	1·57	3·00
46	27·6	4·6	9·2	4·37	4·14	6·44	5·28	4·83	·97	3·91	1·61	3·07
48	28·8	4·8	9·6	4·56	4·32	6·72	5·51	5·04	1·02	4·08	1·68	3·20
50	30·0	5·0	10·0	4·75	4·50	7·00	5·74	5·25	1·07	4·25	1·75	3·33
52	31·2	5·2	10·4	4·94	4·68	7·28	5·97	5·46	1·11	4·42	1·82	3·47
54	32·4	5·4	10·8	5·13	4·86	7·56	6·21	5·67	1·15	4·59	1·89	3·60
56	33·6	5·6	11·2	5·32	5·04	7·84	6·44	5·88	1·19	4·77	1·96	3·74
58	34·8	5·8	11·6	5·51	5·22	8·12	6·67	6·09	1·23	4·94	2·03	3·88
60	36·0	6·0	12·0	5·70	5·40	8·40	6·90	6·30	1·27	5·11	2·10	4·01
62	37·2	6·2	12·4	5·89	5·58	8·68	7·13	6·51	1·31	5·28	2·17	4·14
64	38·4	6·4	12·8	6·08	5·76	8·96	7·36	6·72	1·35	5·45	2·24	4·27
66	39·6	6·6	13·2	6·27	5·94	9·24	7·59	6·93	1·39	5·62	2·31	4·40
68	40·8	6·8	13·6	6·46	6·12	9·52	7·82	7·14	1·40	5·79	2·38	4·53
70	42·0	7·0	14·0	6·65	6·30	9·80	8·05	7·35	1·47	5·96	2·44	4·67
72	43·2	7·2	14·4	6·84	6·48	10·08	8·28	7·56	1·51	6·13	2·51	4·80
74	44·4	7·4	14·8	7·03	6·66	10·36	8·51	7·77	1·55	6·30	2·58	4·93
76	45·6	7·6	15·2	7·22	6·84	10·64	8·74	7·98	1·67	6·46	2·66	5·07
78	46·8	7·8	15·6	7·41	7·02	10·92	8·97	8·19	1·70	6·63	2·73	5·20
80	48·0	8·0	16·0	7·60	7·20	11·20	9·20	8·40	1·73	6·80	2·80	5·33
82	49·2	8·2	16·4	7·79	7·38	11·48	9·43	8·61	1·76	6·97	2·87	5·47
84	50·4	8·4	16·8	7·98	7·56	11·76	9·66	8·82	1·79	7·14	2·94	5·60
86	51·6	8·6	17·2	8·18	7·74	12·04	9·89	9·03	1·82	7·31	3·01	5·73
88	52·8	8·8	17·6	8·37	7·92	12·32	10·12	9·24	1·85	7·48	3·08	5·87
90	54·0	9·0	18·0	8·56	8·10	12·60	10·34	9·45	1·89	7·66	3·15	6·00
92	55·2	9·2	18·4	8·75	8·28	12·88	10·57	9·66	1·92	7·83	3·22	6·14
94	56·4	9·4	18·8	8·94	8·46	13·16	10·80	9·87	1·95	8·00	3·29	6·27
96	57·6	9·6	19·2	9·12	8·64	13·44	11·03	10·08	2·00	8·17	3·36	6·41
98	58·8	9·8	19·6	9·32	8·82	13·72	11·26	10·29	2·05	8·34	3·43	6·54
100	60·0	10·0	20·0	9·50	9·00	14·00	11·49	10·50	2.11	8·51	3·50	6·66

Diameter of Cylinder in inches.	CRANK PIN.		CRANK.				PIPES AND PASSAGES.					
	Diameter of Crank Pin Journal.	Length of Cr'nk Pin Journal.	Breadth of Small eye of Crank.	Length of Small Eye of Crank.	Thickness of Web of Cr'nk at Pin Centre.	Breadth of Web of Crank at Pin Centre.	Horse Power.	Diameter of Waste-water Pipe in inches	Area of Foot Valve Pass'ge in square in.	Area of Injection Pipe in square inches.	Diameter of Feed Pipe in inches.	Safety Valve, when only one used.
20	2·84	3·20	1·26	3·75	2·20	3·20	5	2·69	17	3·15	1·79	5·00
21	2·98	3·36	1·33	3·93	2·31	3·36	10	3·32	26	3·49	1·84	5·23
22	3·12	3·52	1·40	4·12	2·42	3·52	15	3·94	35	3·84	1·89	5·45
23	3·24	3·68	1·46	4·31	2·53	3·68	20	4·56	44	4·18	1·94	5·66
24	3·38	3·84	1·52	4·50	2·64	3·84	25	5·18	53	4·53	1·99	5·86
25	3·52	4·00	1·58	4·68	2·75	4·00	30	5·80	62	4·87	2·04	6·05
26	3·66	4·16	1·65	4·87	2·86	4·16	35	6·42	71	5·22	2·09	6·24
27	3·80	4·32	1·71	5·06	2·97	4·32	40	7·04	80	5·56	2·13	6·42
28	3·94	4·48	1·77	5·25	3·08	4·48	45	7·66	89	5·91	2·18	6·60
29	4·08	4·64	1·83	5·43	3·19	4·64	50	8·28	98	6·25	2·23	6·80
30	4·26	4·80	1·90	5·62	3·30	4·80	55	8·90	107	6·60	2·28	7·07
31	4·40	4·96	1·96	5·81	3·41	4·96	60	9·50	116	6·94	2·32	7·23
32	4·54	5·12	2·02	5·99	3·52	5·12	65	10·06	125	7·29	2·36	7·39
33	4·68	5·28	2·08	6·18	3·63	5·28	70	10·56	134	7·63	2·40	7·55
34	4·83	5·44	2·14	6·37	3·74	5·44	75	10·96	143	7·98	2·44	7·71
35	4·97	5·60	2·21	6·56	3·85	5·60	80	11·31	152	8·32	2·48	7·87
36	5·11	5·76	2·27	6·74	3·96	5·76	85	11·61	161	8·67	2·52	8·03
37	5·26	5·92	2·33	6·93	4·07	5·92	90	11·89	170	9·01	2·56	8·19
38	5·40	6·08	2·39	7·12	4·18	6·08	95	12·09	179	9·36	2·60	8·35
39	5·54	6·24	2·45	7·31	4·29	6·24	100	12·19	188	9·70	2·64	8·51
40	5·69	6·40	2·52	7·50	4·40	6·40	105	12·29	197	10·05	2·68	8·66
41	5·83	6·52	2·58	7·68	4·51	6·56	110	12·56	206	10·39	2·72	8·80
42	5·97	6·68	2·64	7·87	4·62	6·72	115	12·83	215	10·74	2·76	8·94
43	6·11	6·84	2·71	8·05	4·73	6·88	120	13·10	224	11·08	2·80	9·08
44	6·25	7·00	2·78	8·24	4·84	7·04	125	13·37	233	11·43	2·84	9·22
45	6·39	7·16	2·84	8·42	4·95	7·20	130	13·64	242	11·77	2·88	9·36
46	6·54	7·32	2·91	8·61	5·06	7·36	135	13·91	251	12·12	2·91	9·50
48	6·82	7·63	3·03	8·98	5·28	7·68	140	14·18	260	12·46	2·94	9·63
50	7·11	7·95	3·16	9·35	5·50	8·00	150	14·70	278	13·15	3·00	9·89
52	7·39	8·27	3·28	9·72	5·72	8·32	160	15·20	296	13·84	3·07	10·12
54	7·67	8·59	3·41	10·09	5·94	8·64	170	15·65	314	14·53	3·14	10·36
56	7·95	8·91	3·53	10·46	6·16	8·96	180	16·09	332	15·22	3·20	10·60
58	8·24	9·23	3·66	10·84	6·38	9·28	190	16·53	350	15·91	3·26	10·84
60	8·52	9·54	3·78	11·20	6·60	9·60	200	16·97	368	16·60	3·32	11·06
62	8·80	9·86	3·91	11·57	6·82	9·92	210	17·39	386	17·29	3·38	11·29
64	9·09	10·18	4·04	11·94	7·04	10·24	220	17·79	404	17·98	3·44	11·51
66	9·37	10·50	4·17	12·31	7·26	10·56	230	18·19	422	18·67	3·50	11·73
68	9·65	10·82	4·29	12·68	7·48	10·88	240	18·58	440	19·36	3·56	11·95
70	9·94	11·13	4·42	13·06	7·70	11·20	250	18·97	458	20·05	3·61	12·15
72	10·22	11·45	4·54	13·44	7·92	11·52	260	19·34	476	20·74	3·66	12·35
74	10·50	11·77	4·67	13·81	8·14	11·84	270	19·70	494	21·43	3·72	12·55
76	10·79	12·08	4·79	14·19	8·36	12·16	280	20·06	512	22·12	3·77	12·75
78	11·07	12·40	4·91	14·56	8·58	12·48	290	20·42	530	22·81	3·82	12·95
80	11·36	12·72	5·03	14·94	8·80	12·80	300	20·78	548	23·50	3·88	13·14
82	11·64	13·04	5·16	15·32	9·02	13·12	310	21·12	566	24·19	3·93	13·33
84	11·93	13·35	5·29	15·69	9·24	13·44	320	21·46	584	24·88	3·98	13·51
86	12·21	13·67	5·41	16·07	9·46	13·76	330	21·80	602	25·57	4·03	13·69
88	12·50	13·99	5·54	16·44	9·68	14·08	340	22·14	620	26·26	4·07	13·87
90	12·79	14·30	5·67	16·82	9·90	14·40	350	22·46	638	26·95	4·12	14·05
92	13·07	14·62	5·80	17·20	10·12	14·72	360	22·77	656	27·64	4·16	14·23
94	13·35	14·94	5·92	17·58	10·34	15·04	370	23·09	674	28·33	4·21	14·41
96	13·64	15·26	6·05	17·95	10·56	15·36	380	23·40	692	29·02	4·26	14·59
98	13·92	15·58	6·17	18·37	10·78	15·68	390	23·70	710	29·72	4·31	14·76
100	14·20	15·90	6·30	18·75	11·00	16·00	400	24·00	728	30·41	4·37	14·92

I may here repeat that the diameter of cylinder in inches is given in the first vertical column, beginning at 20 inches and ending at 100 inches, while the length of the stroke in feet is given in the first horizontal column, beginning with 2 feet and ending with 9. If, therefore, we wish to find the dimension proper for any given engine, of which we must know the diameter of cylinder and length of stroke, we find in the first vertical column the given diameter in inches, and in the first horizontal column the given length of stroke in feet; and where the vertical column under the given stroke intersects the horizontal column opposite the given diameter, there we shall find the required dimension.*

LOCOMOTIVE ENGINES.

It would be a mere waste of time and space to recapitulate rules similar to the foregoing as applicable to locomotive engines, since the strengths and other proportions proper for locomotives can easily be deduced by taking an imaginary low pressure cylinder of twice the diameter of the intended locomotive cylinder, and therefore of four times the area, when the proportions will become at once applicable to the locomotive cylinder with a quadrupled pressure, or 100 lbs. on the square inch. In locomotive engines the piston rod is generally made $\frac{1}{7}$th of the diameter of the cylinder, whereas by the mode of determining the proportions that is here suggested it would be $\frac{1}{5}$th. But piston rods are made of their present dimensions, not so much to bear the tension produced by the piston, as to bear the compression when they act as a pillar; and properly speaking the proportionate diameter should diminish with every diminution

* For screw or other short-stroke engines working at a high speed, the strengths of shafts given in the foregoing tables should be somewhat increased, and the length of bearing at least doubled. In some recent screw engines an irregular motion of the engine has been perceived, owing to the elasticity of the shaft. For such engines a correspondent suggests the formula $\sqrt[3]{\left(\frac{D}{7}\right)^3 + \frac{D^2 \times R}{90}}$ = diameter of journal in inches; where D = diameter of the cylinder in inches and R = radius of crank in inches.

in the length of the stroke. In very short cylinders a proportion of $\frac{1}{14}$ of the diameter of the cylinder would suffice in the case of low pressure engines, which answers to ^{1}th of the diameter in locomotives where the stroke is always very short. But in high pressure engines of any considerable dimensions, carrying 10C lbs. on the inch, the diameter of the piston rod should be $\frac{1}{5}$th of the diameter, answering to $\frac{1}{10}$th of the diameter in low pressure engines of the common total pressure of 25 lbs. per square inch.

CHAPTER V.

PROPORTIONS OF STEAM-BOILERS.

In proportioning boilers two main requirements have to be kept in view: 1st. The provision of a sufficient quantity of grate-bar area to burn—with the intended velocity of the draught—the quantity of coals required to generate the necessary quantity of steam; and 2d. The provision of a sufficient quantity of heating surface in the boiler, to make sure that the heat will be properly absorbed by the water, and that no wasteful amount of heat shall pass up the chimney. Even the quantity of heating surface, however, proper to be supplied for the evaporation of a given quantity of water in the hour will depend to some extent upon the velocity of the draught through the furnace: for upon that velocity will depend the intensity of the heat within the furnace, and upon the intensity of the heat will depend the quantity of water which a given area of surface can evaporate. The first point therefore to be investigated is the best velocity of the draught, and the circumstances which determine that velocity. Here, too, there are two guiding considerations. The first is, that if the velocity of the draught be made too great, the small coals or cinders will be drawn up into the chimney and be precipitated as sparks, causing in many cases serious annoyance. The second consideration is, that the temperature of the escaping smoke should be as low as possible, and should in no case exceed 600°. While, therefore, it is desirable in land and marine boilers to have a rapid draught through the furnace—such as is produced in locomotives by the blast-pipe—in order that the heat may be sufficiently intense to enable a small amount of surface to accomplish the required evaporation, it is at the

same time inadmissible to have such a rapid draught in the chimney as will suck up and scatter the small particles of the coal; nor is it desirable that the velocity of the air passing through the grate-bars should be so great as to lift small pieces of coal or cinder and carry them into the flues. No furnace has yet been constructed which reconciles the conditions of a high temperature with a moderate velocity of the entering air: but such a furnace may be approximated to by making the opening through the fire-bridge very small, and by insuring the necessary flow of air through these small openings by the application of a horizontal steam-jet at each opening; as by this arrangement a high temperature may be kept up in the furnace, at the same time that the contraction of the area through or over the bridge will not so much impair the draught as to prevent the requisite quantity of coal from being burnt.

The exhaustion which a chimney produces is the effect of the greater rarity of the column of air within the chimney than that of the air outside. If the air be heated until it is expanded to twice its volume, then, its density being half of what it was before, each cubic inch of the hot air will weigh only half as much as a cubic inch of cold air; and if the hot air be enclosed in a balloon, it will ascend in the cold air with a force of ascent equal to half the weight of the balloon full of cold air. As water is about 773 times heavier than air at the freezing-point, it will require 773 cubic inches of air, heated until they expand to twice their volume, to have ascensional force sufficient to balance a cubic inch of water: or if a syphon-tube be formed with a column of water 1 inch high in one leg, it will require a column of the hot air 1546 inches (or nearly 129 feet) high, in the other leg, to balance the column of water 1 inch high. In other words, a chimney heated until the density of the smoke is only half that of the air entering the furnace, and which will be the case at a temperature under 600°, will, if 129 feet high, produce an exhaustion of 1 inch of water. In land boilers the ordinary exhaustion or suction of chimneys is such as would support a column of from 1 to 2 inches of water. But in steam-vessels the height of the chimney is limited, and the deficient height has to

be made up for by an increased area. In practice, the diameter of the chimney of a steam-vessel is usually made somewhat less than the diameter of the cylinder, there being supposed to be one chimney and two cylinders, with the piston travelling at the speed usual in paddle vessels.

Boulton and Watt's rule for proportioning the dimensions of the chimneys of their land engines is as follows:—

BOULTON AND WATT'S RULE FOR FIXING THE PROPER SECTIONAL AREA OF A CHIMNEY OF A LAND BOILER WHEN ITS HEIGHT IS DETERMINED.

RULE.—*Multiply the number of pounds of coal consumed under the boiler per hour by* 12, *and divide the product by the square root of the height of the chimney in feet: the quotient is the proper area of the chimney in square inches at the smallest part.*

Example.—What is the proper sectional area of a factory chimney 80 feet high, and with a consumption of coal in the furnace of 300 lbs. per hour?

Here $300 \times 12 = 3,600$; and divided by 9 (the square root of the height nearly) we get 400, which is the proper sectional area of the chimney in square inches. If therefore the chimney be square, it will measure 20 inches each way within.

BOULTON AND WATT'S RULE FOR FIXING THE PROPER HEIGHT OF THE CHIMNEY OF A LAND BOILER WHEN ITS SECTIONAL AREA IS DETERMINED.

RULE.—*Multiply the number of pounds of coal consumed under the boiler per hour by* 12, *and divide the product by the sectional area of the chimney in square inches: square the quotient thus obtained, which will give the proper height of the chimney in feet.*

Example.—What is the proper height in feet of the chimney of a boiler which burns 300 lbs. of coal per hour, the sectional area of the chimney being 400 square inches?

Here $300 \times 12 = 3,600$, which divided by 400 (the sectional

area) = 9, the square of which is 81; and this is the proper height of the chimney in feet.

These rules, though appropriate for land boilers of moderate size, are not applicable to powerful boilers with internal flues, such as those used in steam-vessels, in which the sectional area of the chimney is usually adjusted in the proportion of 6 to 8 square inches per nominal horse-power. This will plainly appear from the following investigation:—

In a marine boiler suitable for a pair of engines of 110-horse-power, the area of the chimney, allowing 8 square inches per nominal horse-power, would be 880 square inches. Supposing the boiler to consume 10 lbs. of coal per nominal horse-power per hour, or say 10 cwt. (or 1120 lbs.) of coal per hour, and that the chimney was 46 feet high, then, by Boulton and Watt's rule for land engines, the sectional area of the chimney should be $1120 \times 12 \div \sqrt{46} = 13{,}440 \div$ say $7 = 1{,}920$ square inches. This, it will be observed, is more than twice the area obtained by allowing a sectional area of 8 square inches per nominal horse-power. Here, therefore, is a discrepancy which it is necessary to get to the bottom of.

In Peclet's 'Treatise on Heat' an investigation is given of the proper dimensions of a chimney, which investigation is recapitulated and ably expanded by Mr. Rankine. But it gives results similar to those deduced from Boulton and Watt's rule for their small land boilers, and the expressions are much more complicated. Thus if w = the weight of fuel burned in a given furnace *per second;* V_0 = the volume of air at 32° required per lb. of fuel, and which in the case of common boilers with a chimney draught is estimated at 300 cubic feet; T_1 = the *absolute* temperature of the smoke discharged by the chimney, and which is equal to the temperature shown by the thermometer + 461·2°; T_0 = the *absolute* temperature of the freezing-point, or 461·2° + 32°; A = the sectional area of the chimney in square feet; and u = the velocity of the current in the chimney in feet per second:

$$\text{Then } u = \frac{w\, V_0\, T_1}{A\, T_0}$$

If now l = the length of the chimney and of the flue leading to it in feet; m = the mean hydraulic depth of the smoke, or the area of the flue divided by its perimeter, and which for a round flue and chimney is $\frac{1}{4}$ of the diameter; f = a coefficient of friction, the value of which for a current of gas moving over sooty surfaces Peclet estimates at 0·012; G a factor of resistance for the passage of the air through the grate, and which in the case of furnaces burning 20 to 24 lbs. of coal per hour on each square foot, Peclet found to be 12; h = the height of the chimney in feet: Then by a formula of Peclet's

$$h = \frac{u^2}{2\,g}\left(1 + \mathrm{G} + \frac{f\,l}{m}\right)$$

which formula, with the value that Peclet assigns to the constants, becomes

$$h = \frac{u^2}{2\,g}\left(13 + \frac{0{\cdot}012\,l}{m}\right)$$

and by transposition and reduction

$$u = \sqrt{\frac{64\frac{1}{3}\,h}{(13 + \frac{\cdot 012\,l}{m})}}$$

where $64\frac{1}{3}$ is twice the power of gravity, or $32\frac{1}{6}$.

If now the chimney be made 46 feet high and the flue leading to it be 3 feet diameter and 54 feet long, then $64{\cdot}3 \times 46 = 2957{\cdot}8$; $\cdot 012 \times 100 = 1{\cdot}2$; $m = \frac{1}{4}$th of 3, or $\frac{3}{4}$, or ·75, and $1{\cdot}2 \div \cdot 75 = 1{\cdot}6$.

Hence the equation becomes

$$u = \sqrt{\frac{2957{\cdot}8}{14{\cdot}6}} = 14{\cdot}23$$

$$\text{But } u = \frac{w\,\mathrm{V_o T_1}}{\mathrm{A\,T_o}}$$

$$\text{Hence } \frac{w\,\mathrm{V_o T_1}}{\mathrm{A\,T_o}} = 14{\cdot}23$$

Now if 1,120 lbs. of coal be consumed per hour, ·31 lbs. will be consumed per second $= w$; and if the temperature of

the chimney be 600°, then $600° + 461° = 1061° = T_1$, and $461° + 32° = 493° = T_0$.

$$\text{Hence } \frac{\cdot 31 \times 300 \times 1061}{493\ A} = 14\cdot 6$$

$A = 14$ square feet, or 2,016 square inches; whereas 1,920 square inches is the area given by Boulton and Watt's rule. Peclet's rule, consequently, gives areas much too great for boilers with internal flues, though it will answer pretty well for small land boilers with external flues: but even here it has the disadvantage of being too complicated for common use. It is clear that the friction of the smoke passing through internal flues must be much less than the friction of smoke passing through external flues like that which surrounds a wagon-boiler. For as only one side of the external flues is efficient in heating, the flue with the same friction per foot in length will require to be nearly three times as long as in the case of an internal flue of the same area, to give the required amount of heating surface. In steam vessels much heat is wasted, from the height of the chimney being necessarily so limited that but a small portion of the ascensional force due to the temperature of the smoke is obtained. Thus, if a height of chimney of 129 feet will produce an exhaustion of an inch of water when the heat is sufficient to expand the air into twice its volume, as will be the case at a temperature considerably under 600°, then it is clear that another height of 129 feet, added to the first, would produce an exhaustion equal to a column of *two* inches of water without any additional expenditure of heat; and this increase would go on until the velocity of the draught became such that the friction of the additional height balanced its ascensional force. In steam-vessels, where the chimney is necessarily short, a great part of the exhausting or rarefying effect of the heat is lost; and in steam-vessels, therefore, a chimney-draught is a more wasteful expedient for promoting combustion than it is in the case of a land boiler, where a much larger proportion of the ascensional power of the heat may be made available.

The proportion of heating surface per nominal horse-power

obtaining in marine boilers varies very much in different examples, being in some boilers 12 square feet, in others 17 square feet, in others 20 square feet, in others 30 square feet, and in some as much as 35 square feet per nominal horse power. In fact, the proportion of heating surface required will depend upon the intended ratio in which the nominal is to exceed the actual power, which is now often as much as 8 or 9 times, and also upon the measure of expansive action which is proposed to be adopted. In marine boilers, as in land boilers, about 9 square feet, or 1 square yard, of heating surface will be required to boil off a cubic foot of water in the hour, and in Boulton and Watt's modern marine tubular-boilers they allow 10 square feet of heating surface to evaporate a cubic foot of water in the hour, 10 square inches of sectional area of tubes, 7 square inches of sectional area of chimney, and 14 square inches of area over the furnace bridges. The proportions of modern flue-boilers are not very different, except that there is greater sectional area of flue. But no attempt has yet been made to connect the proportions proper for small land boilers, with those proper for large marine boilers, or to construct a rule that would be applicable to every class of flue-boilers.

Great confusion has been caused by referring to so indefinite a unit as the nominal power of a boiler, and it is much better to make the number of cubic feet which the boiler can evaporate the measure of its power. This again depends upon the intensity of the draught. But it may be reckoned that 5 or 6 square feet of surface will evaporate a cubic foot per hour in locomotive boilers, and 9 or 10 square feet in land and marine boilers.

The main dimensions and proportions of Boulton and Watt's wagon-boilers of different powers are given in the following table:—

PROPORTION OF BOULTON AND WATT'S WAGON BOILERS.

Horse Power.	Length of Boiler.		Breadth of Boiler.		Depth of Boiler.		Mean Height of Flue.	Breadth of Flue.	Sectional Area of Flue.	Sectional Area of Flue per H. P.
	ft.	in.	ft.	in.	ft.	in.	in.	in.	sq. in.	sq. in.
2	4	0	3	2	4	1	20	9	130	90
3	5	3	3	4	4	4	21	9	189	63
4	6	0	3	6	4	7	22	10	220	55
6	7	0	3	9	5	1½	27	10	270	45
8	8	0	4	0	5	6	31	12	372	44
10	9	0	4	3	5	9½	35	12	400	40
12	10	0	4	6	6	0	36	13	468	39
14	10	0	4	9	6	2½	39	13	507	36
16	11	9	5	0	6	6	40	14	560	35
18	12	8	5	2	6	8	42	14	588	32
20	13	6	5	4	6	11	44	14	616	30
30	16	0	5	6	7	3	45	15	720	24
45	19	0	6	0	8	5	53	16	795	17

These proportions enable us to establish the following rule, which is applicable to flue-boilers of every class:—

TO DETERMINE THE PROPER SECTIONAL AREA OF THE FLUE IN FLUE-BOILERS.

RULE.—*Multiply the square root of the number of pounds of coal consumed per hour by the constant number* 300, *and divide the product by the square root of the height of the chimney in feet: the quotient is the proper sectional area of the flue in square inches.*

Example 1.—What is the proper sectional area of the flue in a flue-boiler burning 100 lbs. of coal per hour, the chimney being 49 feet high.

Here $\sqrt{100} = 10$, and $10 \times 300 = 3000$; which divided by 7 (the square root of 49) $= 428$ square inches, which is the proper area of the flue in this boiler.

Example 2.—What is the proper sectional area of the flue in a flue-boiler burning 30 lbs. of coal per hour, the chimney being 81 feet high?

Here $\sqrt{30} = 5{\cdot}48$, and $5{\cdot}48 \times 300 = 1644$; which divided by 9 (the square root of 81) $= 183$ nearly, which is the proper area of the flue in square inches.

Example 3.—What is the proper area of the flue in a flue-boiler burning 1,000 lbs. of coal per hour, and with the chimney 49 feet high?

Here $\sqrt{1000} = 31{\cdot}78$, which $\times\ 300 = 9534$, and dividing by 7 (which is the square root of 49), we get 1,362, as the proper area of the flue in square inches. This is equivalent to 13·62 square inches per horse-power.

It is the universal experience with boilers of every class, that large boilers are more economical than small, or, in other words, that a given quantity of coal will boil off more water in boilers of large power than in boilers of small power. Nevertheless, for purposes of classification, it may be convenient to assume the efficiencies as equal.

The proper proportions of flue-boilers from 1 to 100 horses power are given in the following Table:—

PROPER PROPORTIONS OF FLUE-BOILERS OF DIFFERENT POWERS.

Horse Power.	Pounds of Coal consumed per hour.	Sectional Area of Flue in B. & W.'s boilers.	Sectional Area of Flue by rule, with chimney 49 feet high.	Sectional Area of Flue by rule, with chimney 81 feet high	Heating Surface per H. P.	Sectional Area of Flue per square ft. of heating surface.
	lbs.	sq. in.	sq. in.	sq. in.	sq. ft.	sq. in.
1	10	....	123	106		
2	20	180	191	149	15	6·0
3	30	189	235	183	13	4·8
4	40	220	270	210	11	5·0
5	50	....	303	235		
6	60	270	331	258	10·7	4·2
7	70	....	358	278		
8	80	372	383	296	10·2	4·3
9	90	....	406	316		
10	100	400	428	333	10	4·0
11	110	....	463	360		
12	120	468	469	365	9·8	3·9
13	130	....	488	380		
14	140	507	507	394	9·3	3·6
15	150	....	524	408		
16	160	560	541	421	9·7	3·5
17	170	....	554	431		
18	180	588	575	446	9·8	3·2
19	190	....	590	459		
20	200	616	606	471	10	3·0
30	300	720	724	577	9·8	2·4
45	450	795	909	707	9·6	1·7
60	600	....	1,049	818		
75	750	....	1,173	912		
100	1,000	1,300	1,362	1,059	8	1·6

Mr. Watt reckoned that in his boilers 8 lbs. of coal would evaporate a cubic foot of water in the hour, which is equivalent to an actual horse-power in the case of engines working without expansion. Good Welsh coal, however, it has been found, will evaporate 10 lbs. of water for each pound of coal, which is equivalent to 1·6 cubic feet of water, or 1·6 horse's power in the case of an engine working without expansion; and if such a measure of expansion be used as will double the efficiency of the steam, then 10 lbs. of coal burned in the furnace will generate 3·2 actual horses' power. To attain this measure of efficiency, however, the steam would have to be cut off between $\frac{1}{3}$ and $\frac{1}{4}$ of the stroke, and in the best boilers and engines working with the usual rates of expansion it will not be safe to reckon more than 2 (or at most $2\frac{1}{2}$) actual horses' power as obtainable by the evaporation of a cubic foot of water. When, therefore, engines work up to five times their nominal power, as they now often do, it can only be done by passing through them twice the quantity of steam that answers to their nominal power —or, in other words, by making the boilers of twice the proportionate size, unless where some expedient for producing an artificial draught is employed.

The proper height of chimney where the sectional area of the flue is known can easily be deduced from the foregoing rule.

$$\text{For if } A = \frac{\sqrt{P} \times 300}{\sqrt{h}} \text{ then } h = \frac{(\sqrt{P} \times 300)}{A}$$

which formula put into words is as follows:—

TO FIND THE PROPER HEIGHT OF A CHIMNEY IN FEET WHEN THE NUMBER OF POUNDS OF COAL CONSUMED PER HOUR AND ALSO THE SECTIONAL AREA OF THE FLUE ARE KNOWN.

RULE.—*Multiply the square root of the number of pounds of coal consumed per hour by the constant number* 300, *and divide the product by the sectional area of the flue in square inches; the square of the quotient is the proper height of the chimney in feet.*

Example 1.—What is the proper height of the chimney of a boiler consuming 100 lbs. of coal per hour, and with a sectional area of flue of 428 square inches.

Here $\sqrt{100} = 10$, and $10 \times 300 = 3000$, which divided by $428 = 7$, the square of which is 49, which is the proper height of the chimney in feet.

Example 2.—What is the proper height of the chimney of a flue-boiler consuming 100 lbs. of coal per hour, and with a sectional area of flue of 333 square inches?

Here $\sqrt{100} = 10$, and $10 \times 300 = 3000$, which divided by $333 = 9$, the square of which is 81, which is the proper height of the chimney in feet.

In flue-boilers, the sectional area of the chimney will be the same as that of the flue of a boiler of half the power. Hence in the foregoing Table the proper sectional area of the chimney of a 20-horse boiler—the chimney being 49 feet high—will be the same as the sectional area of the flue of a 10-horse boiler, namely 428 square inches, with a height of chimney of 49 feet; and the proper sectional area of the chimney of a 30-horse boiler will be the same as that of the flue of a 15-horse boiler, namely, 524 square inches, with a height of chimney of 49 feet. If the chimney be 91 feet high, then the values will become 333 and 408 square inches respectively. As then the area of the chimney should be the same as that of the flue of the boiler of half the power, it is needless to give a separate rule for finding the area of the chimney, as such rule will be in all respects the same as that for finding the proper area of the flue, except that we take *half* the number of pounds of coal burned per hour instead of the whole.

In marine tubular boilers the total capacity or bulk of the boiler, exclusive of the chimney, is about 8 cubic feet for each cubic foot of water evaporated per hour—divided in the proportion of 6·5 cubic feet devoted to the water, furnaces, and tubes, and 1·5 cubic foot occupied as a receptacle or repository for the steam. The common diameter of tube in marine boilers is about 3 inches, and the length is 28 or 30 times the diameter. In locomotive boilers the usual diameter of the tubes is 2 inches, and

the length is about 60 times the diameter. The area of the blast orifice in locomotives is about $\frac{1}{10}$th of the area of the chimney. The fire-bars are commonly $\frac{1}{2}$ inch thick, and the air-spaces are made 1 inch wide for fast trains. The main dimensions of marine and locomotive boilers required for the evaporation of a cubic foot of water, are given in the following Table:—

PROPORTIONS OF MODERN BOILERS REQUIRED TO EVAPORATE A CUBIC FOOT OF WATER PER HOUR.

Proportion required per Cubic Foot evaporated per hour.	Marine Flue.	Marine Tubular.	Locomotive.
Square feet of heating surface............	8	9 to 10	6
Square inches of fire-grate...............	70	70	18
Square inches sectional area of flue or tubes	13	10	3·1
Square inches sectional area of chimney...	6	7	2·4
Square feet of heating surface per square foot of fire grate......................	16·48	18·54	48
Pounds of coal or coke consumed on each square foot of fire grate per hour........	16	16	62

The quantity of coal or coke burned on each square foot of fire-grate in the hour to evaporate a cubic foot of water will of course very much depend on the goodness of the coal or coke. In the above Table the average working result of 8 lbs. of water evaporated by 1 lb. of coal, or a cubic foot of water evaporated by 7·8 lbs. of coal, is taken.

The efficiency of a steam vessel is measured by the expenditure of fuel necessary to transport a given weight at a given speed through a given space, and one of the most efficient steam vessels of recent construction is the steamer Hansa, built by Messrs. Caird & Co., to ply between Bremen and America. In this vessel there are two inverted direct-acting engines, with cylinders 80 inches diameter and $3\frac{1}{2}$ feet stroke. There are four tubular boilers, with four furnaces in each, containing a total grate surface of 350 square feet, and a heating surface of 9,200 square feet; besides which there is a superheater, containing a heating surface of 2,100 square feet. The steam is of 25 lbs. pressure on the square inch, and it is condensed by being discharged into a vessel traversed by 3,584 brass tubes, 1 inch external diameter, and 7 feet long. Each tube having 1·75 square

feet of cooling surface, the total cooling surface will be 6,272, or about two-thirds of the amount of heating surface. The cooling water is sent through the tubes by means of two double acting pumps, 21 inches diameter and 24 inches stroke, worked from the forward end of the crank-shaft. It is much better to send the water through the tubes than to send the steam through them. But standing and hanging bridges of plate-iron should be introduced alternately in the chamber traversed by the tubes, so as to compel the current of steam to follow a zigzag course; and the steam should be let in at that end of the chamber at which the water is taken off, so that the hottest steam may encounter the hottest water. It would further be advantageous to inject the feed water into a small chamber in the eduction-pipe, so as to raise the feed-water to the boiling-point before being sent into the boiler; or the feed-pipe might be coiled in the eduction-pipe so as to receive the first part of the heat of the escaping steam. A length of 7 feet appears to be rather great for a pipe an inch diameter, as the water at the end of it will become so hot as to cease to condense any steam, unless the velocity of the flow be so great as to involve considerable resistance from friction. Short pipes, with an abundant supply of cold water, will enable a very moderate amount of refrigerating surface to suffice, as plainly appears from Mr. Joule's experiment, already recited.

If we reckon the engines of the Hansa at 700 horses' power, there will be half a square foot of grate-bars per nominal horse-power, and 13·1 square feet of heating surface per nominal horse-power in the boiler, besides 3 square feet in the super-heater, making in all 16·1 square feet of heating surface per nominal horse-power, or 32·2 square feet of heating surface per square foot of fire-grate. If we take 9 square feet as evaporating a cubic foot of water per hour, then the total evaporation of the boilers in cubic feet will be $9,200 \div 9 = 1,022$ cubic feet per hour; and if we reckon 8 lbs. of coal as necessary to evaporate a cubic foot, then the consumption of coal per hour will be 8,176 lbs, or 3·6 tons per hour, supposing the boiler to be working at its greatest power. This is 11·6 lbs. of coal per nominal horse-power, reckoning the power at 700; and at this rate of

consumption 23·2 lbs. of coal will be burned every hour on each square foot of fire-grate, to generate the steam rqeuired for a nominal horse-power, or it will be 16 lbs. on each square foot every hour to evaporate a cubic foot—there being nearly 1·5 cubic feet of water evaporated for the production of each nominal horse-power.

INDICATIONS TO BE FULFILLED IN MARINE BOILERS.

In all boilers the expedients for maintaining a proper circulation of the water, so that the flame may act upon solid water, and not upon a mixture of water and steam, have been greatly neglected; and the consequence is that a much larger amount of surface is required than would otherwise be necessary. The metal of the boiler is often bent and buckled by being overheated, and priming takes place to an inconvenient extent. In all tubular boilers the water should be *within the tubes*, and those tubes should be vertical, so as to enable the current of steam and water to rise upward as rapidly as possible. The best form of steamboat boiler hitherto introduced is the haystack boiler, for which we are indebted to the fertile ingenuity of Mr. David Napier, and in which boiler the prescribed indications are well fulfilled. In the haystack boiler, which is much used in the smaller class of river-boats on the Clyde—but which, like the oscillating engine at the earlier period of its history, has not yet been employed in seagoing vessels—the tubes are vertical, with the water within them; and the smoke on its way to the chimney imparts its heat to the water by impinging upon the outsides of the tubes. The late Lord Dundonald (another remarkable mechanical genius) proposed a similar plan of boiler; and boilers on his principle —in which the furnace flue of a common marine flue-boiler is filled with a grove of small vertical tubes on which the smoke impinges on its way to the chimney—have been much used on the Continent with good results, and were also introduced in the Collins line of steamers navigating the Atlantic. The Clyde haystack boilers are generally made of the form of an upright cylinder with a hemispherical top, from the centre of which the chimney ascends. The furnace is circular, with a water-space

all around it, and with a circular crown; so that the furnace forms, in fact, a short cylinder, divided in some cases into four quarters by vertical water-spaces crossing one another. Suitable passages are provided to conduct the smoke from the furnace into a cylindrical chamber situated above it—the diameter of this cylinder being the same as that of the shell of the boiler, less the breadth of a water-space which runs round it; and the height of this cylinder being equal to the length of the tubes. The tubes are set in circles round the chimney; and the smoke, which is delivered from the furnace near the exterior of the cylindrical chamber, has to make its way among the vertical tubes before it can reach the chimney. The lower tube-plate and the furnace crown are stayed to one another by frequent bolts, and the cylindrical chamber containing the tubes is also bolted at intervals to the shell of the boiler. The water-space intervening between the lower tube-plate and furnace crown is made very wide, so as to hold a large body of water, and also to enable a person to reach in should repairs be required. The only weak part of this boiler is the root of the chimney, which sometimes has collapsed from becoming overheated by the flame ascending the chimney before the steam has been generated; and the small pressure of the air shut within the boiler when heated has caused the root of the chimney to collapse. This risk is easily prevented by placing several rings of T-iron around the root of the chimney, within the steam-chest, and also by carrying down the plating of the chimney for some distance into the tube-chamber, so as to constitute a hanging-bridge that would hinder the hottest part of the smoke from escaping, and retain it in the tube-chamber, until it had given out the principal part of its heat to the water. In all boilers of this construction these precautions should be adopted; and it would further be useful to place a short piece of pipe in the mouth of every upright tube, so as to continue the tube up to the water-level, whereby the column being elongated its ascensional force would be increased, and the circulation of the water be rendered more rapid.

As this species of boiler is likely to come into use both for steam-vessels and for locomotives, it will be proper to indicate

the forms which appear to be most suitable for those objects. In steam-vessels it is desirable to combine the introduction of a species of boiler adapted for working at a higher pressure, with arrangements for burning the smoke, which will be best done by maintaining a high temperature in the furnace; and a high degree of heat will be best kept up in the furnace by forming it of firebrick instead of surrounding it with water in the usual manner. If, therefore, a square box of iron be taken and lined with firebrick, and if it be divided longitudinally and transversely by these brick walls, and afterwards be arched over, we shall have four furnaces, requiring merely the introduction of the firebars to enable them to be put into operation. Suppose that on the top of each of these square boxes a barrel of vertical tubes is placed, the barrel being sufficiently sunk into the brickwork to establish a communication for the smoke between a hole at each of the four top corners of the box and corresponding perforations in the barrel, we shall then have the smoke from each of the four furnaces into which the box is divided escaping from one corner into the chamber containing the tubes, and after travelling among them passing to the chimney. In such a boiler the circulation of the water could be maintained by forming the external water-space very thick, and by placing a diaphragm-plate in it; so that the water and steam could rise upward on the side of the water-space next to the tube chamber, while the solid water descended on that side of the water-space next to the boiler-shell. The intervening plate would enable these currents to flow in opposite directions without interfering with one another.

In a boiler of this kind the grate-bars should have a sufficient declivity to enable the coal to advance itself spontaneously upon them; and if there are two lengths of firebars in the furnace, the front length should be set closer together than the others, so as partially to coke the coal as on a dead-plate, before it enters into combustion. This coking would be affected by the radiant heat of the furnace, to which heat the coal would be exposed. The openings through which the smoke would escape to the tube-chamber might be perforations or lattice openings in the brickwork, so as to bring every particle of the smoke into intimate

contact with the incandescent material of which the furnace is composed; and these perforations should not have too much area, else the heat would escape to the tubes too rapidly, and the temperature of the furnace would fall. To maintain a sufficient draught to bring in the requisite supply of air to the fuel, a jet-pipe of steam could be introduced at the bottom of the chimney; which jet-pipe would open into a short piece of pipe of larger diameter, also pointing up the chimney, and it into another larger piece, and so on. The jet at each of these short pieces of pipe would draw in smoke and form with the previous jet a new jet, which would become of larger and larger volume and less velocity at successive steps, until the dimensions of the jet had enlarged to an area perhaps equal to half the area of the chimney. It will be sufficient if the length of each piece of pipe be a little greater than its diameter; and the lower end of each piece, or that end facing the current of smoke, should be opened a little into a funnel shape, the better to catch the smoke and carry it forward, to form with the steam a jet continually enlarging its dimensions. By this mode of construction a powerful draught will be created by the jet with a very small expenditure of steam. The area through the cylindrical hanging-bridge at the root of the chimney should not be large, and the bridge itself should be perforated with holes in some places, so as to establish a sufficient current of the smoke upward among the tubes to prevent the heat and flame being swept past direct to the bottom of the chimney without rising among the tubes to impart its heat to them.

In the case of locomotive boilers formed with upright tubes, the fire-box would be the same as at present; but that part of the boiler called the barrel, and which is now filled with longitudinal tubes, would be formed with flat sides and bottom and a semicircular top, so that it would have the same external form as the external fire-box, and this vessel would be traversed by a square flue, in which the vertical tubes would be set. The sides and bottom of this flue would be affixed to the shell by staybolts in the same manner as the internal and external fire-boxes are stayed to one another; and the top, being semicircular, would

not require staying, while the upper tube-plate forming the top of the square internal flue would be strutted asunder and prevented from collapsing by the tubes themselves, some of which should be screwed into the plates or formed with internal nuts, to make them more efficient in this respect. Such a boiler would have various advantages over ordinary locomotive boilers, and might be made of any power that was desired without any limitation being imposed by the width of the gauge of the railway. Such boilers might also be used for steam-vessels by merely increasing the area of the fire-grate.

STRENGTH OF BOILERS.

The proportions which a boiler should possess in order to have a safe amount of strength will be determined partly by the pressure of the steam within the boiler, and partly by the dimensions and configuration of the boiler itself. The best proportions of the riveted joints of the plates of which boilers are made are as follows:—

BEST PROPORTIONS OF RIVETED STEAM-TIGHT JOINTS.

Thickness of Plate in Inches.	Proper Diameter of Rivets in Inches.	Proper Length in inches of Rivets from Head.	Proper distance from Centre to Centre of Rivets in inches.	Proper Quantity of Lap in inches in Single Riveted Joints.	Proper Quantity of Lap in inches in Double Riveted Joints.
$\frac{3}{16}$	$\frac{3}{8}$	$\frac{7}{8}$	$1\frac{1}{4}$	$1\frac{1}{4}$	$2\frac{1}{16}$
$\frac{1}{4}$	$\frac{1}{2}$	$1\frac{1}{8}$	$1\frac{1}{2}$	$1\frac{1}{2}$	$2\frac{1}{2}$
$\frac{5}{16}$	$\frac{5}{8}$	$1\frac{1}{8}$	$1\frac{5}{8}$	$1\frac{7}{8}$	$3\frac{1}{8}$
$\frac{3}{8}$	$\frac{3}{4}$	$1\frac{5}{8}$	$1\frac{3}{4}$	$2\frac{1}{16}$	$3\frac{1}{2}$
$\frac{1}{2}$	$\frac{13}{16}$	$2\frac{1}{4}$	2	$2\frac{1}{4}$	$3\frac{3}{4}$
$\frac{5}{8}$	$\frac{15}{16}$	$2\frac{3}{4}$	$2\frac{1}{2}$	$2\frac{3}{4}$	$4\frac{1}{16}$
$\frac{3}{4}$	$1\frac{1}{8}$	$3\frac{1}{4}$	3	$3\frac{1}{4}$	$5\frac{5}{16}$

If the strength of the plate iron be taken at 100, then it has been found experimentally that the strength of a single-riveted joint will be represented by the number 56, and a double riveted joint by the number 70. According to the experiments of Messrs. Napier and Sons, the average tensile strength of rolled bars of Yorkshire iron was found to 61,505 lbs. per square inch of section,

and the average strength of bars made by nine different makers (and purchased promiscuously in the market) was found to be 59,276 lbs. per square inch of section. The tensile strength of cast steel bars intended for rivets was found to be 106,950 lbs per square inch of section, of homogeneous iron 90,647 lbs., of forged bars of puddled steel 71,486 lbs. and of rolled bars of puddled steel 70,166 lbs. per square inch of section. The strength of Yorkshire plates Messrs. Napier found to be—lengthwise 55,433 lbs., crosswise 50,462 lbs., and the mean was 52,947 lbs. per square inch of section. The tensile strength of ordinary *best* and *best-best* boiler plates, as manufactured by ten different makers, was found to be—lengthwise 50,242 lbs., crosswise 45,986 lbs., and the mean was 48,114 lbs. per square inch of section. Plates of puddled steel varied from 85,000 lbs. to 101,000 lbs. per square inch of section, and homogeneous iron was found to have a tensile strength of about 96,000 lbs. per square inch of section.

Experiments have been made to determine the strength of bolts employed to stay the flat surfaces of boilers together; and it has been found that an iron bolt $\frac{3}{4}$ths of an inch diameter, like the staybolt of a locomotive, screwed into a copper plate $\frac{5}{8}$ths of an inch thick, and not riveted, bore a strain of 18,260 lbs. before it was stripped and drawn out. When the end of the bolt was riveted over it bore 24,140 lbs. before giving way, when the head of the rivet was torn off, and the bolt was stripped and drawn through the plate. When the bolt was screwed into an iron plate $\frac{3}{8}$ths of an inch thick, and the head riveted as before, it bore a load of 28,760 lbs. before giving way, when the stay was torn through the middle. When the staybolt was of copper screwed into copper plate and riveted, it broke with a load of 16,265 lbs., after having first been elongated by the strain one-sixth of its length. Locomotive fire-boxes are usually stayed with $\frac{3}{4}$-inch bolts of iron or copper pitched 4 inches asunder, and tapped into the metal of the outer and inner fire-boxes, and the stays are generally screwed from end to end. These stays give a considerable excess of strength over the shell, but it is necessary to provide for the risk of a bad bolt.

With these data it is easy to tell what the scantlings of a boiler should be to withstand any given pressure. If we take the strength of a single-riveted joint at 34,000 lbs. per square inch, then in a cylindrical boiler the bursting strength in pounds will be measured by the diameter of the boiler in inches multiplied by twice the thickness of the plate in inches, and by the pressure of the steam per square inch in pounds; and this product will be 34,000 lbs. Thus in a cylindrical boiler 3 feet or 36 inches diameter and half an inch thick, if we suppose a length of one inch to be cut off the cylinder we shall have a hoop ½ an inch thick and 1 inch long. If we suppose one-half of the hoop to be held fast while the steam endeavours to burst off the other half, the separation will be resisted by two pieces of plate iron 1 inch long and ½ an inch thick; or, in other words, the resisting area of metal will be one square inch, to tear which asunder requires 34,000 lbs. The separating force being the diameter of the boiler in inches multiplied by the pressure of the steam on each square inch, and this being equal to 34,000 lbs., it follows that if we divide the total separating force in pounds by the diameter in inches, we shall obtain the pressure of the steam on each square inch that would just burst the boiler. Now 34,000 divided by 36 (which is the diameter of the boiler in inches) gives 944·4 lbs. as the pressure of the steam on each square inch that would burst the boiler. A certain proportion of the bursting pressure will be the safe working pressure, and Mr. Fairbairn considers that one sixth of the bursting pressure will be a safe working pressure; but in my opinion the working pressure should not be greater than between one-seventh and one-eighth of the bursting pressure. The rule which I gave in my 'Catechism of the Steam Engine,' for determining the proper thickness of a single-riveted boiler, proceeds on the supposition that the working pressure should be $\frac{1}{16}$ of the bursting pressure. That rule is as follows:—

TO FIND THE PROPER THICKNESS OF THE PLATES OF A SINGLE-RIVETED CYLINDRICAL BOILER.

RULE.—*Multiply the internal diameter of the boiler in inches by the pressure of the steam in lbs. per square inch above the*

atmosphere, and divide the product by 8,900: *the quotient is the proper thickness of the plate of the boiler in inches.*

Example 1.—What is the proper thickness of the plating of a single-riveted cylindrical boiler of $3\frac{1}{2}$ feet diameter, and intended to work with a pressure of 80 lbs. on the square inch?

Here 42 inches (which is the diameter) multiplied by $80 = 3360$, and this divided by $8900 = \cdot377$, or a little over $\frac{3}{8}$ of an inch. The decimal ·375 is $\frac{3}{8}$ of an inch.

Example 2.—What is the proper thickness of a single-riveted cylindrical boiler 3 feet diameter, intended to carry a pressure of 100 lbs. on the square inch?

Here 36 inches $\times$ $100 = 3600$, which divided by $8900 = \cdot4$, or, as nearly as possible, $\frac{3}{8}$ and $\frac{1}{32}$.

As the double-riveted joint is stronger than the single-riveted in the proportion of 70 to 56, it follows that 56 square inches of sectional area in a double-riveted boiler will be as strong as 70 square inches in a single-riveted. This relation is expressed by the following rule:—

TO FIND THE PROPER THICKNESS OF THE PLATES OF A DOUBLE-RIVETED CYLINDRICAL BOILER.

RULE.—*Multiply the internal diameter of the boiler in inches by the pressure of the steam in pounds per square inch above the atmosphere, and divide the product by the constant number* 11140: *the quotient will be the proper thickness of the boiler in inches when the seams are double-riveted.*

Example 1.—What is the proper thickness of the plates of a double-riveted cylindrical boiler 42 inches diameter, and intended to work with a pressure of 80 lbs. per square inch?

Here $42 \times 80 = 3360$, and this divided by $11140 = \cdot3016$, or about $\frac{5}{16}$ of an inch, which is the proper thickness of the plates when the boiler is double-riveted.

Example 2.—What is the proper thickness of a double-riveted cylindrical boiler 3 feet diameter, intended to carry a pressure of 100 lbs. on the square inch?

Here 36 inches × 100 = 3600, which divided by 11140 = ·322, or a little more than $\frac{5}{16}$ of an inch, which will be the proper thickness of the plates of the boiler when the seams are double-riveted.

If T = the thickness of the plate in inches, D = the diameter of the cylinder or shell of the boiler in inches,, and P = the pressure of the steam per square inch: Then

$$T = \frac{DP}{8900}$$ is the formula for the thickness of single-riveted boilers, and

$$T = \frac{DP}{11140}$$ is the formula for double-riveted boilers.

Moreover, in single-riveted boilers—

$$D = \frac{8900\ T}{P} \text{ and}$$

$$P = \frac{8900\ T}{D}$$

So also for double-riveted boilers—

$$D = \frac{11140\ T}{P} \text{ and}$$

$$P = \frac{11140\ T}{D}$$

These formulæ put into words are as follows:—

TO FIND THE PROPER DIAMETER OF A SINGLE-RIVETED BOILER OF KNOWN THICKNESS OF PLATES AND KNOWN PRESSURE OF STEAM.

RULE.—*Multiply the thickness in inches by the constant number* 8900, *and divide by the pressure of the steam in lbs. per square inch. The quotient is the proper diameter of the boiler in inches.*

Example 1.—What is the proper diameter of a single-riveted cylindrical boiler composed of plates ·377 inches thick, and intended to work with a pressure of 80 lbs. on the square inch?

Here ·377 × 8900 = 3355·3, which divided by 80 = 41·94 inches, or 42 inches nearly, which is the proper diameter in inches.

Example 2.—What is the proper diameter of a single-riveted boiler composed of plates ·4 inches thick, and intended to work with a pressure of 100 lbs. on the square inch?

Here ·4 × 8900 = 3560, which divided by 100 = 35·6 inches, which is the proper diameter of the cylindrical shell of the boiler in this case.

TO FIND THE PRESSURE TO WHICH A SINGLE-RIVETED CYLINDRICAL BOILER MAY BE WORKED WHEN ITS DIAMETER AND THE THICKNESS OF ITS PLATING ARE KNOWN.

RULE.—*Multiply the thickness of the plating in inches by the constant number* 8900, *and divide the product by the diameter of the boiler in inches. The quotient is the pressure of steam per square inch at which the boiler may be worked.*

Example 1.—What is the highest safe-working pressure in a single-riveted boiler 42 inches diameter, and composed of plates ·377 of an inch thick?

Here ·377 × 8900 = 3355·3, which divided by 42 = 79·8 lbs. per square inch, which is the highest safe pressure of the steam.

Example 2.—What is the highest safe-working pressure in the case of a single-riveted boiler 36 inches diameter, and composed of plates ·4 of an inch thick?

Here ·4 × 8900 = 3560, which divided by 36 = 99 lbs. per square inch.

The rules for double-riveted boilers are in every case the same as those for single-riveted, only that the constant 11140 is used instead of the constant 8900. It will therefore be unnecessary to repeat the examples for the case of double-riveted boilers.

Mr. Fairbairn has given the following table as exhibiting the bursting and safe-working loads of single riveted cylindrical boilers. But I have already stated that I consider Mr. Fairbairn's margin of safety too small. The working pressure, however, which he gives for single-riveted boilers would not be too great for double-riveted boilers, as will appear by comparing those pressures with the pressures which the foregoing rules indicate may be safely employed.

TABLE SHOWING THE BURSTING AND SAFE-WORKING PRESSURE OF CYLINDRICAL BOILERS, ACCORDING TO MR. FAIRBAIRN.

Diameter of Boiler.	Working pressure for ⅜-inch plates.	Bursting pressure for ⅜-inch plates.	Working pressure for ½-inch plates.	Bursting pressure for ½-inch plates.
ft. in.	lbs.	lbs.	lbs.	lbs.
3 0	118	708¼	157½	944½
3 3	109	653¾	145¼	871¾
3 6	101	607	134¾	809⅛
3 9	94⅛	566⅛	125¾	755⅛
4 0	88⅛	531	118	708½
4 3	83¼	500	111	666⅛
4 6	78¾	472	104¾	629⅛
4 9	74⅛	447½	99½	596¼
5 0	70¾	425	94¼	566⅛
5 3	67½	404¾	89¾	539⅛
5 6	64¼	386¼	85¼	515
5 9	61⅛	369⅛	82	492¾
6 0	59	354	78¾	472
6 3	56⅛	340	75⅛	453½
6 6	54½	326¾	72⅛	435¾
6 9	52½	314¾	69¾	419⅛
7 0	50⅛	303⅛	67½	404⅛
7 3	48¾	293	65	396¾
7 6	47	283¼	62¾	377⅛
7 9	45⅛	274	60¾	365⅛
8 0	44	265¾	59	354
8 3	42¾	257⅛	57	343½
8 6	41⅛	250	55⅛	333⅛
8 9	40⅛	242¾	54	323¾
9 0	39¼	236	52¼	314¾
9 6	37	223⅛	49½	298½
10 0	35¼	212⅛	47	283¼

It will be useful to compare some of the figures of this table with the results given by the rules just recited. For example, according to Mr Fairbairn, a single-riveted boiler, 5 feet diameter, and formed of ½-inch plates, may be habitually worked with safety to a pressure of 94½ lbs. on the square inch. Now, by our rule, $\cdot5 \times 8900 = 4450$, which divided by 60, the diameter of the boiler in inches, gives 74 lbs. as the safe pressure at which the boiler may be worked. If the boiler be double-riveted, then we have $\cdot5 \times 11140 = 5570$, which, divided by 60, gives 93 lbs. as the pressure per square inch at which the boiler may be safely worked. This differs very little from Mr. Fairbairn's result of 94½ lbs., and his table may therefore be used if the results be regarded as applicable to double-riveted boilers, but as applied to single-riveted boilers his proportions, I consider, are too weak. The following diameters of boilers with the corresponding thick-

ness of plates, it will be seen, are all of equal strengths, their bursting pressure being 450 lbs. per square inch, which answers to 34,000 lbs. per square inch of section of the iron. Diameter 3 ft., thickness ·250 inches; $3\frac{1}{2}$ ft., ·291; 4 ft., ·333; $4\frac{1}{2}$ ft., ·376; 5 ft., ·416; $5\frac{1}{2}$ ft., 458; 6 ft., ·500; $6\frac{1}{2}$ ft., ·541; 7 ft., ·583; $7\frac{1}{2}$ ft., ·625; and 8 ft., ·666.

The collapsing pressure of cylindrical flues follows a different law from the bursting pressure, being dependent, not merely upon the diameter and thickness of the tube, but also upon its length; and Mr. Fairbairn gives the following formula for computing the collapsing pressure. If $T =$ the thickness of the iron, $P =$ collapsing pressure in lbs. per square inch, $L =$ length of tube in feet, and $D =$ diameter of tube in inches; then

$$P = 806{,}300 \ \frac{T^{2\cdot19}}{L\,D}$$

and as to multiply the logarithm of any number is equivalent to raising the natural number to the power which the logarithm represents, we may for $T^{2\cdot19}$ write 2·19 log. T. With this transformation the equation becomes

$$P = 806{,}300 \ \frac{2\cdot19 \log. T}{L\,D}.$$

If now we take the thickness of the plate of the circular flue at ·291 inches, and if we make the diameter of the flue 12 inches and its length 10 feet, the equation will become

$$P = 806{,}300 \ \frac{2\cdot19 \log. \cdot291}{120}.$$

Now ·291 being a number less than unity, the index of its logarithm will be negative, and for such a number as ·291 the index will be $\bar{1}$, the minus being for the sake of convenience written on the top of the figure; whereas for such a number as ·0291 the index will be $\bar{2}$; for ·00291 the index will be $\bar{3}$, and so on. It does not signify, so far as the index is concerned, what the significant figures are, but only at what decimal place they begin; and ·1 has the same index as ·291, and ·01 as ·0291. Now the ogarithm of 291, as found in the logarithmic tables, is 463893, and the index being $\bar{1}$, the whole logarithm is $\bar{1}\cdot463893$. In multi-

plying a logarithm with a negative index, as it is the index alone that is negative, while the rest of the logarithm is positive, we must multiply the quantities separately, and then adding the positive and negative quantities together, as we would add a debt and a possession, we give the appropriate sign to that quantity which preponderates. Now ·463893 multiplied by $2·19 = 1·01592567$, and $\bar{1}$ multiplied by 2·19 gives $\bar{2}·19$, which is a negative quantity. Adding these products together, we in point of fact subtract the 2·19 from the 1·01592567, which leaves $\bar{2}·82592567$. Now if we turn to the logarithmic tables, we shall find that the number answering to the logarithm 82592567, or the number answering to the nearest logarithm thereto (which is 825945), is 6698; but as the index is negative, this quantity will be a fraction, and the index being $\bar{2}$, the number will begin in the second place of decimals—or, in other words, it will be 0·6698. Now 806300 multiplied by $·06698 = 54004·974$, which, divided by 120, gives 450 lbs. as the collapsing pressure. If we allow the same excess of strength to resist collapse that we allowed to resist bursting—namely, 7·6 times—a tube of the dimensions we have supposed will be safe in working at a pressure of 60 lbs. on the square inch. But the strength of tubes to resist collapse may easily be increased by encircling them with rings of T iron riveted to the tube. Cylindrical flues of different dimensions, but of equal strength to resist collapse, are specified in the following table:—

CYLINDRICAL FLUES OF EQUIVALENT STRENGTH, THE COLLAPSING PRESSURE BEING 450 POUNDS PER SQUARE INCH.

Diameter of Flue in inches.	Thickness of plates in decimal parts of an inch.		
	For a Flue 10 feet long.	For a Flue 20 feet long.	For a Flue 30 feet long.
12	·291	·399	·480
18	·350	·480	·578
24	·399	·548	·659
30	·442	·607	·730
36	·480	·659	·794
42	·516	·707	·851
48	·548	·752	·905

If $P = 806300 \dfrac{T^{\cdot 19}}{L D}$, then by transformation

$$T^{2\cdot 19} = \frac{P L D}{806300} \text{ and}$$

$$T = \sqrt[2\cdot 19]{\frac{P L D}{806300}}.$$

If now we put P the collapsing pressure = 450 lbs., L = 10 feet, and D = 12 inches, the expression becomes

$$T = \sqrt[2\cdot 19]{\frac{54000}{806300}} = \frac{\log. \cdot 06734}{2\cdot 19}.$$

In like manner the quantities L and D can easily be derived from the formula, and in fact the equations representing them will be

$$L = \frac{806300 \ T^{2\cdot 19}}{P D} \text{ and}$$

$$D = \frac{806300 \ T^{2\cdot 19}}{P L}.$$

It is unnecessary to put these equations into words, as the rule for finding the collapsing pressure of flues is not much required, seeing that in the case of all large internal flues they may be strengthened by hoops of T iron, so as to be as strong as the shell.

PRACTICAL EXAMPLE OF A LOCOMOTIVE BOILER.

It will be useful to compare the results given by these computations with the actual proportions of a locomotive boiler of good construction, and I shall select as the example one of the outside-cylinder tank engines constructed by Messrs. Sharp and Co. for the North-Western Railway. The diameter of cylinder in this locomotive is 15 inches, and the length of the stroke 20 inches. The pressure of the steam in the boiler is 80 lbs. per square inch. The barrel of the boiler is 3 feet 6 inches diameter, and 10 feet 8½ inches long, and it is formed of iron plates ⅜ths thick. The junction of the plates is effected by a riveted jump-joint, which is equal in strength to a single riveted-joint. The rivets are ¾

inch in diameter. The external fire-box is of iron $\frac{3}{8}$ths thick and the internal fire-box is $\frac{7}{16}$ths thick, except the part of the tube-plate where the tubes pass through, which is $\frac{3}{4}$ inch thick. The internal and external fire-boxes are stayed together by means of copper stay-bolts, $\frac{7}{8}$ inch in diameter, and pitched 4 inches apart. The roof of the fire-box is supported by means of seven wrought-iron ribs $1\frac{1}{4}$ inches thick and $3\frac{5}{8}$ inches deep, which rest at the ends on the sides of the fire-box, while the fire-box crown, being bolted to the ribs, is kept up. The ribs are widened out at the bolt-holes, and are also made somewhat deeper there, so that only a surface of about $\frac{1}{4}$ inch round each bolt bears on the boiler crown, to which it is fitted steam-tight. To assist in keeping up the crown, the cross-ribs are also connected with the roof of the external fire-box. The water space left between the outside and inside fire-box is about 3 inches, and the inside fire-box should always be made pyramidical, to facilitate the disengagement of the steam from the surface of the metal. There is a glass tube and three gauge-cocks, for ascertaining the level of the water in the boiler. The lowest gauge-cock is set 3 inches above the roof of the internal fire-box, the next 3 inches above that, and the next 3 inches above that, so that the highest cock is 9 inches above the top of the internal fire-box.

There is a lead plug $\frac{7}{8}$ths of an inch diameter screwed into the top of the fire-box. But the usual course now is to place the lead plug in a cupped brass plug rising a little way above the furnace crown, so that the lead may melt before the plating of the crown gets red-hot, should the supply of water be from any cause intercepted.

The boiler is fitted with 159 brass tubes, 10 feet $7\frac{3}{4}$ inches long, $1\frac{3}{4}$ inches external diameter, and $\frac{1}{16}$th of an inch thick, fixed in with ferules only at the fire-box end. Such tubes last from four to five years, and they are now made thickest at the fire-box end, where the wear is greatest. The part of the boiler above the tubes is supported by eight longitudinal stays, running from end to end of the boiler. The back tube-plate is of iron $\frac{5}{8}$ths of an inch thick. The smoke-box is $\frac{1}{4}$ inch thick, and the chimney, which is 15 inches diameter at bottom and $12\frac{1}{2}$ inches

at top, and rises 13 feet 3 inches above the rails, is $\frac{1}{8}$th of an inch thick. The damper for regulating the draught is placed at the front of the ash-pan, and there is another similar damper at the back of the ash-pan to be used when the engine is made to travel backward, which tank engines can the better do, as they have no tender. The surface of the fire-grate is 10$\frac{3}{4}$ths square feet. The steam ports for admitting the steam to the cylinder are 11 inches by 1$\frac{5}{8}$ths, and consequently each has an area of 17·875 square inches. The branch steampipe leading to each cylinder has $\frac{1}{4}$ less area than this. The blast-pipe is 6$\frac{1}{2}$ inches diameter, tapering to 5$\frac{1}{4}$ inches diameter at the top, and within it is a movable piece of taper pipe, which may be raised up when it is desired to contract the blast orifice. The consumption of coke in these engines is 25 lbs. per mile. The evaporation in locomotive boilers is 7$\frac{1}{2}$ to 8 lbs. of water per lb. of coke, and in locomotive boilers working without expansion the evaporation of a cubic foot of water in the hour will be about equivalent to an actual horse-power. Now if the speed be supposed to be 30 miles an hour, a mile will be performed in two minutes; and as the consumption per two minutes is 25 lbs., the consumption per one minute will be the half of 25 lbs., or say 12 lbs. per minute; and the consumption in 60 minutes, or one hour, will be consequently 720 lbs. of coke; and if 8 lbs. of water are evaporated by 1 lb. of coke, the water evaporated per hour will be 8 times 720, or 5760 lbs. Now if we take a cubic foot of water at 62$\frac{1}{2}$ lbs., and as the evaporation of a cubic foot in the hour is equivalent to a horse-power, 5760 divided by 62$\frac{1}{2}$ = 92, will be the number of actual horse-power exerted by this engine under the circumstances supposed.

Practically, however, locomotives of this class are capable of exerting much more than 92 actual horse-power; for all modern locomotives work, to a certain extent, expansively, whereby a given bulk of water raised into steam is enabled to exert more power, and further, the consumption of coal per mile may be increased beyond 25 lbs., with a corresponding increase of the power generated. In all boilers, indeed, whether land, marine, or locomotive, the evaporative power will be

greatly increased by every expedient which increases the velocity of the draft, and if arrangements be simultaneously made for increasing the temperature of the furnace, by contracting the escaping orifice over the bridge or through the flues, the expenditure of fuel to accomplish any given evaporation will not be increased. In this way marine boilers have been constructed with only 12 square feet of heating surface per nominal horse power, and in which the consumption was only $2\frac{1}{3}$ lbs. of coal per actual horse power, as will be seen by a reference to page 52 of the Introduction to my 'Catechism of the Steam Engine.'

CHAPTER VI.

POWER AND PERFORMANCE OF ENGINES.

The manner of determining the nominal power of an engine has been already explained, and it now remains to show in what manner its actual or indicator horse-power may be determined.

Construction of the Indicator.—The common form of indicator applicable to engines moving at low rates of speed I have already described in my 'Catechism of the Steam-Engine.' But in the case of engines moving at high rates of speed, and, in fact, in the case of all engines to which the steam is quickly admitted, the diagrams formed by this species of indicator are much distorted, and the accuracy of the result impaired, by the momentum of the piston of the indicator itself, which is shot up suddenly by the steam to a point considerably higher than what answers to the actual pressure. The recoil of the spring again sends the piston below the point which properly represents the pressure; and in interpreting the diagram the true curve is supposed to run midway between the crests and hollows of the waving line produced by these oscillations. Latterly an improved form of indicator, called Richards' indicator, has been introduced, which is represented in fig. 5, of which the main peculiarity is that its piston is very light and has a very small amount of motion, so that its momentum is not sufficiently great to disturb the natural line of the diagram. The motion of the piston of the indicator is multiplied sufficiently to give a diagram of the usual height by means of a small lever jointed to the top of the piston rod. To the end of this lever a small link, carry-

Fig. 5.

RICHARDS' INDICATOR. (By Elliot Brothers, Strand.)

ing the pencil, is attached, and from the lower end of this small link a small steel radius bar proceeds to a fixed centre on a suitable part of the instrument, so as to form a parallel motion whereby the pencil is constrained to move up or down in a vertical direction. The paper is placed upon the drum, shown in the figure with a graduated scale, and the string causing this drum to turn round and back again on its axis is put into connection with some part partaking of the motion of the piston in the usual manner. To withdraw the pencil from the paper, the whole parallel motion and the arms carrying it are turned round upon the cylinder, and the pencil is thus made readily accessible. The action of this indicator is precisely the same as that of the common indicator, which, having been described in my 'Catechism of the Steam-Engine,' need not be further noticed here. But in this indicator, as the spring is very stiff, and the travel of the piston correspondingly small, there are no inconvenient oscillations of the pencil such as occur when a long and slender spring is employed.

Method of applying the Indicator.—The drum being put into communication with some part of the engine possessing the same motion as the piston, but sufficiently reduced in amount to be suitable for the small size of the instrument, the drum will begin to be turned round when the piston begins its forward stroke; and the string having drawn it round in opposition to the tension of the spring coiled at the bottom of it, it will follow that when the string is relaxed, as it will be on the return stroke of the piston, the drum will turn back again to its original position, and its motion and that of the string will be an exact miniature of the motion of the piston. The pencil, if now suffered to press against the paper, will describe a straight line. But if the cock which connects the cylinder of the indicator with the cylinder of the engine be now opened, the pencil will no longer trace a straight line, but being pressed upward during the forward stroke by the steam, and being sucked downward by the vacuum during the return stroke, if the engine is a condensing one, or being pressed downward by the spring when the pressure of the steam is withdrawn, as it will be during the return stroke, it is

quite clear that the pencil must now describe a figure containing a space or area, and the figure is what is called the indicator diagram, and the amount of the space is the measure of the amount of the power exerted at each stroke by the engine. This will be more clearly understood by a reference to fig. 6, which is an indicator diagram taken from a steam fire-engine constructed by Messrs Shand, Mason and Co., with two high-pressure engines of 6¼ inch cylinders and 7 inches stroke, with a pressure on the

Fig. 6.

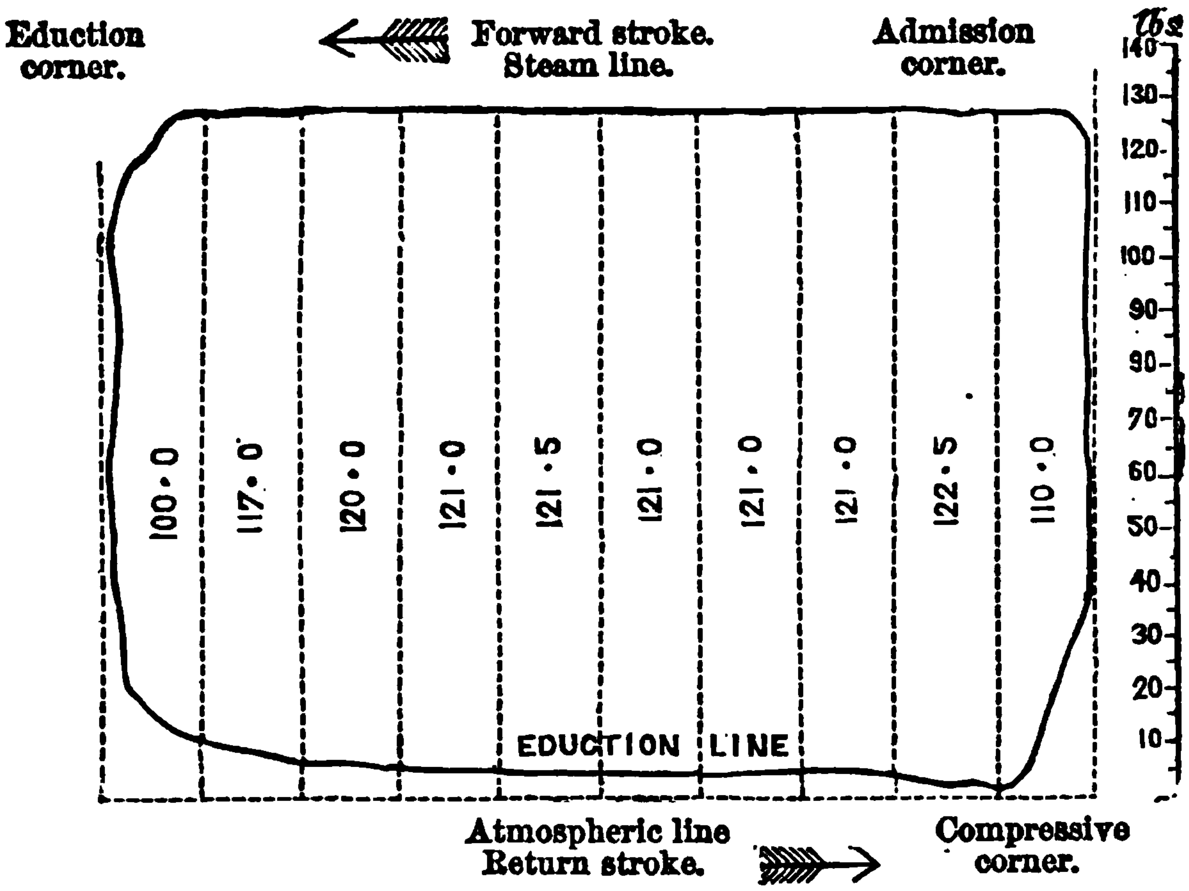


DIAGRAM ILLUSTRATIVE OF THE MODE OF COMPUTING THE HORSE-POWER.

boiler of 145 lbs. per square inch, and making 156 revolutions per minute. The total weight of this engine is 24 cwt. 2 qrs., and by a reference to the diagram it will be seen that the mean pressure urging the piston is 117·5 lbs. per square inch, which mean pressure is ascertained by adding together the pressure at each division or ordinate, and dividing by the number of ordinates, which in this case is 10. The mean pressure multiplied by the areas of the cylinders and by the speed of the piston in feet per minute, and divided by 33000 lbs., gives 18·3 horses as

the power actually exerted by this engine. The weight of the engine is consequently only 1·3 cwt. per actual horse-power.

The advantage of taking 10 ordinates instead of 8 or 9 or 11 is, that the division by 10 is accomplished by merely shifting the position of the decimal point; while 10 ordinates are enough to enable the area to be measured accurately enough for all practical purposes. Thus the total amount of the pressures in the diagram, fig. 6, taken at 10 places, is 1175 lbs., and the tenth of this, or 117·5 lbs. per square inch, is the mean pressure on the piston throughout the stroke. It is clear that when we have got the mean pressure on each square inch of the piston, we have only to ascertain the number of square inches in it, and the distance through which it moves in a minute, to determine the power, and the indicator enables us to determine the mean pressure on the piston throughout the stroke in the manner just explained. The indicator is sometimes applied to the air-pump and to the hot well, to determine the varying pressures within them at different parts of the stroke; and it is virtually the stethoscope of the engine, as it enables us to tell whether all its internal motions and pulsations are properly performed.

Mode of reading Indicator Diagrams.—In the preceding diagram the piston moves in the forward stroke in the direction shown by the arrow, and backward on the return stroke in the direction shown by the arrow. In all diagrams the top indicates the highest pressure, and the bottom the lowest pressure. But it is quite indifferent whether the diagram is a right-hand or left-hand diagram; and where two diagrams are shown on the same piece of paper, as is often done, that which represents the performance of one end of the cylinder is generally right-hand, and that which represents the performance of the other end of the cylinder is generally left-hand. This arrangement, however, is quite immaterial, that which alone determines the power exerted being with any given scale the area shut within the diagram.

In fig. 6, the steam being supposed to be let in upon the piston of the engine, presses the piston of the indicator up to the point shown at the 'admission corner,' and as the piston moves forward the steam continues to press upon it with undiminished

pressure, until close to the end of the stroke, at the 'eduction corner,' the eduction passage is opened; and as the steam consequently escapes into the atmosphere there is no longer the same pressure on the spring of the indicator as before, and its piston consequently descends. As, however, the steam cannot instantaneously get away, the pressure does not descend quite so low as the atmospheric line. The eduction passage, it appears by the diagram, begins to be opened when about nine-tenths of the forward stroke has been completed, and it also begins to be shut when about nine-tenths of the return stroke has been com-

Fig. 7.

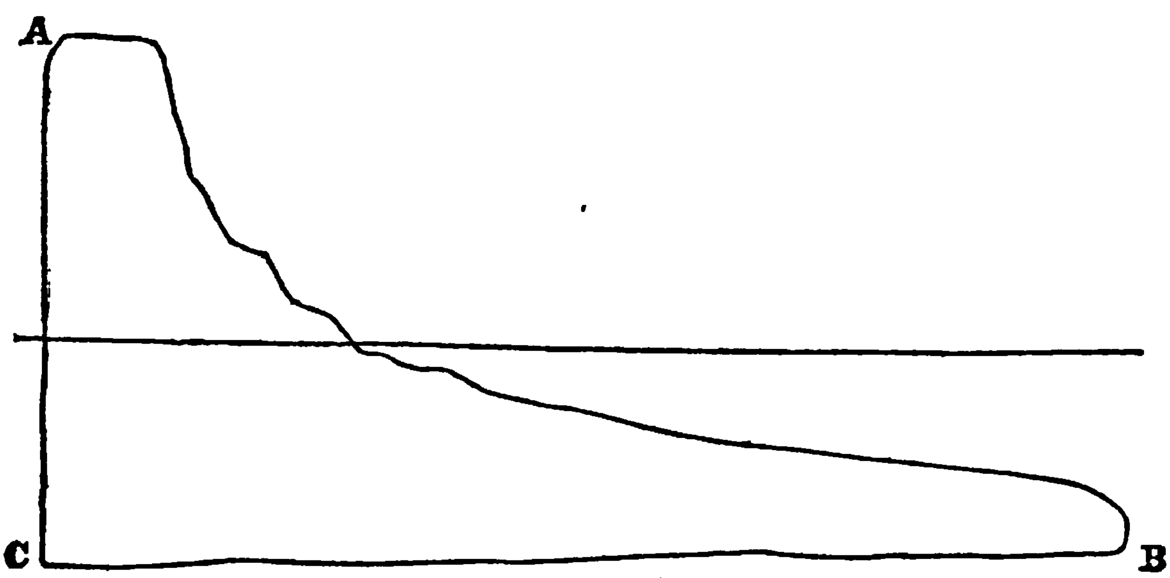


DIAGRAM TAKEN FROM STEAMER 'ISLAND QUEEN.'

pleted, as appears by a reference to the 'compression' corner, which shows that the back pressure begins to rise before the termination of the stroke. The area comprehended between the atmospheric line and the bottom of the diagram shows the amount of back pressure resisting the piston, which in this diagram is of the average amount of 5·1 lbs.; and this increased back pressure at the 'compression corner' is produced by the compression of the steam shut within the cylinder, which is accomplished by the piston as it approaches the end of its stroke.

Various examples of Indicator Diagrams.—In the engine of which the diagram is given in fig. 6, the steam works with very little expansion; but in fig. 7 we have a diagram taken from the steamer 'Island Queen which shows a large amount of expan-

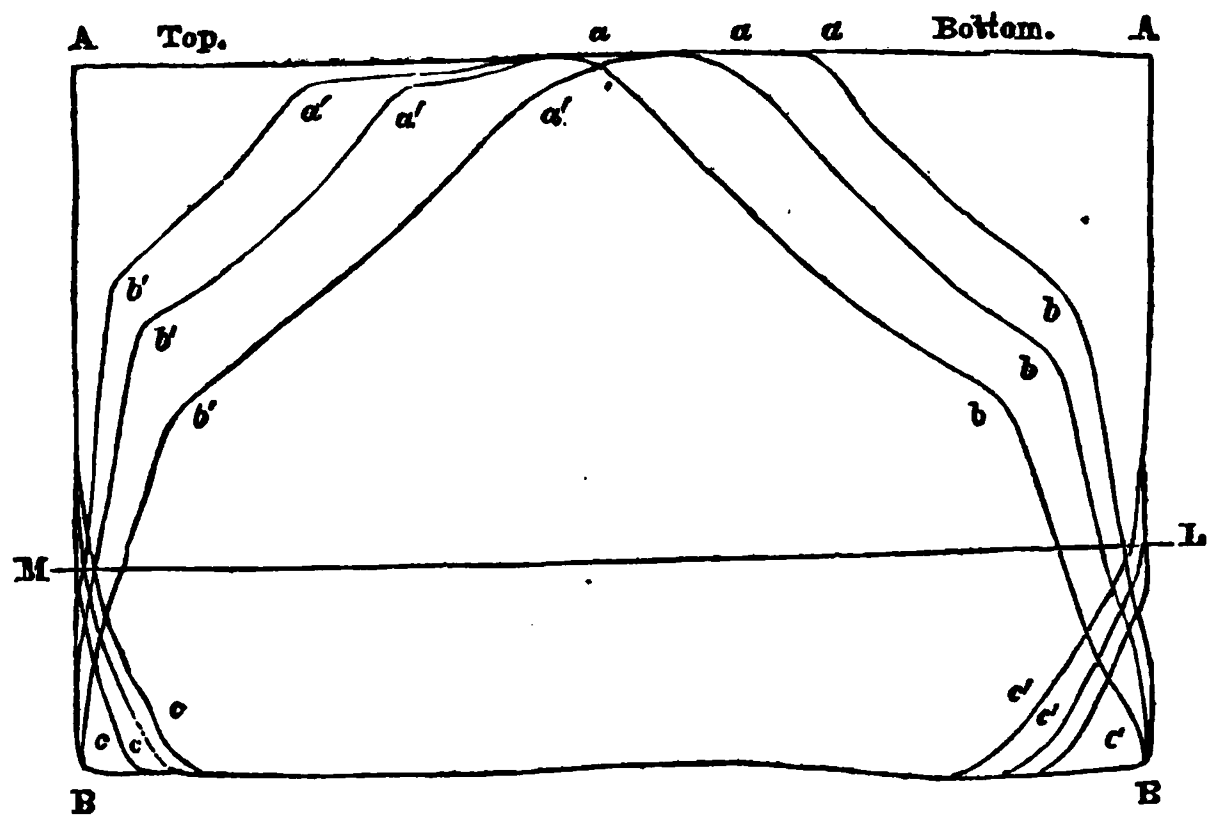
sion. This diagram is a left-hand diagram, the former one, shown in fig. 6, being a right-hand diagram. A is the admission corner, and the steam is only admitted until the piston reaches the position answering to that of a vertical line drawn through *a*, and which is about one-eighth of the stroke. The steam being shut off from the cylinder at *a*, thereafter expands until the end of the stroke is nearly reached, when the eduction passage is opened, and the pencil then subsides to the point B, at which point the piston begins to return. The straight line drawn across the middle of the diagram is the atmospheric line; and it is traced by the pencil before the cock of the indicator communicating with the cylinder is opened. The distance of the line B C below the atmospheric line shows the amount of vacuum obtained in the cylinder, and the height of A *a* above the atmospheric line shows the pressure of the steam subsisting in the cylinder. This diagram, which is a very good one, is obtained with the aid of a separate expansion valve. The pressure of the steam was 22 lbs. per square inch, the vacuum 14¼ lbs., and the number of revolutions per minute 17.

In some high-pressure engines, where the steam is allowed to escape suddenly through large ports, and a large and straight pipe, there is not only no back pressure on the piston, but a partial vacuum is created within the cylinder by the momentum of the escaping steam. In ordinary condensing engines the momentum of the steam escaping into the condenser might in some cases be made to force the feed-water into the boiler, in the same manner as is done by a Giffard's injector, which is an instrument that forces water into a boiler by means of a jet of steam escaping from the same boiler. This instrument will not act if the temperature of the feed-water be above 120° Fahr., as in such case the steam will not be condensed with the required rapidity. As the steam is water in a state of great subdivision, and as the particles of this water are moved with the velocity of the issuing steam, which is very great, we have in effect a very small jet of water issuing with a very great velocity, and this small stream would consequently balance a very high head of water, or, what comes to the same thing, a very great pres-

sure. Precisely the same action takes place when the steam escapes to the condenser; and under suitable arrangements the boiler might be fed by aid of the power resident in the educting steam, and indeed the function of the air-pump might also be performed by the same agency.

In fig. 8 we have an example of the diagrams taken from the top and the bottom of the cylinder disposed on the same piece of paper, those on the left-hand side being taken from the top of the cylinder, and those on the right-hand side being taken

Fig. 8.

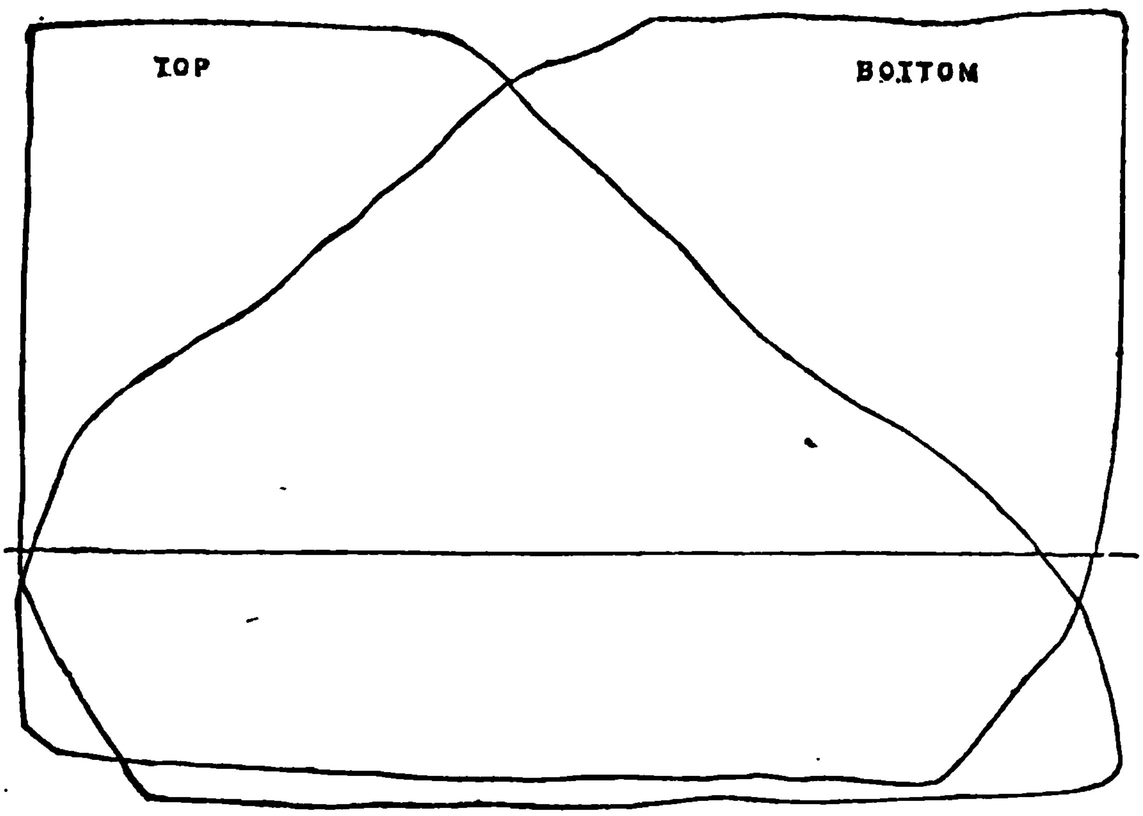


DIAGRAMS TAKEN AT MOORINGS FROM HOLYHEAD PADDLE-STEAMER 'MUNSTER.'

from the bottom of the cylinder. There are three diagrams taken from each end with different degrees of expansion. A is the admission corner of the three diagrams, taken from the top of the cylinder, and *a a a* are the three several points at which the steam is cut off in these three diagrams. Thereafter the steam continues to expand, and the pressure gradually to fall, until the points *b b b* are reached, when the eduction passage is opened to the condenser, and the pressure then falls suddenly to the point B. The line B B′ represents the amount of exhaustion

attained within the cylinder measured downward from the atmospheric line M L; and $c\,c\,c$ represent the three points at which compression begins, answering to the three degrees of expansion. The letters A′, a', b', B′, and c' represent the corresponding points for each of the three diagrams taken from the bottom of the cylinder; and the amount of correspondence in the right-hand and left-hand diagrams shows the amount of accuracy with which the valves are set to get a similar action at each end of the cyl-

Fig. 9.



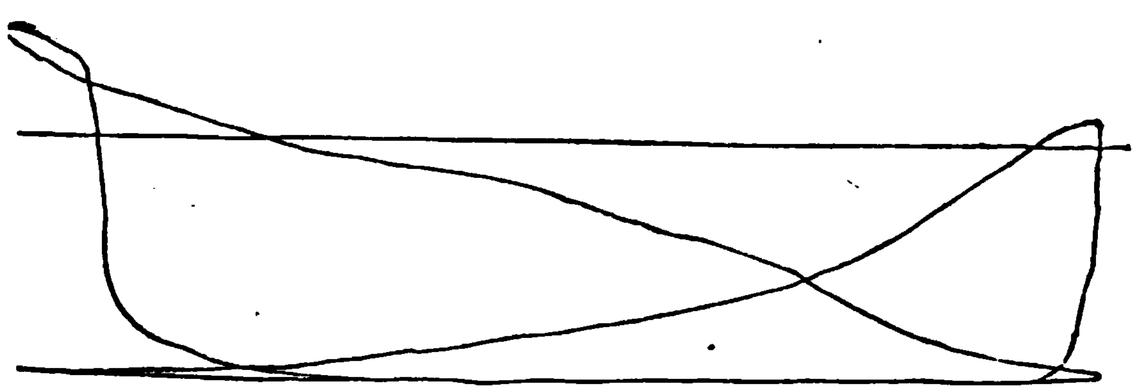
DIAGRAMS TAKEN FROM HOLYHEAD PADDLE-STEAMER 'ULSTER' WHEN UNDER WAY.

inder. The diagrams given above were taken from the Holyhead steam-packet 'Munster,' the engines of which were constructed by Messrs. Boulton and Watt. The cylinders are oscillating, of 96 inches diameter and 7 feet stroke. The pressure of steam was 26·16 lbs. per square inch, vacuum 25½ lbs., and the number of strokes per minute 9—the vessel having been at moorings at the time. It will be seen by these diagrams that the amount of lead upon the eduction side, or the equivalent distance which the piston is still from the end of the stroke when eduction begins to take place, corresponds in every instance with the amount of

the compression, since, in fact, by shifting the eccentric round to let the steam out of the cylinder before the end of the stroke, the valve will be equally shifted to shut the educting orifice before the end of the stroke, and thus to keep within the cylinder any vapour left in it when the valve has been shut, and which is thereafter compressed by the piston until the end of the stroke is reached, or until the valve opens the communication with the boiler.

Fig. 9 represents a diagram taken from the top, and another taken from the bottom of one of the cylinders of the Holyhead paddle-steamer 'Ulster,' a vessel of the same power and dimensions as the 'Munster,' and the engines also by Messrs. Boulton

Fig. 10.



DIAGRAMS FROM STEAMER 'ULSTER' AT 4 1-2 STROKES.
(STEAM THROTTLED BY THE LINK.)

and Watt. When these diagrams were taken the pressure of the steam in the boiler was 26 lbs. per square inch, the vacuum in the condenser 13 lbs. per square inch, and the engine was making 23 strokes per minute. The mean pressure on the pistons, obtained by taking a number of ordinates, as in fig. 6, reckoning up the collective pressure at each, and dividing by the number of ordinates, was 28·27 lbs. It is immaterial what number of ordinates is taken, except that the more there are taken the more accurate will be the result.

In fig. 10 we have diagrams taken from top and bottom in the same engine, when slowed to 4½ strokes per minute, partly by closing the throttle valve, and partly by shifting the link towards its mid-position. In these diagrams nearly the whole areas are

below the atmospheric line. But on the left-hand corner of one of the figures a loop is formed, which often appears in engines employing the link, and the meaning of which it is necessary to explain. The extreme point of the diagram in every instance answers to the length of the stroke; and if the steam is pent up in the cylinder by the eduction passage being shut before the end of the stroke, or if it be suffered to enter from the boiler before the stroke is ended, the pencil will be pushed up to its highest point before the stroke is ended, and as the paper still continues to move onward the upper part of the loop is formed. If the pressure within the cylinder when the piston returns were to be precisely the same as when the piston advances during this part of its course, the loop would be narrowed to a line. But as the advance of the piston when the valve is very little opened somewhat compresses the steam, and as its recession when the valve is very little opened somewhat wire-draws it, the pressures while the piston advances and retires through this small distance, although the cylinder is open to the boiler by means of a small orifice, will not be precisely the same; and the higher pressure will form the upper part of the loop, and the lower pressure the lower part. In fig. 10, by following the outline of the left-hand diagram, it will be seen that the steam begins to be compressed within the cylinder when about three-fourths of the stroke has been completed; and the pencil consequently begins to rise somewhat above its lowest point. But as the vapour within the cylinder is very rare, the rise is very little until, when the piston is about one-eighth part of its motion, or about 8 inches from the end of the stroke, the steam-valve is slightly opened, when the piston of the indicator is compelled to ascend to the point answering to the pressure within the cylinder thus produced. As the opening from the boiler continues, and the piston by advancing against the steam, instead of receding from it, compresses rather than expands the steam admitted into the cylinder, the pressure continues to rise somewhat to the end of the stroke; when the piston of the engine, having to move in the opposite direction, the steam within the cylinder will be expanded, and any still entering will be wire-drawn in the contracted passage,

and the pressure will fall. Under such circumstances a loop will necessarily be formed at the corner of the diagram, such as is shown to exist at the left-hand corner of fig. 10. The reason why there is no corresponding loop at the right-hand corner of the right-hand diagram is simply because the valve is somewhat differently set at one end of the engine from what it is at the other; and the angles of the eccentric rods will generally cause

Fig. 11.

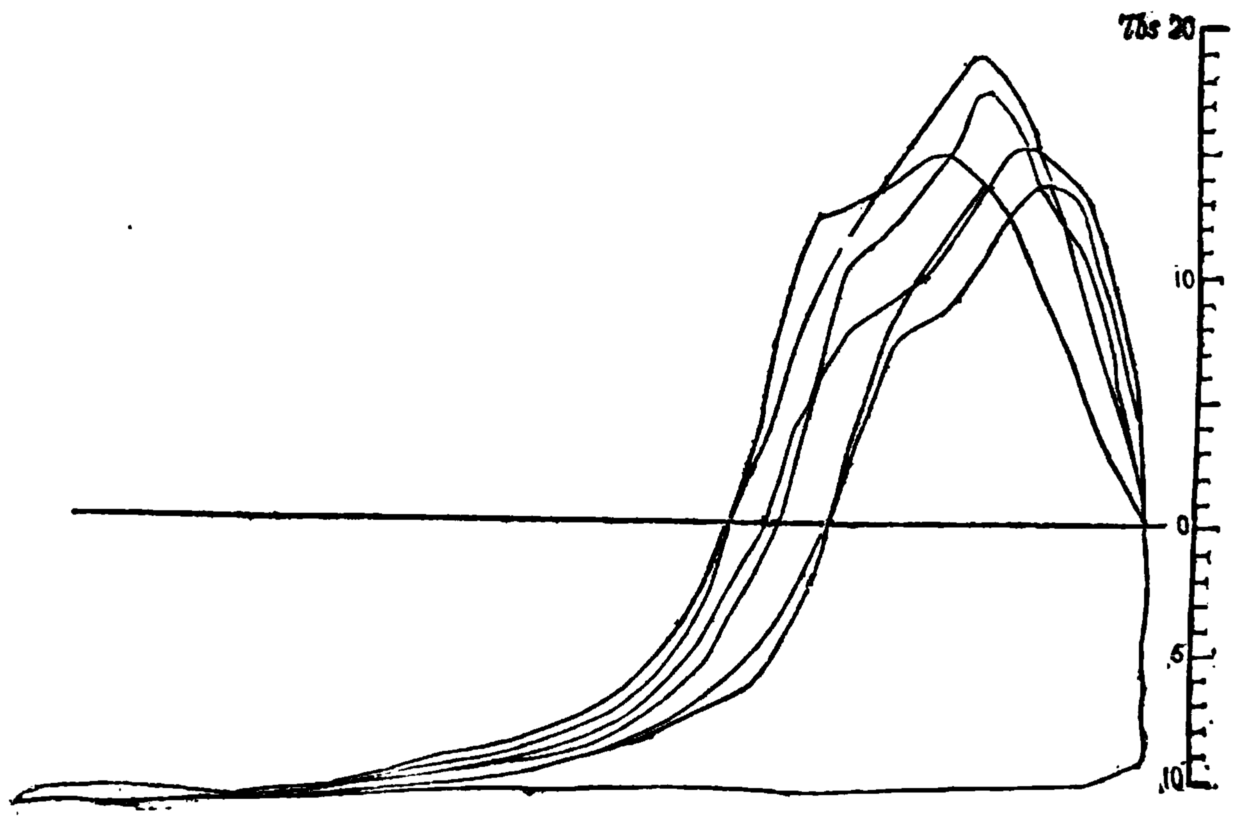


DIAGRAM FROM AIR-PUMP OF STEAMER 'ULSTER.'
(19 REVOLUTIONS PER MINUTE.)

some small difference in the action of the valve at the different ends of the engine.

Diagrams from the Air-Pump.—Fig. 11 is a diagram taken from the air-pump of the 'Ulster,' when the engine was making 19 revolutions per minute. In this diagram the pencil begins to ascend from that point which marks the amount of exhaustion existing in the air-pump, and it rises very slowly until about two-thirds of the stroke of the pump has been performed, when it shoots rapidly upwards, indicating that at this point the water is encountered which has to be expelled. Midway between the

atmospheric line and the highest point of ascent, the delivery valve begins to open, and somewhat relieves the pressure; and there is consequently a wave in the diagram on that point. But the inertia of the water in the hot-well has then to be encountered, and an amount of pressure is required to overcome this inertia, which is measured by the highest point to which the pencil ascends. So soon as the water in the hot-well and waste-water pipe has been put into motion, the motion is continued by its own momentum, without a sustained pressure being required to be exerted by the bucket of the pump; and the pressure in the pump consequently falls, as is shown by the descent of the piston of the indicator towards the end of the stroke.

Fig. 12.

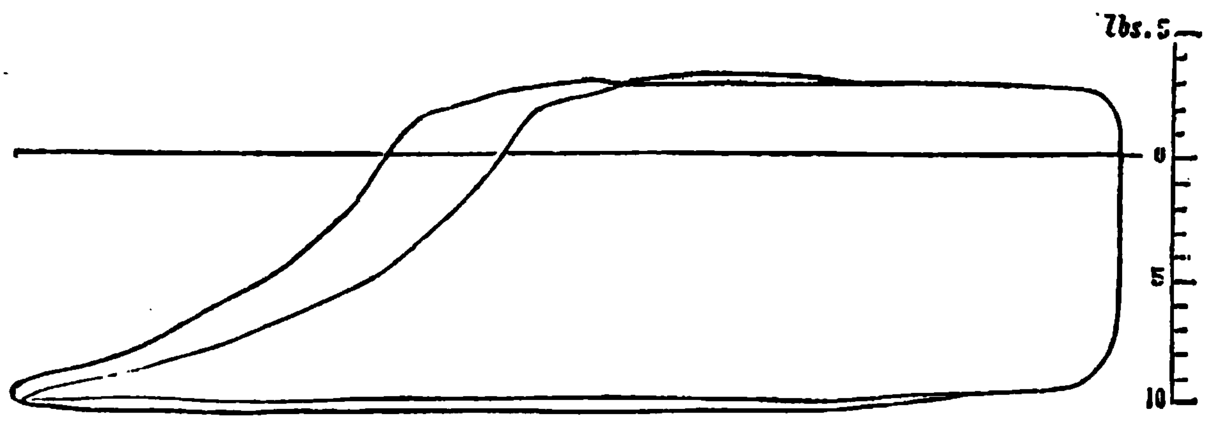


DIAGRAM FROM AIR-PUMP OF STEAMER 'ULSTER.'
(7 STROKES PER MINUTE.)

The effect of partially closing the throttle-valve of an engine so as to diminish the speed, will be to reduce the momentum of the water in the hot-well, and correspondingly to reduce the maximum pressure which the pump has to exert. But the effect will also be to fill the pump with water through a larger proportion of its stroke; and if the engine were to be slowed very much by shutting off the steam, without correspondingly shutting off the injection, the air-pump at its reduced speed would be unable to deliver all the water, which would consequently overflow into the cylinder and probably break down the engine. In fig. 12 we have an air-pump diagram taken from the steamer 'Ulster,' when the speed of the engine was reduced to

six strokes per minute; and it will be observed that we have no longer the same amount of maximum pressure in the pump, nor the same sudden fluctuations. The pump, however, is filled for a greater proportion of its stroke; and the maximum pressure once created, is constant, and does not rise much above the pressure of the atmosphere, being, in fact, the simple pressure due to the pressure of the atmosphere, and that of the column of water intervening between the level of the air-pump and that of the waste-water pipe.

Diagram illustrative of the evils of Small Ports.—Fig. 13 is a diagram taken from a pumping-engine in the St. Katherine's Docks, and is introduced mainly to show the detrimental effect

Fig. 13.

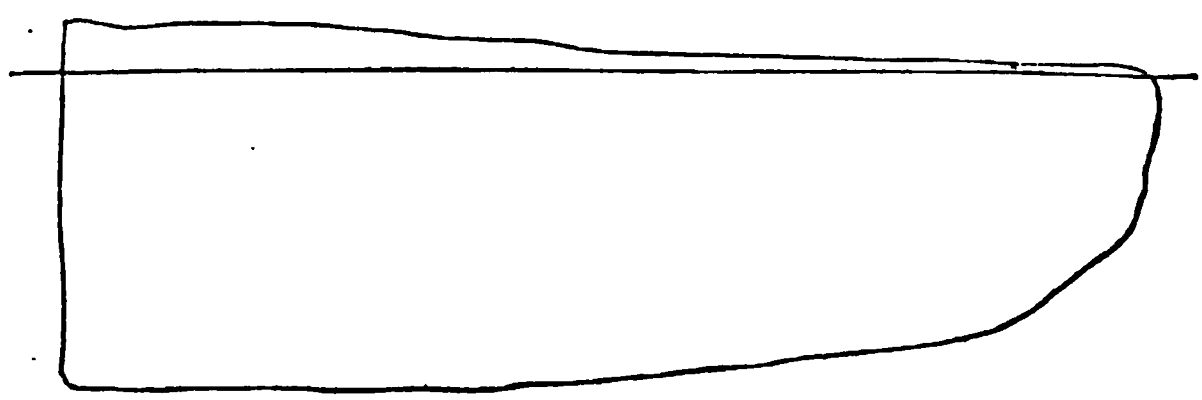


DIAGRAM TAKEN FROM PUMPING-ENGINE, ST. KATHERINE'S DOCKS.

of an insufficient area of the eduction passages. The steam is supposed to enter at the left-hand corner, but as the speed of the piston accelerates, as it does towards the middle of the stroke, the pressure falls, from the port being small and the steam wire-drawn. Towards the other end of the stroke the pressure would again rise, but that it is hindered from doing so by the condensation within the cylinder, which is considerable, as the engine works at the low speed of 12 strokes per minute, lifting the water 9½ feet. The eduction corner of the diagram is very much rounded away, from the inadequate size of the ports; and the eduction will also be impeded by any condensed water within the cylinder, which, unless got rid of by other arrangements, will have to be put into motion by the escaping steam. The mean

pressure exerted on the piston of this engine is only 12·45 lbs. per square inch, although it operates without expansion; and it may be taken as a fair example of eneligible construction.

Diagrams showing the momentum of the Indicator piston.—Fig. 14 is a pair of diagrams taken from one of the engines of H. M. S. 'Orontes.' This vessel, which is 300 feet 1 inch long, 44 feet 8 inches broad, and 2,823 tons, has horizontal direct acting engines of 500 horse-power, constructed by Messrs Boul-

Fig. 14.

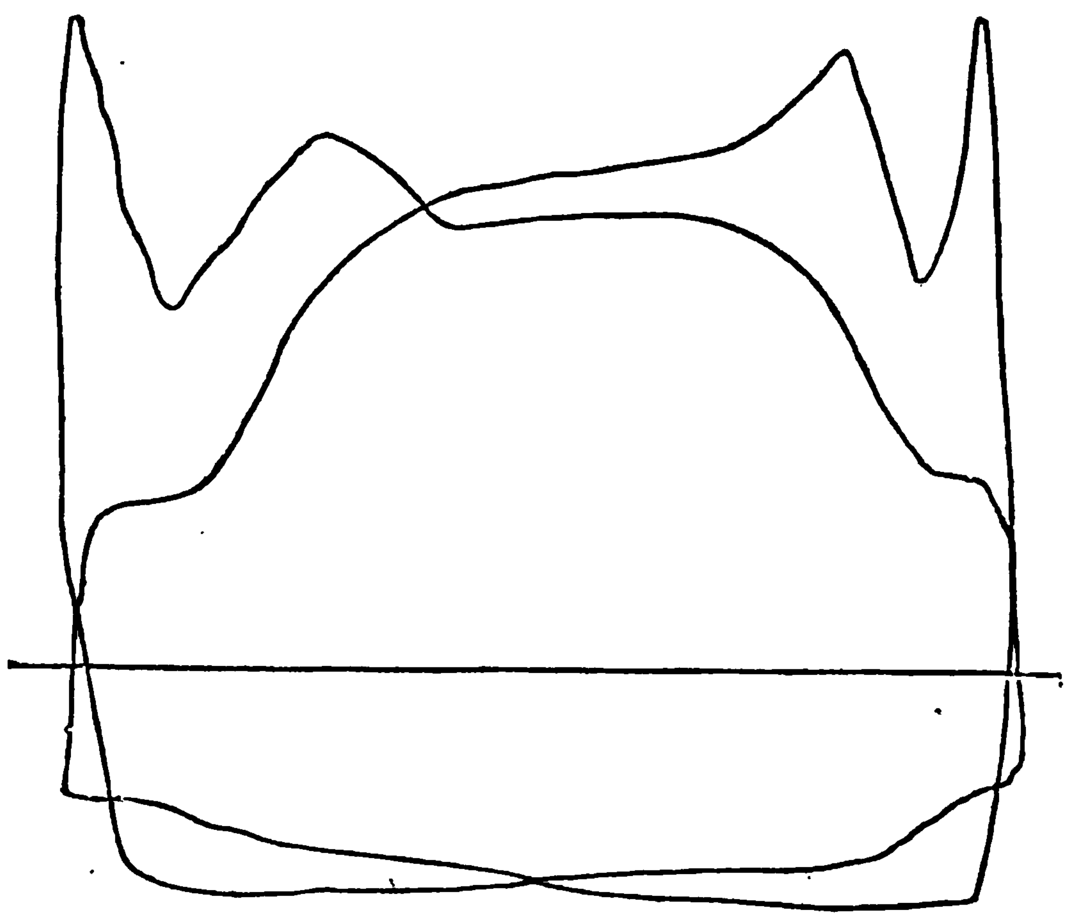


DIAGRAM TAKEN FROM H.M. TROOP-STEAMER 'ORONTES.'

ton and Watt. With a midship section of 644 square feet, and a displacement of 3,400 tons, the vessel attained a speed on her official trial, of 12·622 knots, with a pressure of steam in the boiler of 25 lbs. per square inch, 61 revolutions per minute, the engines exerting 2,249 horse-power. On one occasion the speed obtained was 13·3 knots. With an area of immersed section of 781 square feet, and a displacement of 4249 tons, the speed attained was 12·354 knots, with 2,143 horse-power. There are two horizontal

engines of 71 inches diameter, and 3 feet stroke. The screw is 18 feet diameter, 25 feet pitch, and 4 feet long, and the slip of the screw was found to vary between 13 and 16 per cent. When the diagrams represented in fig. 14 were taken, the pressure of the steam in the boiler was $21\frac{1}{2}$ lbs. of the vacuum, in the condenser $11\frac{1}{4}$ lbs., and the engine was making 60 revolutions per minute. If ordinates be taken in the case of these diagrams, and the mean pressure be thus determined, it will be found to amount to 25·22 lbs. per square inch. In these diagrams the waving line formed by the pencil, owing to the momentum of the piston of the indicator, is very plainly shown; and although such irregularities will not materially impair the accuracy of the result, if a sufficient number of ordinates be taken correctly to measure the irregularity, yet it is greatly preferable to employ an indicator which will be as free as possible from the disturbing influence of the momentum of its own moving parts. In this engine, as in most of Messrs. Boulton and Watt's engines, there is a great similarity in the diagrams taken from each end of the cylinder —a result mainly produced by giving a suitable length to the eccentric rods, by moving up or down the links vertically by a screw, instead of by a lever moving in the arc of a circle, and placing the projecting side of the eccentric suitably with the curvature of the link, since, if placed in one position, it will aggravate the distortion produced by the angle of the eccentric rods, and if placed in the opposite position it will correct this distortion.

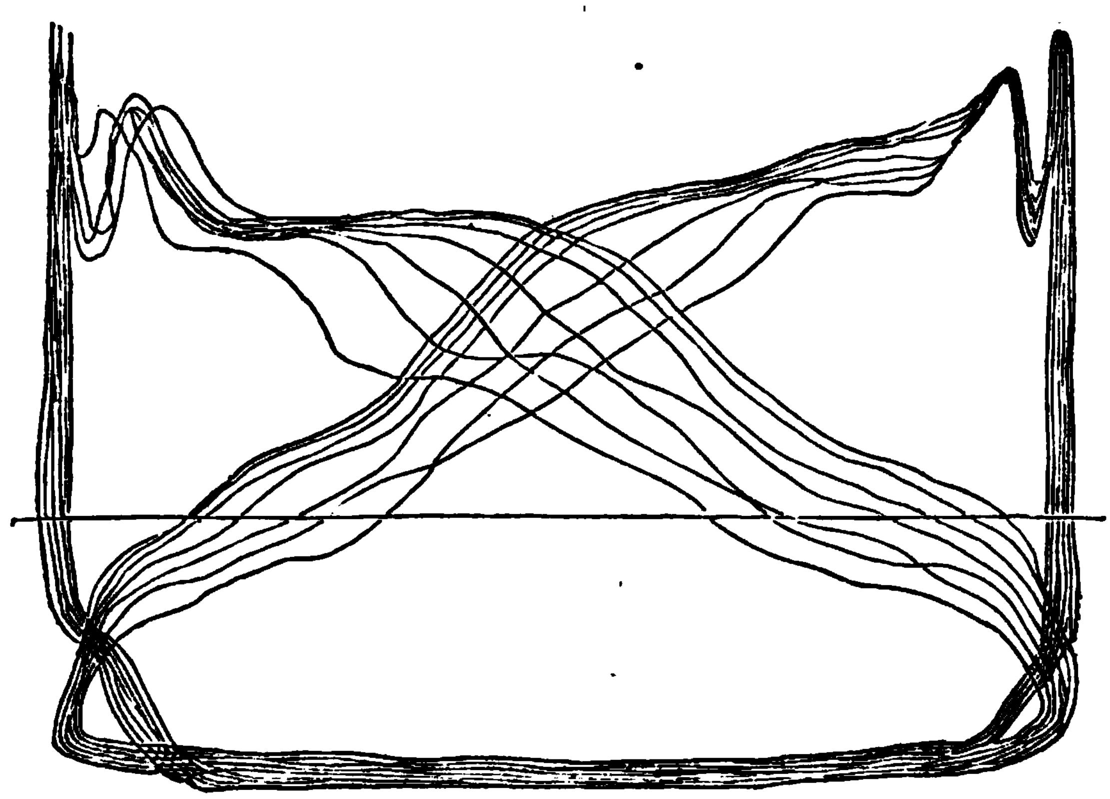
Fig. 15 represents a series of diagrams from each end of one of the engines of the 'Orontes,' formed by allowing the pencil to rest on the paper during many revolutions, instead of only during one. These diagrams show small differences between one another, mainly in the mean pressure of the steam.

Fig. 16 represents two diagrams taken from the engines of the iron-clad screw steamer 'Research, fitted with horizontal engines, with 50-inch cylinders, and 2 feet stroke. With a pressure of steam in the boiler of 22 lbs., and with a vacuum in the condenser of $12\frac{3}{4}$ lbs. per square inch, the mean pressure on the piston shown by the diagrams is 24·55 lbs., the engine making

85 revolutions per minute. This engine is fitted with surface condensers. The serrated deviation at *a* is caused by the momentum of the piston of the indicator.

In fig. 17 we have two diagrams, taken from opposite ends of one of the engines of H.M.S. 'Barossa.' This vessel is 225 feet long, 40 feet 8 inches broad, and 1,702 tons burden. With a mean draught of water 15½ feet or thereabout, the area of midship section is 466 square feet, and the displacement 1,780 tons. The vessel is propelled by two horizontal engines, with cylinders

Fig. 15.

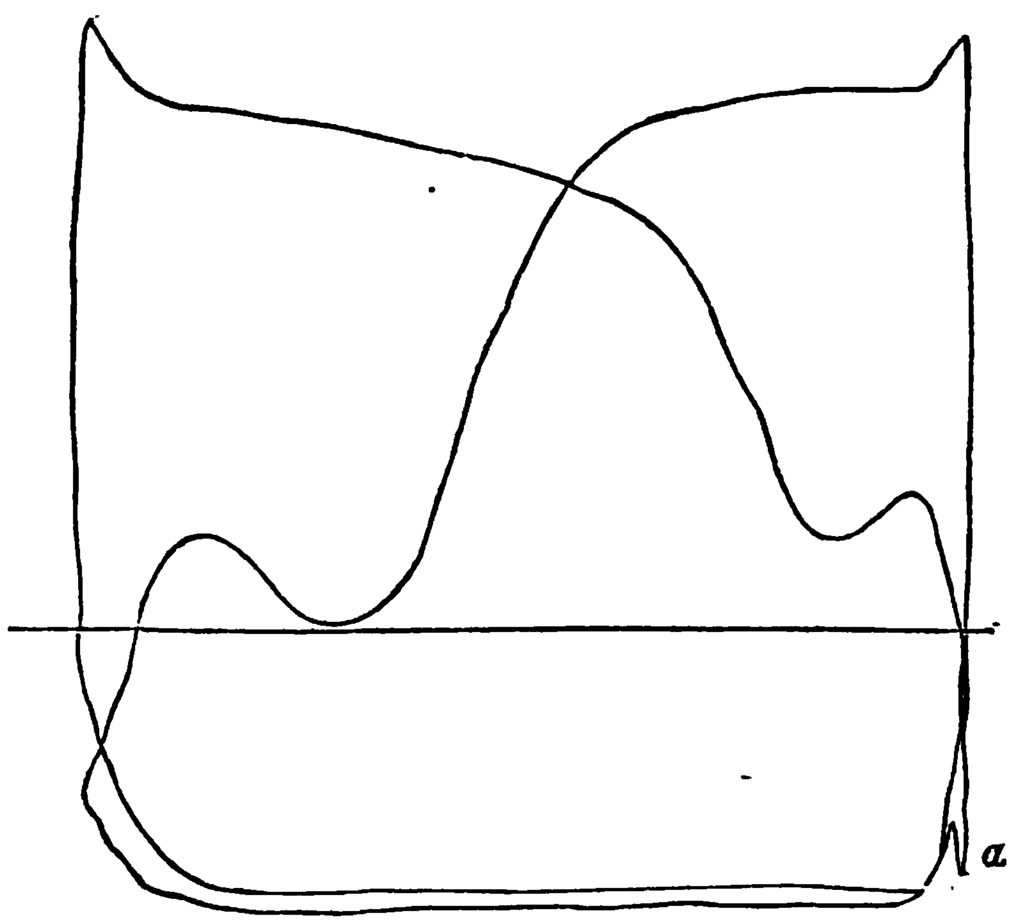


DIAGRAMS TAKEN FROM SCREW STEAMER 'ORONTES.'

of 64 inches diameter and 3 feet stroke, the nominal power being 400 horses. On the official trial this vessel realised a speed of 11·92 knots, with a pressure of steam in the boiler of 20 lbs. per square inch, and with an indicated power of 1798·2 horses, the engine making 66 revolutions per minute. The screw is 16 feet diameter, 24 feet pitch, and 3 feet long, and the slip at the time of trial was 23·71 per cent. When the diagrams shown in fig. 17 were taken, the pressure of steam in the boiler was 19 lbs. per square inch; vacuum in condenser 12½ lbs. per square inch, the revolutions 66 per minute, and the mean pres-

sure on the piston 22·3 lbs. per square inch. The area of a cylinder of 64 inches diameter is 3216·2 square inches, the double of which (as there are two cylinders) is 6433·8 square inches, and as there 22·3 lbs. on each square inch, there will be a total pressure of 6433·8 times 22·3, or 143,473·74 lbs. urging the pistons, and as the length of the double stroke is 6 feet, the power exerted will be equal to 6 times 143,473·74 lbs., or 860,840·44

Fig. 16.



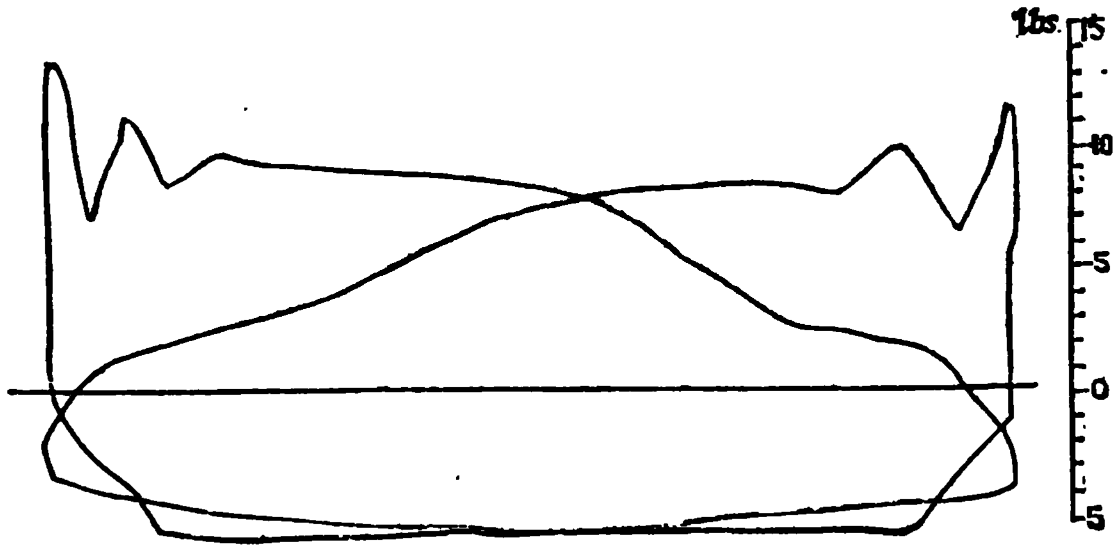
INDICATOR DIAGRAMS FROM IRON-CLAD STEAMER 'RESEARCH.'

foot-pounds per stroke, and as there are 66 strokes per minute, there will be 66 times this, or 56,797,869·04 foot-pounds exerted per minute. As an actual horse-power is 33,000 foot-pounds per minute, we shall, by dividing 56,797,869·04 by 33,000, get the actual power exerted by this engine at the time the above diagrams were taken, and which, by performing the division, we shall find to be 1721·1 horses.

Various Diagrams.—Fig. 18 is a diagram taken from the air-pump of the 'Barossa,' which is a double-acting pump. The

injection was all on at the time this diagram was taken, and the vacuum was only 11 lbs. per square inch. In my 'Catechism of the Steam-Engine,' published in 1856, I drew attention to the fact of the existence of very imperfect vacuums in engines with

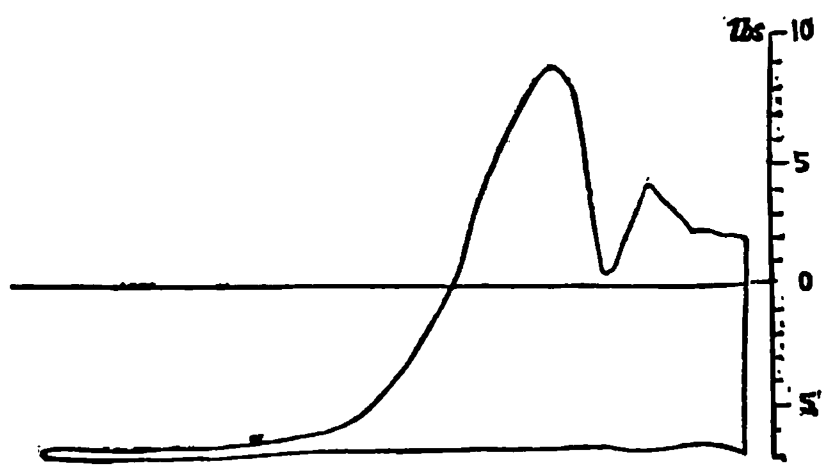
Fig. 17.



DIAGRAMS TAKEN FROM H. M. STEAMER 'BAROSSA.'

double-acting air-pumps, the buckets of which move at a high rate of speed; and I also pointed out the cause of this imperfect vacuum, which I showed to be consequent on the lodgment of

Fig. 18.



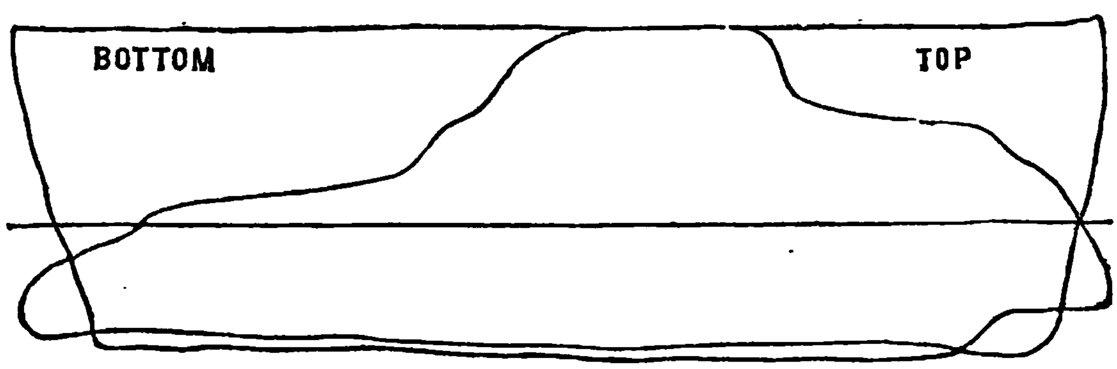
AIR-PUMP DIAGRAM FROM H. M. STEAMER 'BAROSSA.'

large quantities of water between the foot and delivery-valves at the end of the pump, into which water the pump forced in the air or drew it out without ejecting it from the pump at all. I consequently recommend that in all pumps of this class the bucket

and valve-chambers should be so contrived that every particle of water would be forced out of the pump at every stroke. But up to the present time I do not find that this recommendation has been generally adopted, and in nearly every species of direct-acting screw-engine operating by a jet in the condenser, the vacuum is much worse than it was in the old class of paddle-engines, or even in the land engines made by Watt nearly a century ago.

In fig. 19 we have an example of diagrams taken from the top and bottom of one of the paddle-engines of the steamer 'Great Eastern,' constructed by Messrs. J. Scott Russell and Co. These engines are oscillating engines of 74 inches diameter of cylinder, and 14 feet stroke, making 10 revolutions per minute, and there are

Fig. 19.



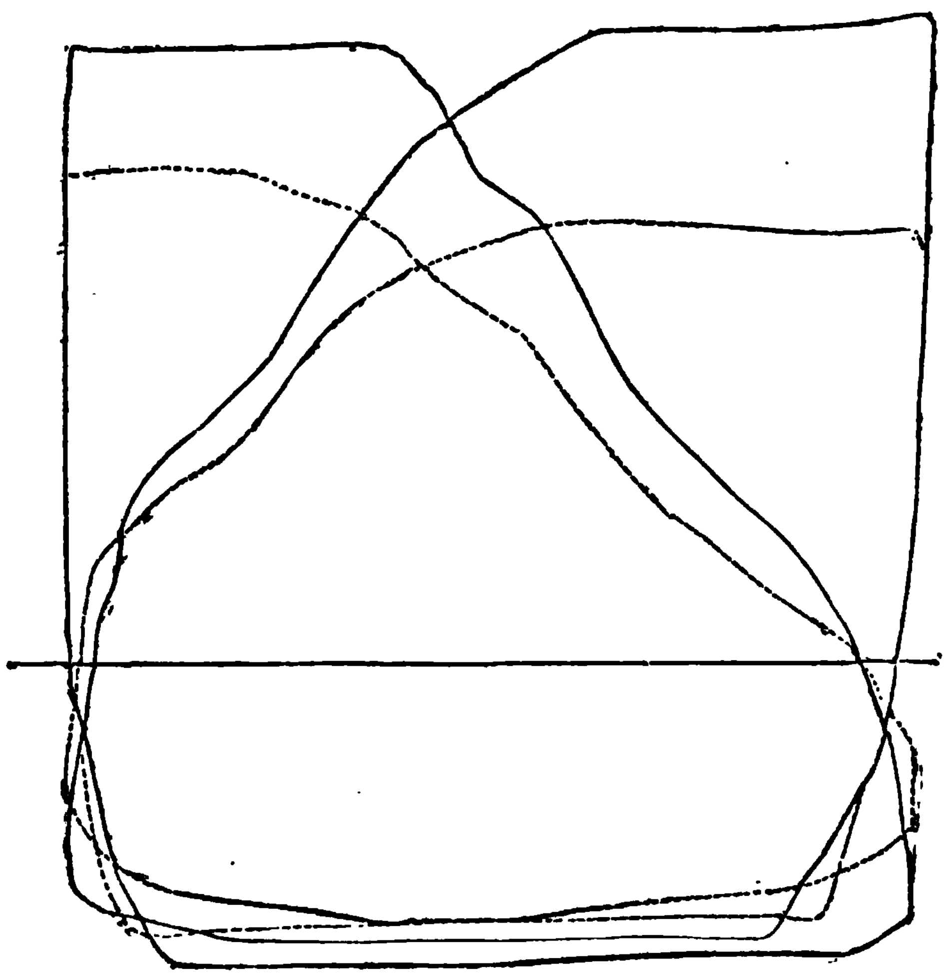
DIAGRAMS FROM PADDLE-ENGINES OF 'GREAT EASTERN.'

four cylinders, or two to each wheel. The mean pressure on the piston which these diagrams exhibit is 22·2 lbs. per square inch, from which, with the other particulars, it is easy to compute the power.

In fig. 20 we have two different pairs of diagrams. The larger pair is taken from one of the engines of the paddle-steamer 'Ulster,' and the smaller pair—represented in dotted lines—is taken from the engines of the paddle-steamer 'Victoria and Albert.' In the case of the 'Ulster' the pressure of steam in the boiler when the diagram was taken was 26 lbs. per square inch, and the vacuum in the condenser 13 lbs. per square inch. The number of strokes per minute was 23, the mean pressure on the piston 28·77 lbs. per square inch, and indicated horse-power

4,100. The 'Victoria and Albert' has two oscillating engines, with 88-inch cylinders and 7-feet stroke. The pressure of the steam in the boilers when the diagrams were taken was 26 lbs. per square inch; of the vacuum 12½ lbs. per square inch; the mean pressure on the piston 22·87 lbs. per square inch, and the number of strokes per minute 25·4. The area of an 88-inch cylinder is 6082·1 square inches, and the area of two such cylinders is the

Fig. 20.



COMPARATIVE DIAGRAMS FROM 'ULSTER' AND 'VICTORIA AND ALBERT.'

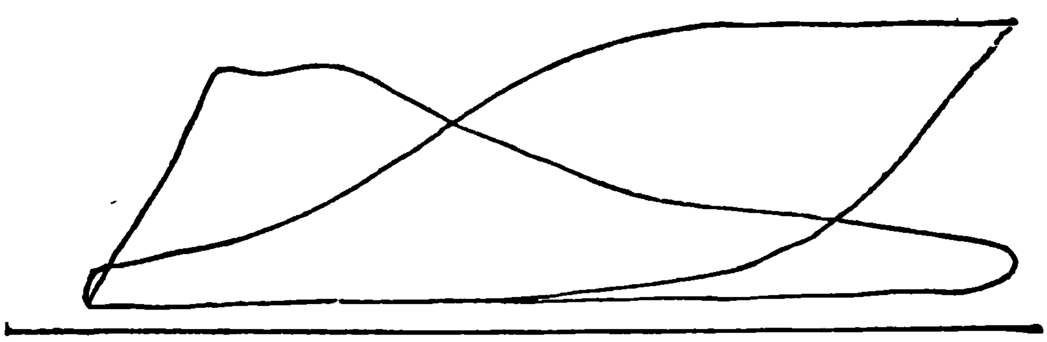
double of this, or 12,164·2 square inches, and as there are 22·87 lbs. on each square inch, the total pressure urging both pistons will be 12,164·2 times 22·87 or 278,195 lbs. Now, as the length of the stroke is 7 feet, and as the piston traverses it each way in each revolution, the piston will travel 14 feet for each revolution, and 278,195 multiplied by 14 will give 3,894,730 as the number of foot-pounds exerted in each stroke; or, as there are 25·4 strokes each minute, there will be 25·4 times 3,894,-

730, or 98,926,142 foot-pounds exerted each minute. Dividing this by 33,000, we get the power exerted by this engine as equal to 2997·7 actual horse-power.

In the diagrams of the 'Victoria and Albert,' it will be remarked there is a greater disparity in the period of the admission of the steam than in the case of the diagrams of the 'Ulster,' arising from the valves not being so accurately set.

Diagram showing wrong setting of Valves.—In fig. 21 are given two diagrams, taken from an engine making 200 strokes per minute, applied to work the exhausting apparatus employed by the Pneumatic Despatch Company to shoot letters and parcels through a tube. These diagrams show that the valve is wrongly set, and that at one end of the cylinder the steam is ad-

Fig. 21.



DIAGRAMS FROM ENGINE OF PNEUMATIC DESPATCH COMPANY.

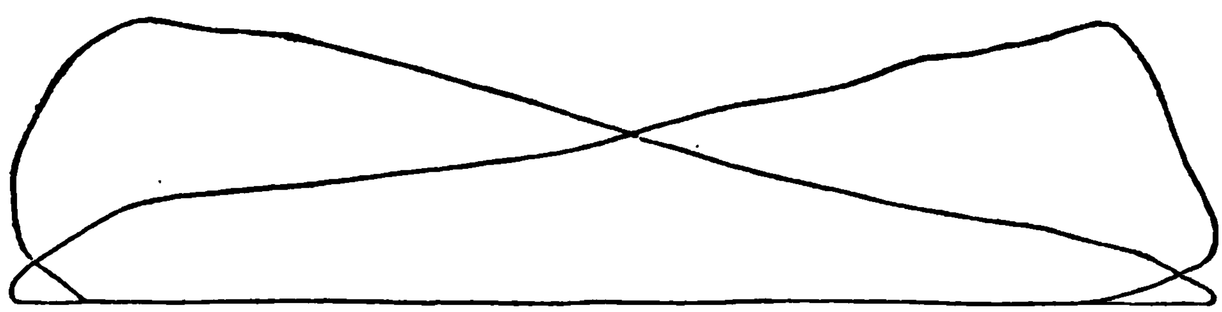
mitted too soon, and at the other end too late. By following the right-hand diagram it will be seen that the eduction passage is closed when about half the stroke has been performed, and that the steam is admitted in front of the piston when about one-fourth of the stroke has still to be performed, whereas the left-hand diagram shows that a considerable part of the stroke has been performed before that end of the cylinder begins to get steam. The action in this case would be amended by shifting round the eccentric. The mean pressure on the piston shown by these diagrams is only 10·79 lbs. per square inch.

Diagram showing the necessity of large Ports for high speeds of Piston.—Fig. 22 represents two diagrams taken from the same engine with the unequal action at the different ends of the cylinder corrected. But the diagrams show that the engine has

not enough lead in the valves, and, moreover, that the passages are too small for the speed with which the engine works. It would be an advantage to increase either the width or the amount of travel of the valve of this engine, or both; as also to give more lead, so that the steam would be able to attain and maintain its proper pressure at the beginning of the stroke, and until it is purposely cut off. The mean pressure of steam on the piston shown by the diagrams represented in fig. 22 is 13·36 lbs. per square inch.

Diagrams illustrative of the action of the Link Motion.—In fig. 23 we have a diagram taken from a horizontal engine, with 27-inch cylinder and 3-feet stroke, constructed by Messrs. Boulton and Watt, employed to work the Portsmouth Floating

Fig. 22.



DIAGRAMS FROM ENGINE OF PNEUMATIC DESPATCH COMPANY.

Bridge. The steam is cut off by the link so as to make the admission almost the least possible, so as to test the engine itself before the chains which draw the bridge backward and forward had been applied. With the steam cut off thus early there is necessarily a very large amount of expansion, and also a very large amount of cushioning; and it will be observed that the steam begins to be compressed at not much less than half-stroke. With this amount of expansion the link is 2¼ inches from the centre. The pressure of steam in the boiler was 22 lbs., and that of the vacuum in the condenser 11 lbs. per square inch, when this diagram was taken; and the engines ran without the chains at 40 revolutions per minnte.

Fig. 24 is another diagram taken from the same engine with

the link in the same place. Pressure of steam in boiler, 21 lbs. per square inch; pressure of vacuum in condenser, 11¼ lbs. per square inch; number of revolutions per minute, 35. In this diagram, and also in the last, we have a small loop formed at the top of the diagram, from causes already explained.

In fig. 25 we have another diagram taken from the same engine, but in this case the steam is not shut off by the link but by the throttle-valve, and there is consequently very little cushioning, and the loop at the top of the diagram almost dis-

Fig. 23.

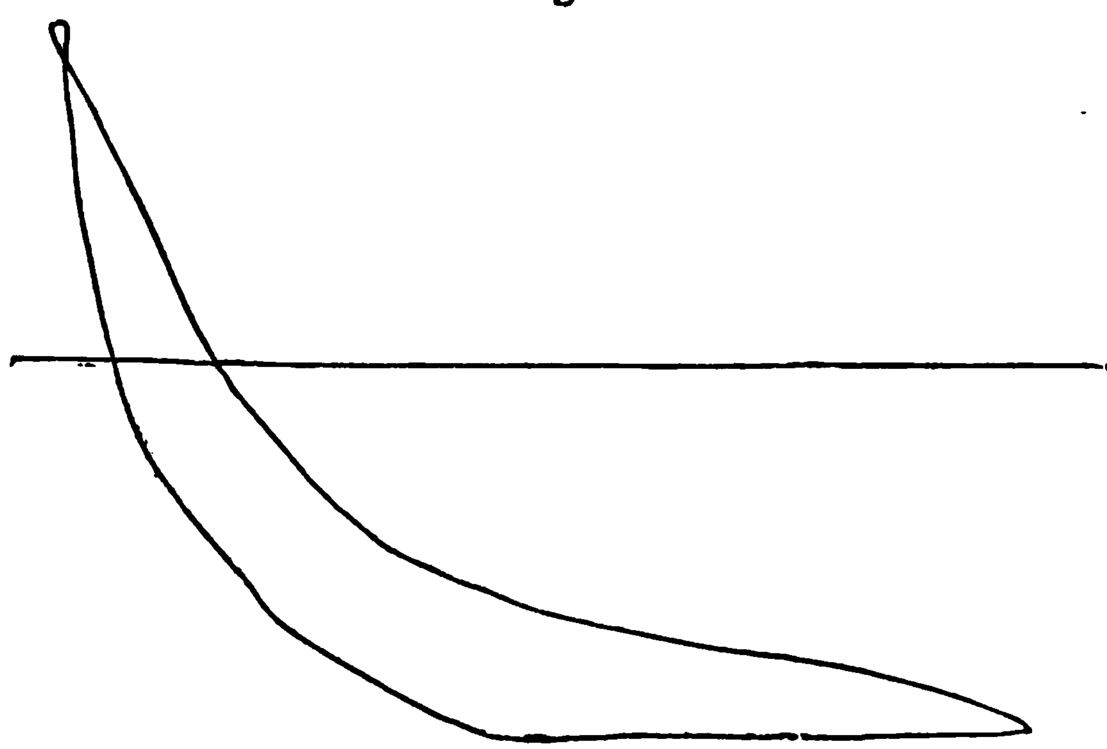


DIAGRAM FROM ENGINE OF PORTSMOUTH FLOATING BRIDGE.
(ENGINE THROTTLED BY LINK.)

appears. When the diagram was taken the pressure of steam in the boiler was 22 lbs., and of the vacuum in the condenser 11¼ lbs. per square inch, and the number of revolutions per minute was 38.

Figs. 26, 27, and 28 are diagrams taken by Richards' indicator from Allen's engine, in the United States department of the International Exhibition of 1862. In this engine the diameter of the cylinder was 8 inches; length of stroke, 24 inches; pressure of steam in boiler, 49 lbs. per square inch; revolutions per minute, 150.

Diagrams illustrative of action of Air-pump and Hot-well. —Fig. 29 is a diagram taken from the air-pump of the Duke of

Fig. 24.

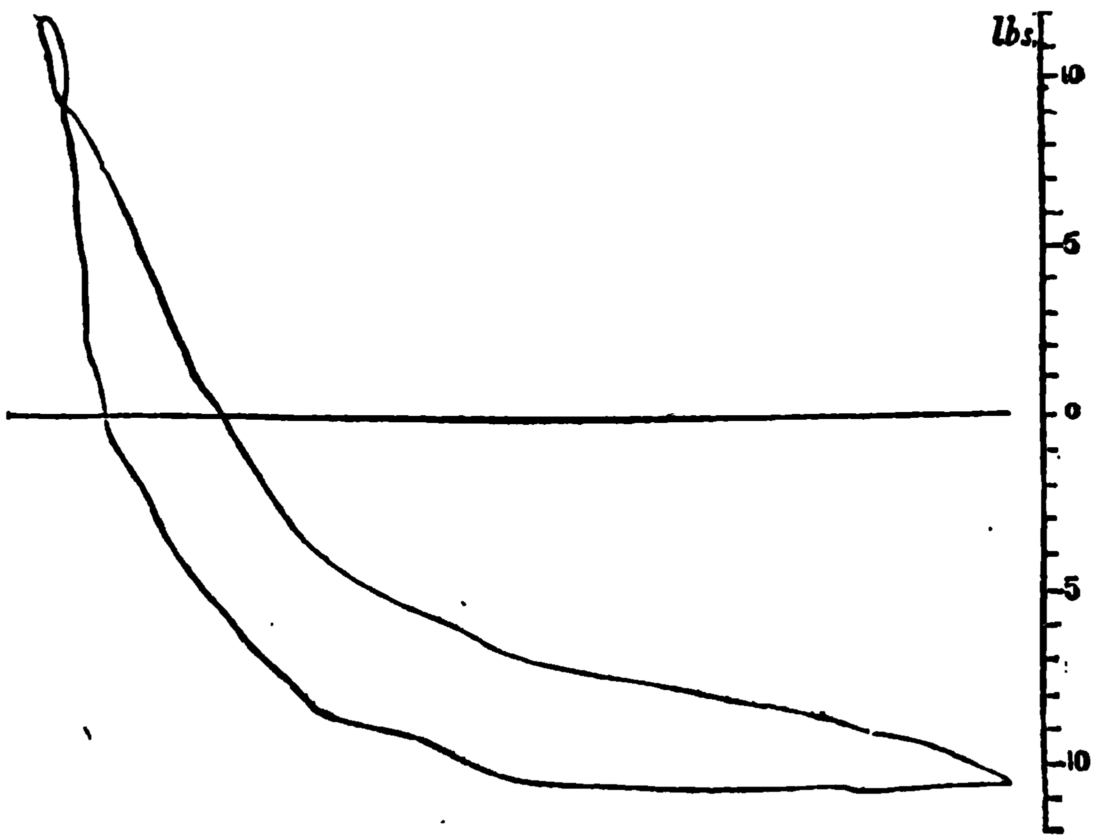


DIAGRAM FROM ENGINE OF PORTSMOUTH FLOATING BRIDGE.
(ENGINE THROTTLED BY LINK.)

Sutherland's yacht 'Undine,' a vessel fitted with two inverted angular engines, with cylinders 24 inches diameter and 15 inches

Fig. 25.

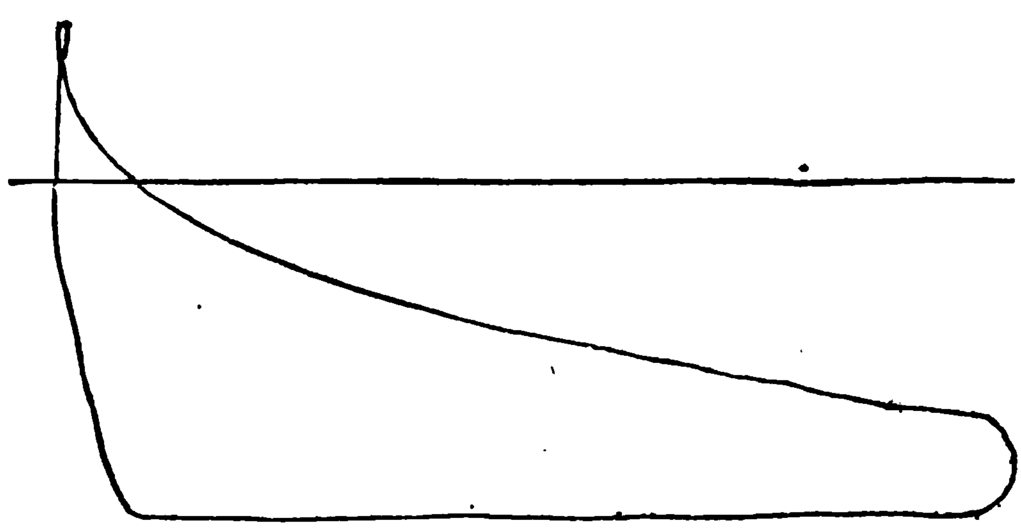


DIAGRAM FROM ENGINE OF PORTSMOUTH FLOATING BRIDGE.
(ENGINE THROTTLED BY THROTTLE-VALVE.)

stroke. When this diagram was taken the ordinary amount of injection was on, and the engine was working at moorings at 72 strokes per minute. There was also an air-vessel on the hot-well. In fig. 30 we have a diagram taken from the air-pump of the same engine, with an extra amount of injection put on.

Fig. 26.

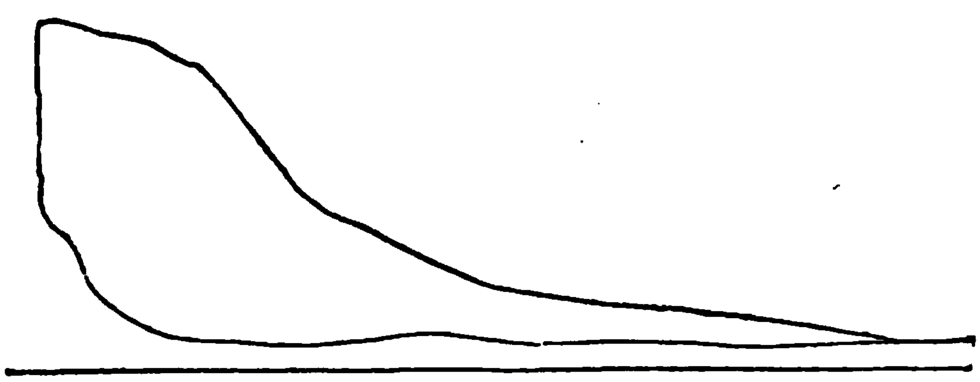
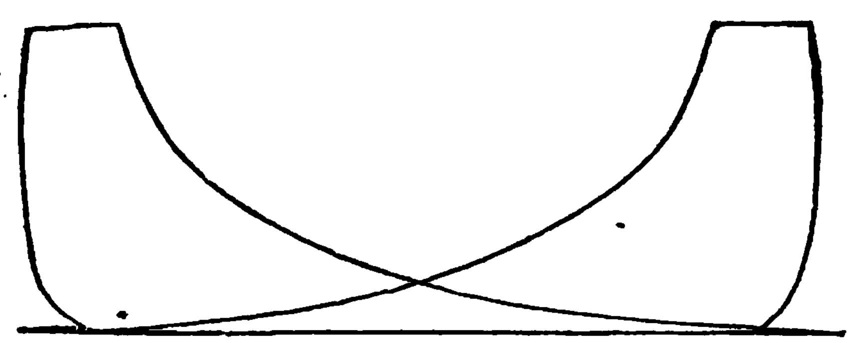


DIAGRAM FROM ALLEN'S ENGINE.

The pump appears to be quite too small for the work it has to do, as is seen by the different configuration of the diagram from that of the diagrams represented in figs. 11 and 18, which are also diagrams taken from air-pumps. In those diagrams, however, the stroke of the bucket is more than half performed, be-

Figs. 27 and 28.



DIAGRAMS FROM ALLEN'S ENGINE.

fore the pressure rises above the atmospheric line; whereas in fig. 30, the pressure rises above the atmospheric line the moment the bucket begins to ascend, showing that at that time the whole of the pump barrel is filled with water. The vacuum must always be inferior where the air-pump is gorged with water.

Unlike the previous diagrams taken from air-pumps, we see in these figures the pressure or resistance has to be encountered from the beginning, or nearly the beginning of the stroke; and the vacuum is not good, and the pump overloaded. There is a

Fig. 29.

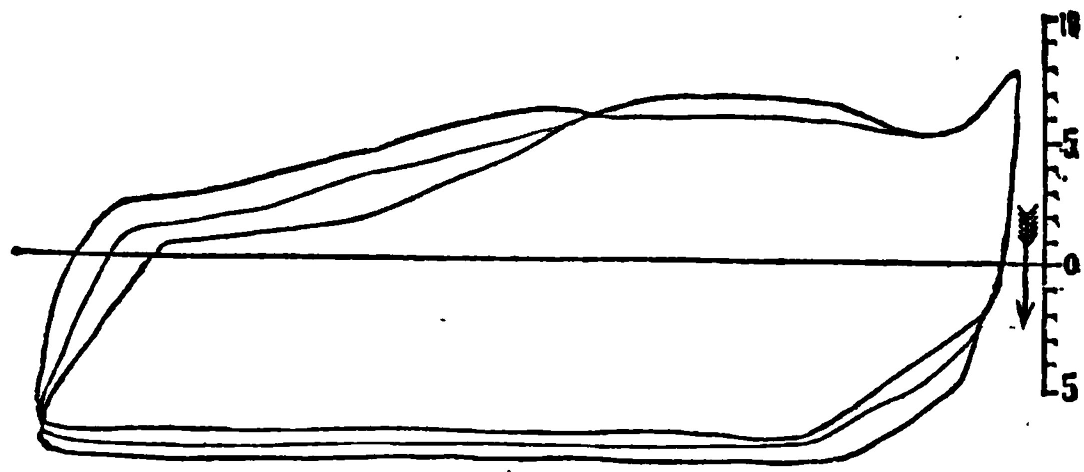


DIAGRAM FROM AIR-PUMP OF DUKE OF SUTHERLAND'S YACHT. (ORDINARY INJECTION.)

worse vacuum with the increased injection than with the ordinary injection, showing that it is not the too great heat of the condenser which makes the vacuum bad, but a deficient capacity of pump, or an imperfect emptying of it every stroke.

Fig. 30.

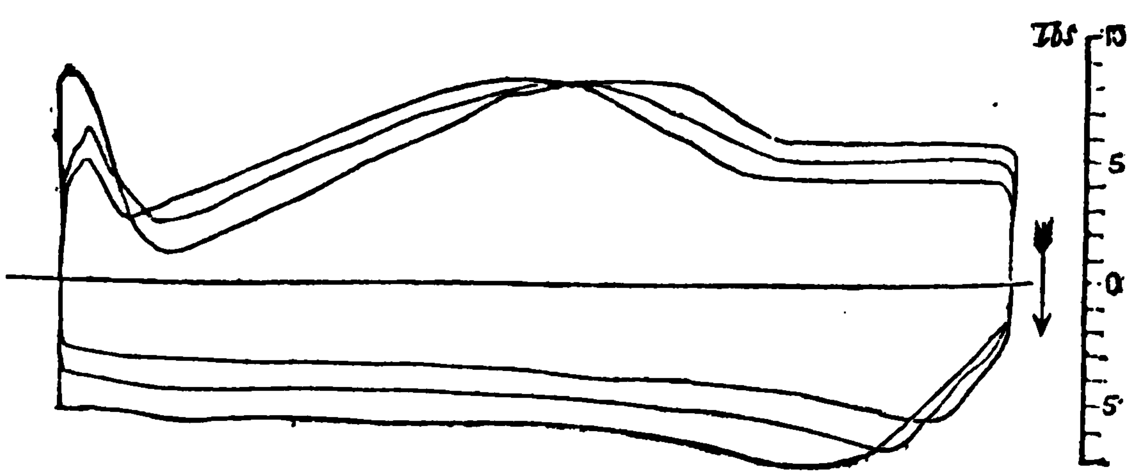


DIAGRAM FROM AIR-PUMP OF DUKE OF SUTHERLAND'S YACHT. (EXTRA INJECTION PUT ON.)

In fig. 31 we have a diagram illustrative of the diminished load upon the air-pump, caused by putting an air-vessel on the hot-well. A is the atmospheric line, and B is the line represent-

ing the ordinary pressure existing in the hot-well when the air-vessel is in operation. By letting out the air the pressure rises to C, showing that the pressure on the pump is less with the air-vessel than without it. If the air-vessel be discarded, an increased velocity must be given to the water passing through the waste-water pipe to enable the bucket to ascend, and this implies a waste of power.

Fig. 31.

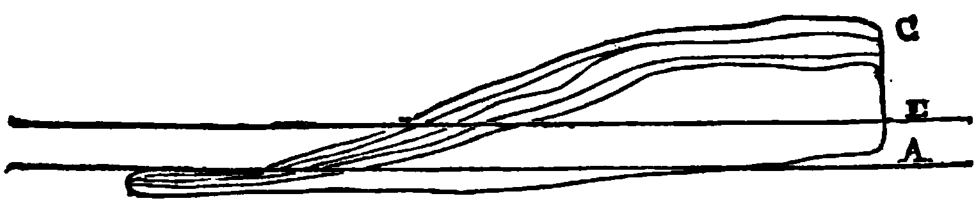


DIAGRAM FROM HOT-WELL OF DUKE OF SUTHERLAND'S YACHT.
(AIR-VESSEL ON.)

In fig. 32 we have a diagram taken from the hot-well of the Duke of Sutherland's yacht after the air-vessel has been removed. In this diagram the pressure begins to rise pretty quickly, as the bucket of the pump ascends; and the maximum pressure, when reached, is maintained pretty uniform to the end of the stroke. It does not then, however, suddenly fall, but only gradually, owing to the momentum of the water; and the

Fig. 32.

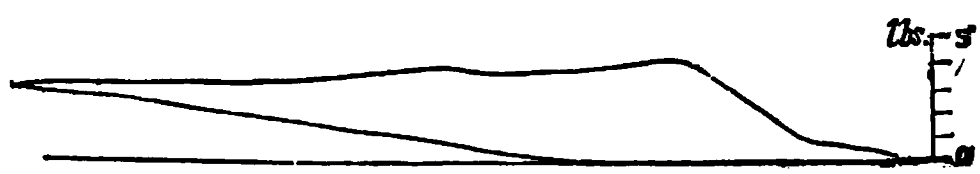


DIAGRAM TAKEN FROM HOT-WELL OF DUKE OF SUTHERLAND'S YACHT.
(AIR-VESSEL OFF.)

pencil does not again come down to the atmospheric line until nearly half the downward stroke of the pump has been completed.

In fig. 33 we have a diagram taken from the hot-well of the steamer 'Scud,' a vessel fitted with two single-trunk engines, that is, trunk engines with the trunks projecting only at one end, and not at both, as in Messrs. Penn's arrangement. The

engines are angular, working up to the screw-shaft, and the cylinders are 68 inches diameter, and 4½ feet stroke. The trunks are 41 inches diameter. These engines made 42 strokes per minute, and worked up to 8½ times the nominal power. The diagram shows an increase of pressure in the hot-well at

Fig. 33.

DIAGRAM TAKEN FROM HOT-WELL OF STEAMER 'SCUD.'

each end of the stroke of the double-acting pump, and the pressure runs up slowly at each end of the stroke, when it slowly falls, forming the loop shown in the diagram.

Diagram from Pump of Water-works.—Fig. 34 is a diagram taken from the pump of a pumping-engine at the Cork Water-

Fig. 34.

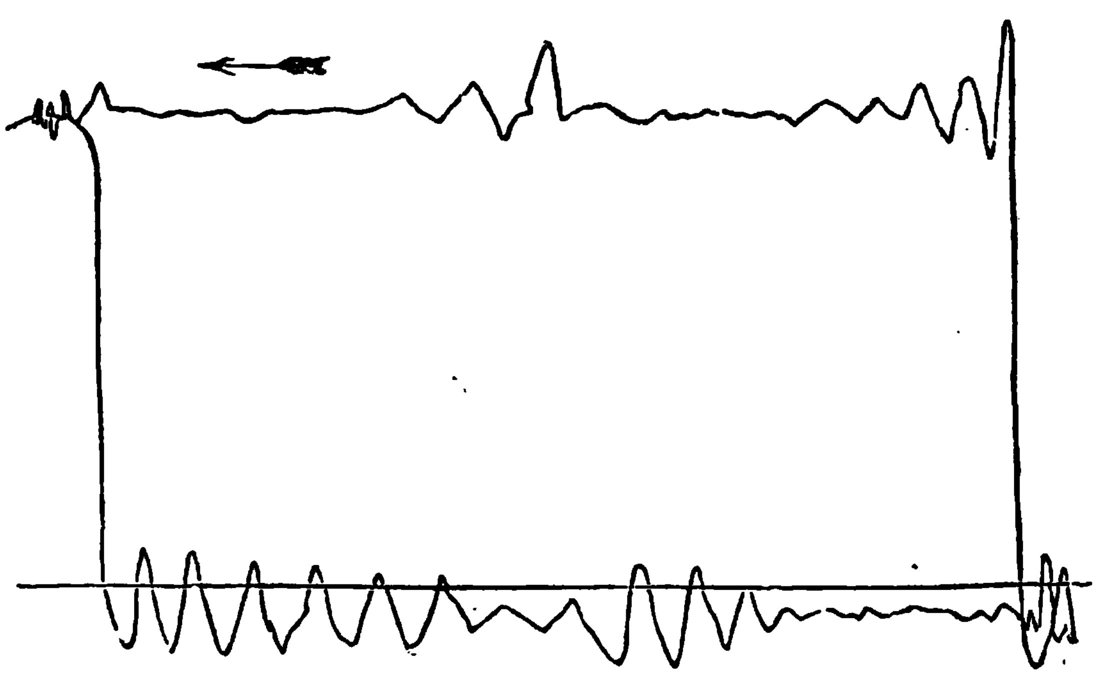


DIAGRAM TAKEN FROM PUMP OF CORK WATER-WORKS.

works. This engine, in common with most pumping-engines of modern construction, is a rotative engine—an innovation first effectually introduced by Mr. David Thomson. The engines make 31 revolutions per minute, and work with steam of 40 lbs. on the square inch. When the plunger is ascending, the pump

is sucking; and when the piston is descending it is forcing, and the diagram shows that both operations are accomplished with much regularity, and without any of those sudden fluctuations which always occasion a loss of power.

Having now shown in what manner the indicator may be applied to ascertain the performance of ordinary engines, I shall proceed to describe the manner of its application in the case of double-cylinder engines. In this class of engines the steam having pressed the first piston to the end of its stroke in the manner of a high-pressure engine, escapes, not into the atmosphere, but into another engine of larger dimensions, where it expands, and acts as low-pressure steam on the piston of the second engine, being finally condensed in the usual manner. The pressure urging the first, or high-pressure piston, is consequently the difference of pressure between the steam in the boiler and that in the second cylinder; and the pressure urging the second, or low-pressure piston, is the difference of pressure between the steam on the eduction side of the high-pressure cylinder and that of the vapour in the condenser. There will be a small difference between the pressures in the communicating parts of the high and low-pressure engines, just as there is a small difference between the vacuum in the cylinder and that in the condenser. But in well-constructed high-pressure engines this difference will not sensibly detract from the power.

Diagrams from Double-cylinder Engines.—In proceeding to determine the power of a double-cylinder engine, we first determine by a diagram and a computation, such as I have already given examples of, the power exerted by the high-pressure engine; and then, in like manner, we determine the power exerted by the low-pressure engine. The total power is obviously the sum of the two.

An example of the diagrams taken from the high and low-pressure cylinders of a double-cylinder engine, at the Lambeth Water-works, constructed by Mr. David Thomson, and erected under his direction, will next be given. In a paper read by Mr. Thomson before the Institution of Mechanical Engineers, and a copy of which he has forwarded to me, the main particulars of

these engines are recited; and some of the most material points of that paper I shall here recapitulate, as these engines constitute a very superior example of the double-cylinder class of engine.

These engines are beam-engines, having the double cylinders at one end of the beam, and a crank and connecting-rod at the other end. Four engines of 150 horse-power each are fixed side by side in the same house, arranged in two pairs, each pair working on to one shaft, with cranks at right-angles, and a fly-wheel between them. The strokes of the crank and of the large cylinder are equal; while the small cylinder, which receives the steam direct from the boiler, has a shorter stroke, and its effective capacity is nearly one-fourth that of the large cylinder. The pumps are connected direct to the beams near the connecting-rod end by means of two side rods, between which the crank works. The pumps are of the combined plunger and bucket construction, and are thus double-acting, although having only two valves. This kind of pump, which is now in general use, was first introduced by Mr. Thomson at the Richmond and the Bristol Water-works in the year 1848. The following are the principal dimensions of the engines:—Diameter of large cylinder, 46 ins.; diameter of small cylinder, 28 ins.; stroke of large cylinder, 8 ft.; stroke of small cylinder, 5 ft. 6⅜ ins.; diameter of pump-barrel 23⅜ ins.; diameter of pump-plunger, 16½ ins.; stroke of pump, 6 ft. 11⅜ ins.; length of beam between extreme centres, 26 ft. 6 ins.; height of beam-centre from floor, 21 ft. 4 ins. The valves are piston-valves, connected by a hollow pipe, through which the escaping steam passes, and are so constructed that one valve effects the distribution of the steam in each pair of cylinders.

The cylinder-ports are rectangular, with inclined bars across the faces to prevent the packing-rings of the valve from catching against the edges of the ports; and the bars are made inclined instead of vertical, in order to avoid any tendency to grooving the valve-packing. The openings of the port extend two-thirds round the circumference of the valve in the ports of the large cylinder; but they extend only half round in the ports of the

small cylinder. The packing of the valve consists of the four cast-iron rings, which are cut at one side exactly as in an ordinary piston, the joint being covered by a plate inside. A considerably stronger pressure of the rings against the valve-chest is required than was at first expected, because the openings of the steamports extend so far round the valve; and for this purpose springs are placed inside the packing-rings to assist their own elasticity. This construction of valve has the advantage of admitting of great simplicity in the castings of the cylinders; and also allows of the whole of the valve-work being executed in the lathe, which is generally the cheapest and most correct kind of work in an engineering workshop. These valves are worked by cams.

The principal object aimed at in the construction of this piston-valve was a reduction to a minimum of the loss of pressure which the steam undergoes in passing from the small cylinder to the large one. This is here accomplished by making the passage of moderate dimensions and as direct as possible; and also by preventing any communication of this passage with the condenser, so that when the steam from the small cylinder enters the passage, the latter is already filled with steam of the density that existed in the large cylinder at the termination of the previous stroke. In constructing the engines some doubt was entertained as to the best size of passage, in order on the one hand to avoid throttling the steam, and on the other to obviate as much as possible the loss of steam in filling the passage. The size adopted was a pipe 6 inches in diameter, or 1-60th of the area of the large cylinder, for a speed of piston of 230 feet per minute in the large cylinder: and this is believed to be about the best proportion, the entire cubic content of the whole passage in the valve amounting to 3,944 cubic inches. The indicator diagrams show that with this construction of valve there is very little or no throttling of the steam, and also that there is but a very moderate drop in the pressure as the steam passes from the small cylinder into the large one. In this respect the valve completely answered the expectations entertained of it and left little further to be desired on this point.

In figs. 35 and 36 we have diagrams taken simultaneously from the top of the small cylinder and the bottom of the large one, in the double-cylinder engines of the Lambeth Water-works, designed by Mr. Thomson—the high-pressure diagram being placed above, and the low-pressure diagram below, with a small space between the two answering to the loss of pressure in the communicating pipe. The dotted line shows the exhaust-line in the small cylinder reversed, so as to tell by direct measure-

Figs. 35 and 36.

lbs

DIAGRAMS FROM DOUBLE-CYLINDER ENGINES, LAMBETH WATER-WORKS.

(TAKEN SIMULTANEOUSLY FROM TOP OF SMALL CYLINDER AND BOTTOM OF LARGE CYLINDER.)

ment between this bottom and the top of the diagram what is the pressure of the steam on the small piston at every part of its stroke.

The most material of the results which may be deduced from the indicator diagrams of this engine are as follows:—Percentage of stroke at which steam is cut off in small cylinder, 40 per cent.; total expansion at end of stroke in small cylinder, in terms of bulk before expansion, 2·41 per cent.; amount of expansion on passing from small to large cylinder, in terms of bulk

before escaping from small cylinder, 1 18 per cent.; total expansion at end of stroke in large cylinder, in terms of original bulk, 9·66 per cent.; total amount of efficient expansion, in terms of original bulk, 8·19 per cent.; total pressure of steam per square inch at point of cutting off, 41 lbs.; theoretical total pressure at end of stroke of small piston, 17·0 lbs.; actual total pressure shown by diagram, 18·0 lbs.; excess of actual over theoretical in percentage of actual pressure, 6 per cent.; theoretical loss of pressure in passage from small to large cylinder, 2·6 lbs.; actual loss shown by diagram, 4·5 lbs.; theoretical total pressure at end of stroke of large piston, 4·2 lbs.; actual total pressure shown by diagram, 5·5 lbs.; excess of actual over theoretical in percentage of actual pressure, 23 per cent.; mean pressure on crank-pin from both cylinders, 22,400 lbs.; maximum ditto, 36,-058 lbs.; ratio of maximum to mean, 1·61 to 1·00; ratio of maximum to mean pressure on crank-pin in a single cylinder engine with the same total amount of efficient expansion, the clearances and ports bearing the same proportion to the working capacity of the cylinder, namely, 1-40th part (this ratio is calculated from the ordinary logarithmic expansion curve), 2·75 to 1·00; efficiency of steam contained in large cylinder at end of stroke, as shown by diagram, if used without expansion, taken as 1·00; actual efficiency of same steam as employed in both cylinders, as shown by diagram, 2·90; theoretical efficiency of the same steam if expanded to the same degree as the total amount of efficient expansion, 3·10. The engines are fitted with steam-jackets, and these indicator diagrams show that the pressure of the steam at the end of the stroke, instead of falling short of what it ought to be by the theoretical expansion curve, exceeds that amount by about 23 per cent. of the actual final pressure. It might be supposed that the increased pressure at the end of the stroke was due to the heat imparted from the jackets either superheating the steam or converting the watery vapour mixed with it into true steam; and probably the latter is the cause of a small part of the observed effect; but Mr. Thomson considers it less likely that sufficient heat could be communicated from the jackets to produce an increase of 23 per cent. in the actual

final pressure, especially as on several occasions the condensed water from the jackets has been collected and found not to exceed half-a-gallon per hour. The experiments made on the quantities of water passed from the boilers give uniformly the result, that a considerably larger quantity of water passes from the boilers than is accounted for by the indicator diagrams, taking the quantity and pressure of the steam just before it escapes to the condenser as the basis of calculation. In some trials made within a few days of these diagrams being taken, the excess of water thus disappearing from the boilers was about 37 per cent. To suppose that the valve was leaking might account for it;* but besides great care having been taken to avoid this source of error, it can hardly be supposed that the valve was always leaking more than the pistons.

To ascertain the amount of friction in these engines Mr. Thomson made many experiments, and found that, when the engines were new, and working at perhaps little more than half their power, the loss in comparing the work done with the indicator diagrams amounts to as much as 25 per cent. of the indicated power; but in these cases the pistons have been too tight in the cylinders, and when this error has been corrected, and the engines worked up to their regular work, all the losses were brought down to from 12 to 15 per cent. of the indicated power. This includes the friction of both the engines and the pumps, the working of the air-pumps, feed-pumps, cold-water pumps, and pumps for charging the air-vessels with air.

With regard to the economy of fuel attained by these double-cylinder engines, it may be stated that the four pumping-engines at the Lambeth Water-works are fixed in one house, and are employed in pumping through a main-pipe 30 inches diameter and about nine miles in length; and when all the engines are working together at their ordinary speed of 14 revolutions per minute, the lift on the pumps, as measured by a mercurial gauge, is equal to a head of about 210 feet of water. Under these circumstances they were tested by Mr. Field soon after being fin-

* Some of the disappearance of the heat is no doubt imputable to its transformation into power, as explained under the head of thermo-dynamics.

ished, in a trial of 24 hours' duration without stopping. The actual work done by the pumps during this trial was equal to 97,064,894 lbs., raised one foot high for every 112 lbs. of coal consumed; in addition to which this consumption included the friction of the engines and pumps, and the power required to work the air-pumps, feed and charging-pumps, and the pumps raising the water for condensation. The coal used was Welsh, of good average quality.

The economy in consumption of fuel during this trial, and in the subsequent regular working of these engines, together with the satisfactory performance generally of the engines and pump work, induced the Chelsea Water-works Company, and also the New River Company, each to erect in 1854 a set of four similar engines, which were made almost exactly the same as the Lambeth Water-works engines already described, with the exception that a jacket of high-pressure steam was in these subsequent engines provided under the bottoms of the cylinders, which had not been done with the previous engines. The pumps were also different in size to suit the different lifts.

The New River engines were tested soon after being completed, and the result reported was 113 million lbs. raised one foot high by 112 lbs. of Welsh coal. But this duty was obtained from a trial of only seven or eight hours' duration, which is too short to obtain very trustworthy results.

The set of engines made for the Chelsea Water-works was the last finished, and on completion the engines were tested by Mr. Field in the same manner as the Lambeth engines, by a trial of 24 hours' continuous pumping. The coal used was Welsh, as before, and the duty reported was 103·9 million lbs. raised one foot high by 112 lbs. of coal. This, as in the previous instance, was the duty got from the pumps in actual work done, no allowance being made for the friction of the engines and pumps, and the power required to work the air-pumps, cold-water pumps, &c. At the time of these engines being tested, the loss by friction and by working the air-pumps, &c., averaged about 20 per cent. of the power, as given by the indicator diagrams; so that if the duty had been estimated from the indicator diagrams, as is usual

in marine engines, it would have been $103{\cdot}9 \times \frac{100}{80}$, or about 130 million lbs. raised one foot by 112 lbs. of coal, which is equivalent to a consumption of 1·7 lb. per indicated horse-power per hour.

In figs. 37 and 38 we have diagrams taken from a small engine called Wenham's double-cylinder engine, working with a pressure of 40 lbs. per square inch in the boiler, and exhibited at the Great Exhibition in 1862. The average pressure on the

Fig. 37.

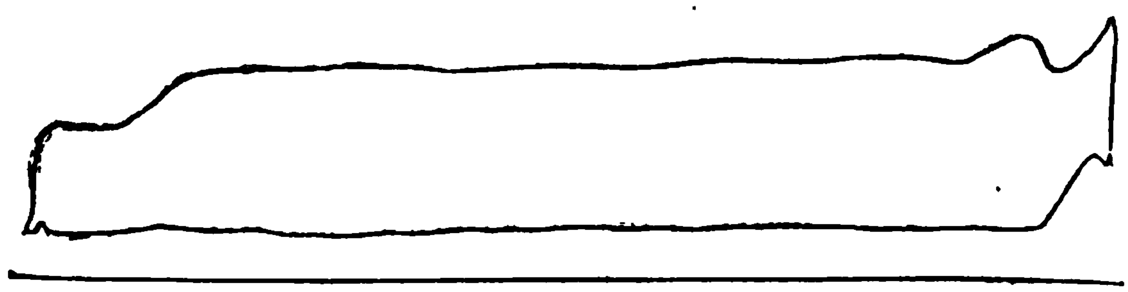


DIAGRAM FROM HIGH-PRESSURE CYLINDER OF WENHAM'S DOUBLE-CYLINDER ENGINE.

(CYLINDER THREE INCHES DIAMETER AND TWELVE INCHES STROKE.)

piston of the high-pressure engine, which is 3 inches diameter and 12 inches stroke, is 26·6 lbs. per square inch, and the power it exerts is 3·16 horses. The average pressure exerted on the

Fig. 38.

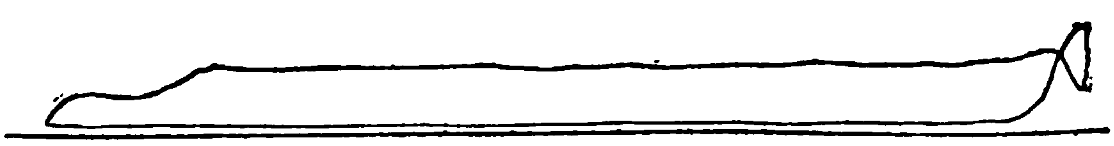


DIAGRAM FROM LOW-PRESSURE CYLINDER OF WENHAM'S DOUBLE-CYLINDER ENGINE.

piston of the low-pressure engine is 8·5 lbs. per square inch, and the power it exerts is 2·37 horses. The steam in passing from one cylinder to the other is heated anew, as had previously been done by me in the engines of the steamer 'Jumna,' of 400 horse-power. The total power developed in both cylinders of Wenham's engine is 6·05 horses.

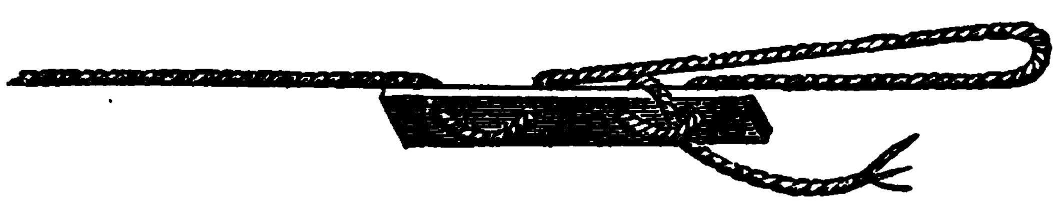
Having now explained how to *interpret* a diagram, the next thing is to explain how to *take* one, and here I cannot do better

than recite the instructions for this operation issued with Richards' indicator by the makers, Elliot Brothers, of the Strand.

To fix the Paper.—Take the outer cylinder off from the instrument, secure the lower edge of the paper, near the corner, by one spring, then bend the paper round the cylinder, and insert the other corner between the springs. The paper should be long enough to let each end project at least half-an-inch between the springs. Take the two projecting ends with the thumb and finger, and draw the paper down, taking care that it lies quite smooth and tight, and that the corners come fairly together, and replace the cylinder. The spring used on this indicator for holding the paper will be found preferable to the hinged clamp. A little practice, with attention to the above directions, will enable any one to fix the paper very readily.

The marking-point should be fine and smooth, so as to draw a fine line, but not cut the paper. It may be made of a brass wire; the best material is gun-metal, which keeps sharp for a long time, and the line

Fig. 39.



made by it is very durable. Lines drawn by German silver points are liable to fade. A large-sized common pin, a little blunted, answers for a marking-point very well indeed; a small file and a bit of emery cloth used occasionally will keep the point in order.

To connect the Cord.—The indicator having been attached, and the correct motion obtained for the drum, and the paper fixed, the next thing is to see that the cord is of the proper length to bring the diagram in its right place on the paper—that is, midway between the springs which hold the paper on the drum. In order to connect and disconnect readily, the short cord on the indicator is furnished with a hook, and at the end of the cord coming from the engine a running loop may be rove in a thin strip of metal, in the manner shown in the preceding cut, by which it can be readily adjusted to the proper length, and taken up from time to time, as it may become stretched by use. On high-speed engines, it is as well, instead of using this, to adjust the cord and take up the stretching, as it takes place, by tying knots in the cord. If the cord becomes wet and shrinks, the knots may need to be untied, but this

rarely happens. The length of the diagram drawn at high speeds should not exceed four and a-half inches, to allow changes in the length of the cord to take place to some extent, without causing the drum to revolve to the limit of its motion in either direction. On the other hand, the diagram should never be drawn shorter than is necessary for this purpose.

To take the Diagram.—Every thing being in readiness, turn the handle of the stop-cock to a vertical position, and let the piston of the indicator play for a few moments, while the instrument becomes warmed. Then turn the handle horizontally to the position in which the communication is opened between the under side of the piston and the atmosphere, hook on the cord, and draw the atmospheric line. Then turn the handle back to its vertical position, and take the diagram. When the handle stands vertical, the communication with the cylinder is wide open, and care should be observed that it does stand in that position whenever a diagram is taken, so that this communication shall not be in the least obstructed.

To apply the pencil to the paper, take the end of the longer brass arm with the thumb and forefinger of the left hand, and touch the point as gently as possible, holding it during one revolution of the engine, or during several revolutions, if desired. There is no spring to press the point to the paper, except for oscillating cylinders; the operator, after admitting the steam, waits as long as he pleases before taking the diagram, and touches the pencil to the paper as lightly as he chooses. Any one, by taking a little pains, will become enabled to perform this operation with much delicacy. As the hand of the operator cannot follow the motions of an oscillating cylinder, it is necessary that the point be held to the paper by a light spring, and instruments to be used on engines of this class are furnished with one accordingly.

Diagrams should not be taken from an engine until some time after starting, so that the water condensed in warming the cylinder, &c., shall have passed away. Water in the cylinder in excess always distorts the diagram, and sometimes into very singular forms. The drip-cocks should be shut when diagrams are being taken, unless the boiler is priming. If when a new instrument is first applied the line should show a little evidence of friction, let the piston continue in action for a short time, and this will disappear.

As soon as the diagram is taken, unhook the cord; the paper cylinder should not be kept in motion unnecessarily, as it only wears out the spring, especially at high velocities. Then remove the paper, and minute on the back of it at once as many of the following particulars as you have the means of ascertaining, viz.:

The date of taking the diagram, and scale of the indicator.

The engine from which the diagram is taken, which end, and which engine, if one of a pair.

The length of the stroke, the diameter of the cylinder, and the number of double strokes per minute.

The size of the ports, the kind of valve employed, the lap and lead of the valve, and the exhaust lead.

The amount which the waste-room, in clearance and thoroughfares, adds to the length of the cylinder.

The pressure of steam in the boiler, the diameter and length of the pipe, the size and position of the throttle (if any), and the point of cut-off.

On a locomotive, the diameter of the driving-wheels, and the size of the blast orifice, the weight of the train, and the gradient, or curve.

On a condensing-engine, the vacuum by the gauge, the kind of condenser employed, the quantity of water used for one stroke of the engine, its temperature, and that of the discharge, the size of the air-pump and length of its stroke, whether single or double acting, and, if driven independently of the engine, the number of its strokes per minute, and the height of the barometer.

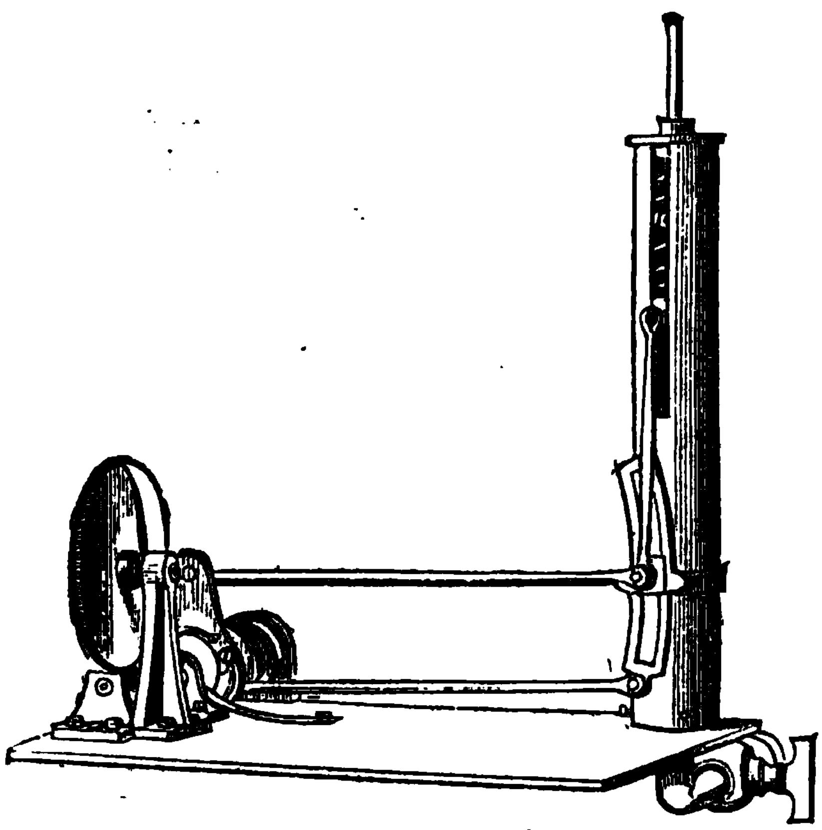
The description of boiler used, the temperature of the feed-water, the consumption of fuel and of water per hour, and whether the boilers, pipes, and engine are protected from loss of heat by radiation, and if so, to what extent.

In addition to these, there are often special circumstances which should be noted.

Counter and Dynamometer.—There are other instruments besides the indicator for telling the performance of an engine—the counter which registers the number of strokes made by an engine being used for this purpose, in the case of pumping-engines, working with a uniform load, and the dynamometer being employed in testing the power exerted by small engines. The dynamometer consists of a moving disc well oiled, and encircled by a stationary hoop, which can be so far tightened as to create sufficient friction to constitute the proper load for the engine. The hoop is prevented from revolving with the disc by an arm extending from it, which is connected with a spring, the tension on which, reduced to the diameter of the disc, represents the load which the friction creates; and the load multiplied by the space passed through per minute by any point on the circumference of the disc will represent the power. Such dyna-

mometers, however, cannot be conveniently applied to large engines; and as in steam-vessels, where economy of fuel is most important, the counter will not accurately register the work done, seeing that the resistance is not uniform, and as without some reliable means of determining the power produced in different vessels relatively with the fuel consumed, it is impossible to establish such a comparison of efficiency as will lead to emulation, and consequent improvement, I have felt it necessary to contrive a species of continuous indicator, or power-meter, for

Fig. 40.



BOURNE'S DUTY METER.

ascertaining and recording the amount of work done by any engine during a given period of time. The outline of one form of this instrument is exhibited in fig. 40; but I prefer that the cylinder should be horizontal instead of vertical, and that it should be larger in diameter, and shorter—this figure being copied from a photograph of an instrument I had converted from a common M'Naught's indicator, for the sake of readiness of construction. In this instrument one end of the indicator cylinder communicates with one end of the main cylinder, and the other end of the indicator cylinder with the other end of the

main cylinder, so that the atmosphere does not press upon the piston of the indicator at all, but that piston is pressed on either side by steam or vapour of precisely the same tension as that which presses on either side of the piston of the engine. The indicator piston is pressed alternately upward and downward against a spring in the usual manner. A double-ended lever vibrating on a central pivot, and with a slot carried along it nearly from end to end, as in the link of a common link-motion, is attached tc the side of the cylinder, and from this slot a horizontal rod extends to the arm of a ring encircling a ratchet-wheel, there being a number of pawls in this ring of different lengths to engage the ratchets. This link is moved backwards and forwards on its centre, 8 or 10 times every stroke of the engine, by means of the lower horizontal rod which is attached at one end to the lower end of the link, and at the other end to a small pin in the side of a drum, which is drawn out by a string, like the drum for carrying the paper in a common indicator, and is, in like manner, returned by a spring; but the dimensions of the drum, and the place of attachment of the string, are such that the drum makes a considerable number of turns—say 10—for each stroke of the engine, and the link makes the same number of reciprocations. If there be an equality of pressure on each side of the piston, the end of the rod moving in the slot will be in the mid position; and as while it is there no amount of vibration of the link will give it any end motion, there will be no motion under such circumstances communicated to the ratchet. If, however, the pressure either upward or downward is considerable, the end of the rod will be moved so much up or down in the link that its reciprocation will give considerable end motion to the rod communicating with the ratchet; and the amount of motion given to the ratchet every stroke will represent the amount of mean pressure urging the piston. The number of revolutions to be made by the drum every stroke having been once definitively fixed, it is clear that the number of revolutions it will make per minute will depend on the number of strokes made per minute by the engine, and the revolutions of the ratchet-wheel will consequently represent both the mean pressure and the speed of

piston—or in other words, it will represent the power. The spindle of the ratchet wheel is formed into a screw, which works into the periphery of a wheel that gives motion to other wheels and hands, like the train of a gas-meter; and on opening the instrument at the end of any given time, such as at the termination of a voyage of an ocean steamer, the power which the vessel has exerted since she started on the voyage will be found to be accurately registered. This being compared with the quantity of coals consumed, which can easily be found from the books of the owners, will give the duty of the engine; and by ascertaining and publishing the duty of different vessels, a wholesome emulation would be excited among engine-makers and engine tenders, and a vast reduction in the consumption of fuel would no doubt be obtained. For many years past I have urged the introduction of that system of registration in the case of steam-vessels which in the case of the Cornish engines speedily led to such unprecedented economy. But the want of a suitable registering apparatus constituted a serious impediment, and I have consequently undertaken to contrive the instrument of which a rough outline is given above.

Heating Surface in modern Boilers.—The quantity of heating surface given in modern boilers per nominal horse-power has been constantly increasing, until, in some of the boilers of recent steam-vessels intended to maintain a high rate of speed, it has become as much as 35 square feet per nominal horse-power; and such vessels exert a power nine times greater than the nominal power. The nominal power, in fact, has ceased to be any measure of the dimensions of a boiler; and the best course will be to consider only the water evaporated. In modern marine boilers it may be reckoned that a cubic foot of water will be evaporated in the hour by 7 lbs. of coal burned on 70 square inches of fire-bars, and the heat from which is absorbed by 10 square feet of heating surface, so that the consumption of coal per hour, on each square foot of grate, will be 14·4 lbs. If the steam be cut off from the cylinder when one-third of the stroke has been performed, as is a common practice, the efficiency of the steam will be somewhat more than doubled, or a horse-power will be

generated with something less than $3\frac{1}{2}$ lbs of coal. In large boilers and engines, however, the efficiency is greater than in small, and there is further benefit obtained from superheating, and from heating the feed-water very hot. In modern steam-vessels of efficient construction, therefore, the consumption of coal is not more than $2\frac{1}{2}$ lbs. per actual horse-power. Boulton and Watt put sufficient lap upon their valves to cut off the steam when two-thirds of the stroke have been performed as a minimum of expansion; and then, by aid of the link-motion, they can expand still more, if required, so as to cut off when one-third of the stroke has been performed.

The area of the back uptake should be 15 square inches per cubic foot evaporated; the area of the front uptake 12 square inches, and the area of the chimney 7 square inches per cubic foot evaporated. These proportions will enable the dimensions of any boiler to be determined when the rate of expansion has been fixed.

The proportion in which the actual exceeds the nominal power varies very much in different engines, but about 4 or $4\frac{1}{2}$ times appears to be the prevalent proportion in 1865, though, as I have stated, in special cases twice this proportion of power is exerted, and the boilers are proportioned to give the increased supply of steam required. For any temporary purpose the power may be increased by quickening the draught through the furnace by a jet of steam in the chimney; but in such case the consumption of fuel per cubic foot of water evaporated will be somewhat increased. The first proportion of heating surface, however, which the flame encounters is very much more efficient than the last portion, in consequence of the higher temperature to which it is subjected; and if the draught be quickened the temperature will be increased, and every square foot of heating surface will thereby acquire a greater absorbing power. The hotter the furnace is, the more heat will be absorbed by the water in the region of the furnace; and the more heat that is absorbed by the furnace the less will be left for the tubes to absorb. It is material, therefore, to maintain high bridges, a rapid draught, and all other aids to a high temperature in the furnace; as the absorption of heat will thus be more rapid, and the combustion will be more perfect,

from the high temperature to which the smoke is exposed. It will increase the efficacy of the heating surface, moreover, if the smoke be made to strike against instead of sliding over it; and this end will be best attained by using vertical tubes, with the water within them, on which the smoke may strike on its way to the chimney. Such tubes, furthermore, are eligible in consequence of the facilities they give for the rapid circulation of the water within the boiler; and this rapid circulation will not merely render the boiler more durable by preventing overheating of the metal, but as the rapidly ascending current, by carrying off the steam and presenting a new surface of water to be acted upon, keeps the metal of the tubes cool, they are in a better condition for absorbing heat from the smoke than if the metal had become overheated from the entanglement of steam in contact with it, which impeded the access of the water, and prevented the rapid absorption of heat which would otherwise take place. In locomotive boilers, where the temperature of the furnace is very high, as much evaporative efficacy is obtained from 7 lbs. of coal, with 5 or 6 square feet of heating surface, as is obtained in land and marine boilers with 9 or 10; and the reason manifestly is, that as the rapidity of the transmission of heat increases as the square of the temperature, a square foot of heating surface in a furnace twice as hot will be four times more effective, so that the tubes are left with comparatively little work to do, from so much of the work having been done in the furnace. Each square foot of tube surface in locomotives will only evaporate as much as each square foot in an ordinary land and marine boiler; but the mean efficacy of the whole heating surface is, nevertheless, raised very high by the greatly increased efficacy of the fire-box surface, from its high temperature. It is desirable to imitate these conditions in marine and land furnaces by making the area fire-grate small, the draught rapid, and the bridges high, to the end that a high temperature in the furnace may be preserved, and a consequently rapid generation of steam promoted. It would also be desirable, and not difficult, to feed the furnaces with hot air instead of with cold, which would conduce more to economy than feeding the boiler with hot instead of cold water; and it would not be dif-

ficult to carry out this improvement, by encircling the chimney with air-casing nearly to the top, and conducting the air which would be admitted by openings around the casings at its upper end, past the smoke-box doors, to the end of the furnaces. The only difficulty which might be apprehended from this procedure would be the increased heat and diminished durability of the furnace-bars. But this difficulty might no doubt be surmounted by making the bars deep and thin, and by not increasing the temperature of the entering air beyond the point which experience proved it could be raised to with impunity. The area of the casing around the chimney would require to be about as great, at the largest part, as the area of the chimney itself. But it could be made conical, or tapering off at the top, and the air might be admitted in vertical slits extending downwards for a certain length, as the heat at the top of the chimney could be abstracted by such a small volume of air as a narrow casing would contain. In this heating of the air entering furnaces there is an expedient of economy available for the engineer which has not yet been brought into force; and its effect will be both to reduce the consumption of the fuel and to render the existing heating surface more effective. If, for example, we take the existing temperature of the furnace to be 3,000° Fahrenheit, and if we increase the temperature of the entering air by 500°, which we might easily do without any new expense, we shall not merely save one-sixth of the fuel, but we shall render the absorbing surface of the furnace more efficacious by raising the temperature from 3,000° to 3,500°. Nor will this probably be the limit of benefit obtained; and as in feeding boilers with boiling water instead of cold, and in surrounding cylinders by steam to keep them hot instead of exposing them to the atmosphere, we obtain a greater benefit than theory would have led us to expect, so in feeding furnaces with hot air instead of cold air we shall in all probability obtain a larger benefit than that which theory indicates. The experience already obtained of the saving effected by using the hot blast in iron smelting furnaces certainly points to the probability of such a realization; and one manifest effect will be, that the combustion of the coal will be rendered more perfect, and less smoke will be produced.

The present system of land and marine boilers, however, is altogether faulty, and must be changed completely. When I planned and constructed the first marine tubular boiler in 1838, and which was adapted for working with a high pressure of steam, and which also had the advantage of surface condensation, the innovation was a step in advance, and it has proved successful and serviceable, though up to the present time the system then propounded by me has not been fully wrought out in practice. But we now want something much better than what would have sufficed for our wants in 1838, and I will here briefly recapitulate what we require and must obtain. First, then, we must have a still higher pressure of steam than I contemplated in 1838; to obtain which with safety we must have two things; a very strong boiler, and absolute immunity from salting. The expedient of surface condensation, which I propounded in 1838, as the means of accomplishing the last disideratum, though effectual for the purpose, and now widely adopted, is less eligible for moderate pressures than the method of preventing salting which I have since suggested, and which consists in the introduction of a small jet in the eduction-pipe, the water of which, though unable wholly to condense the steam, will be itself raised to the boiling point, and be transmitted to the boiler without any means of stopping it off; and the excess of feed-water which, under this arrangement, will always be entering the boiler, will escape through a continuous blow off, and thus prevent the boiler from salting. The column of steam escaping to the condenser will, under suitable arrangements, itself force this water into the boiler; and in locomotives, in like manner, the water may be forced into the boiler by using a portion of the steam escaping from the blast pipe for that purpose, whereby the boiler will be fed with boiling water by the aid of steam otherwise going to waste. In this way marine boilers may be kept from salting; for the sulphate of lime which is deposited from sea water at the temperatures of high-pressure steam, may be separated by filtration in the feed pipe. On the whole, for high pressures a small surface condenser with auxiliary jet seems best. To give a rapid circulation to the water, and render the heating surface efficient

in the highest degree, the tubes should be upright with the water within them; and the furnaces should be fed with coal by self-acting mechanism, which would abridge the labor of firing, and insure the work being better done. To reduce the strain on the engine at the beginning of the stroke, when steam of a high pressure is employed, the stroke should be long, the piston small in diameter, and a considerable velocity of piston should be employed; or, where there are two engines, the steam may be expanded from the cylinder of one engine into the cylinder of the other engine, according to Nicholson's system, whereby twice the expansion will be obtained with only the same apparatus.

Relative surface areas of Boilers and Condensers.—The evaporative power of land and marine boilers per square foot of heating surface, depends very much upon the structure and configuration of the boiler. In some marine engines a performance of six times the nominal power has been obtained with a proportion of heating surface in the boiler of only 12 square feet per nominal horse-power; and as about half of this power was obtained by expanding the steam, 1 cubic foot of water was evaporated by every 4 square feet of heating surface, which is a smaller proportion even than that which obtains commonly in locomotives. In such cases the proportion of cooling surface in the condenser has been made equal to the amount of heating surface in the boiler; and the amount of cooling surface in the condenser relatively to the amount of the heating surface of the boiler should manifestly have reference to the activity of that heating surface. So in like manner it should be influenced by the amount of expansion which the steam undergoes in the cylinder; since the steam, in communicating power, parts with a corresponding quantity of heat. A still more important condition of the action of the condenser is, that the water shall pass through the tubes with rapidity, and that it shall flow in the opposite direction to the steam, so that the hottest steam shall meet the warmest water; as warm water will suffice to condense hot steam, which would be quite inoperative in condensing attenuated vapour. A common proportion of condenser surface in modern engines is ·75 that of the boiler surface. Thus a

boiler with 20 square feet of heating surface will have 15 square feet of heating surface. But the largest part of this surface is required to obtain the last pound or two of exhaustion; and it is preferable to employ a moderate surface to condense the bulk of the steam, and to condense the residual vapour by a small jet of salt water let in from the sea. It is found advisable to admit a small quantity of salt water on other grounds. For the fresh water in the boiler, as it forms no scale, leaves the boiler subject to the corrosive influence produced by placing a mass of copper tubes—on which the sea water acts chemically—in connexion with the mass of wet iron which constitutes the boiler; and, as in Sir Humphrey Davy's arrangement for protecting copper sheathing by iron blocks, the copper tubes are protected at the expense of the boiler, since the communicating pipes and the water within them form an efficient connexion. It would be easy to break the circuit so far as the metal is concerned by interposing glass flanges between the flanges of the pipes. But this would not stop the communication by the water itself, and the best course appears to satisfy the corroding conditions by placing blocks of zinc within the condenser, which might be corroded instead of the tubes or the boiler. The present antidote to the corrosive action consists in the introduction of a certain proportion of salt water into the boiler, which is intended to shield the evaporating surfaces from corrosive action by depositing a coating of scale upon those evaporating surfaces. But in this arrangement we have necessarily an excess of water entering the boiler; for we have not only all the water returned which passes off as steam, but a certain proportion of sea water besides. It will consequently be necessary to provide for the excess being blown out of the boiler; and the question is, whether, as we must introduce such an arrangement, it would not be advisable, with low pressures, to make the proportions such as would enable us to dispense with the surface condenser altogether? If it is retained at all, it should only be retained in such shorn proportions as to condense the grossest part of the steam—the water resulting from which should be sent into the boiler quite hot, and the rarer part of the steam should be con-

densed by a jet of salt water of about the same dimensions as that already employed. It is very necessary to be careful in the case of surface condensers to prevent any leakage of air, which, if mingled with the steam, would form a wall of air against the refrigeratory surface, which would prevent the contact of the steam and hinder the condensation, precisely as it was found to do in the old engines of Newcomen, where air was purposely admitted to form a stratum between the hot steam and the cold cylinder; and which diminished the loss from the condensation of the steam within the cylinder to a very material extent.

Example of modern marine engine and boiler.—As an example of the proportions of marine engines and boilers and condensers of approved modern construction, I may here recapitulate the main particulars of the machinery of the screw steamer 'Rhone,' constructed for the West India Mail Company by the Millwall Iron Company in 1865.

These engines are on the inverted cylinder principle of 500 horse-power. There are two cylinders of 72 inches diameter and 4 feet stroke, and the estimated number of revolutions per minute is 52. The cylinders are supported on massive hollow standards resting on a bed plate of the same construction. There are two air-pumps wrought by links and levers from two pins on the ends of the piston rods. The surface condenser is placed between the two air-pumps, and is fitted with brass tubes placed horizontally, and resting in vertical tube plates. The two end plates have screwed stuffing boxes, with cotton washer packing for each tube. The tubes are divided into three groups or sections, through each of which the condensing water successively passes; and the water enters from the lower end of the condenser and escapes at the upper end, where the steam enters, so that the hottest water meets the hottest steam. The two circulating pumps are placed opposite each other, and are wrought by a crank on the end of the crank shaft. The steam is condensed outside the tubes; and the condensed water flows down to the air-pumps, by which it is pumped to the hot well, and from which it is taken to the boilers in the usual way.

The crank shaft is of Krupp's cast steel in two pieces,

coupled by flanges. The screw shaft is of iron, covered with brass in the stern tube, and working in lignum vitæ bearings in the stern tube and after stern post. The boilers are in four separate parts, and fitted with a superheating apparatus consisting of a series of vertical iron tubes 4½ inches bore, on the plan of Mr. Ritchie, the company's superintending engineer.

The surface condenser has 3,566 tubes, ¾ inches external diameter, and 9 feet 2¼ inches long between the tube plates. The surface of the tubes is 6,525 square feet, or 13·05 square feet per nominal horse-power. The two circulating pumps are double acting 25″ diameter, with a trunk of 17″ diameter on one end of the plungers. The boilers have 20 furnaces 3′ 0⅛″ wide, with fire bars of 6 feet 8 inches in length. The total area of fire grate is 400 square feet, = 0·8 square feet per nominal horse-power. The number of brass tubes in the boiler is 1,180 of 3½ external diameter and 6 feet 8 inches long. The total heating surface in the boilers is 9,800 square feet, or 19·6 square feet per nominal horse-power. In the superheater the surface is 2,160 square feet, or 4·32 square feet per nominal horse-power, making the total heating surface in boiler and superheater 23·92 square feet per nominal horse-power. The area of heating surface in the boiler per square foot of grate is 24·5 square feet, and the area of superheating surface per square foot of grate is 5·4 square feet, making the total heating surface in boiler and superheater 29·9 square feet per square foot of grate. The total area of the condenser surface is ·66 of the total heating surface in the boiler, and ·54 of the total area of the heating surface of boiler and superheater taken together. These engines are very strong, and manifestly embody the results of the long experience of steam navigation which the West India Mail Company must now possess. The workmanship and materials are of the very first quality; and accurate adjustment and conscientious construction are manifested throughout.

Giffard's Injector.—This instrument, which feeds boilers by a jet of steam discharged into the feed pipe, acts on the principle that the particles of water which obtain a high velocity when they flow out as steam retain this velocity when reduced by

condensation to the form of water; and a jet of water of great velocity is capable of balancing a correspondingly high head, or a pressure greater than that which subsists within the boiler. The jet consequently penetrates the boiler, as we can easily understand any jet would do which has a greater velocity than a similar jet escaping from the boiler. These injectors, though very generally employed in locomotives, are not much used for land or marine boilers; and in their present form they occasion much waste, as the steam by which they are actuated is drawn from the boiler, whereas it ought to be the steam, or a portion of it, which escapes to the condenser or the atmosphere. These injectors, like Bourdon's gauges, and other instruments employed in the steam-engine, are not made by engineers, but are a distinct manufacture; and the manufacturers, on being supplied with the necessary particulars, furnish the proper size of instrument in each particular case. The proper diameter of the narrowest part of the instrument to deliver into the boiler any given number of gallons per hour, may be found by dividing the number of gallons required to be delivered per hour by the square root of the pressure of the steam in atmospheres, and extracting the square root of the quotient, which, multiplied by the constant number ·0158, gives the diameter in inches at the smallest part. Contrariwise, if we have the size, and wish to find the delivery, we multiply the constant number 63·4 by the diameter in inches and square the product, which, multiplied by the square root of the pressure of the steam in atmospheres, gives the delivery in gallons per hour. These rules correspond very closely with the tables of the deliveries of different sizes published by the manufacturers, Messrs. Sharp, Stewart, and Co., of Manchester.

POWER REQUIRED TO PERFORM VARIOUS KINDS OF WORK.

The power required to obtain any given speed in a given steamer will be so fully discussed in the next chapter that the subject need not be further referred to here; and in my 'Catechism of the Steam-Engine' I have recapitulated the amount of power, or the size of engine, required to thrash and grind corn, spin cotton, work sugar and saw mills, press cotton, drive piles,

dredge earth, and blow furnaces. The subject, however, is so important that I shall here recapitulate other cases for the most part derived from experiments made with the dynamometer in France by General Morin,* whose researches on this subject have been highly interesting, and have been conducted with much care and ability.

Comparative efficiency of different machines for raising water. —Of the different pumps experimented upon by General Morin, the result of eight experiments made with pumps draining mines showed that the effect utilised was 66 per cent. of the power expended. But in these cases there was considerable loss from leakage from the pipes. At the salt works of Dreuze the useful effect was 52·3 per cent. of the power expended. In fire-engine pumps employed to deliver the water pumped at a height of from 12 to 20 feet, the proportion of the water delivered to the capacity of the pump was, in the pumps of the following makers —Merryweather, Tylor, Perry, Carl-Metz, Letestu, Flaud, and Perrin, respectively, as follows:—·920, ·887, ·910, ·974, ·910, ·920, and ·900; while the percentage of useful effect relatively with the power expended was 39·7, 39·1, 30·2, 28·7, 27·1, 19·4, and 15·5, respectively. With a higher pressure, the efficiency of the whole of the pumps increased; and when employed in throwing water with a spout-pipe the delivery of water relatively with the effective capacity, or space described by the piston, was, when the names are arranged, as follows:—Carl-Metz, Merryweather, Tylor, Letestu, Perry, Flaud, Perrin, and Lamoine, respectively, ·950, ·810, ·565, ·870, ·910, ·912, ·950, and ·900; while the proportion of useful effect, or percentage of work done relatively with the power expended, was 80, 57·3, 54·5, 45·2, 37·8, 33·4, 28·8, and 17·5, in the respective cases. In the membrane pump of M. Brûle the efficacy was found to be 40 to 45 per cent. of the power expended. In the water-works pumps of Ivry, constructed by Cavé, the efficiency was found to be 53 per cent. of the power expended; and in the water-works of St. Ouen, by the same maker, 76 per cent. It is desirable that the buckets of the pumps of water-works should move slowly, otherwise the

* *Aide Mémoire* by General Morin, 5th edition, 1864.

water will go off with considerable velocity, involving a corresponding loss of power. The area through the valves should be half the area of the pump, and the area of the suction and forcing pipes ought to be equal to three-fourths of the area of the body of the pump. Waste spaces should be avoided. The loss of water through the valves before they shut is, in good pumps, about 10 per cent.

In a chain-pump the efficiency was found to be 38 per cent., but in many chain-pumps the efficiency is much more than this. The efficiency of the Persian wheel was found to increase very much with the height to which the water was raised. For heights of 1 yard it was 48 per cent., for 2 yards 57, for 3 yards 63, for 4 yards 66, and for 6 yards and upwards 70 per cent. of the power consumed. For a wheel of pots the efficiency is 60 per cent.; Archimedes screw, 65 per cent.; scoop wheel with flat boards moving in a circular channel, 70 per cent.; improved bucket wheel, 82 per cent., and tympan-wheel, or, as it is sometimes called, Wirtz's Zurich machine, 88 per cent. This machine should dip at least a foot into the water to give the best results. In the belt-pump the efficiency was found to be 43 per cent.; in Appold's centrifugal pump, 65 per cent.; in the centrifugal pump, with inclined vanes, 42 per cent., and with radial vanes, 24 per cent. In Gwynn's pump the efficiency was 30 per cent.

In the Archimedes screw the diameter is usually one-twelfth of the length, and the diameter of the newel or central drum should be one-third of the diameter of the screw. It ought to have at least three convolutions, and the line traced by the screw on the enveloping cylinder should have an angle of 67° to 70° with the axis. The axis itself should make an angle of from 30° to 45° with the horizon. There is a sensible advantage obtained from working hand-pumps by a crank instead of a lever.

Old French Flour Mill at Senelle.—Diameter of millstones, 70 inches; number of revolutions per minute, 70; quantity of corn ground and sifted per hour, 260·7 lbs.; power consumed, 3·34 horses. The power is in all cases the power actually exerted, as ascertained by the dynamometer.

English Flour Mill near Metz.—Diameter of millstones, 51·18

inches; number of revolutions per minute, 110; weight of millstones, 1 ton; corn ground per hour by each pair, 220 lbs.; with two pairs of millstones acting, one bolting machine and one winnowing machine, the power consumed was $8\frac{1}{2}$ horse-power.

English Flour Mill near Verdun.—Diameter of millstones, 51·18 inches; number of revolutions per minute, 110; quantity of corn ground per hour by each pair, or by each revolving millstone, 220 lbs.; with two stones revolving the power consumed was 5·64 horses. The power consumed by one winnowing machine and two bolting machines, with brushes sifting 1,650 lbs. of flour per hour, was $6\frac{1}{2}$ horses. In another mill the number of turns of the millstone was 486 per minute, the quantity of corn ground by each horse-power was 120 lbs., and the quantity of corn ground per hour was 110 lbs. of which 72·7 per cent. was flour, 7·8 per cent. was meal, and 19·5 per cent. was bran. In a portable flour-mill, with machinery for cleaning and sifting, the total weight was 1,000 lbs.

Barley Mill.—Number of revolutions of the millstone per minute, 246; barley ground per hour, 143·68 lbs.; motive force in horses, 3·11; barley ground per hour by each horse-power, 48·2 lbs. The products were, of first and second quality of barley flour, 60·12 per cent., of meal and bran, 30·25 per cent., and of bran and waste, 9·63 per cent.

Rye Mill.—Number of revolutions of the millstone per minute, 448; rye ground per hour, 92·114 lbs.; power expended, 2·86 horses; temperature of flour, 60·8° Fahr.; products 64·9 per cent. of flour, 9·1 per cent. of meal, and 26 per cent. of bran. In another rye mill the revolutions of the millstones per minute were 232; rye ground per hour, 180 lbs. by 2·19 horse-power, and the rye ground per hour by each horse-power was 82·21 lbs. The products were 72·5 per cent. of flour; 17·5 per cent. of meal and fine bran, and 10 per cent. of bran and waste.

Maize Mill.—Number of revolutions of the millstone per minute, 246; maize ground per hour, 73·96 lbs.; motive force in horses, 2·69; maize ground per hour by each horse-power, 27·5 lbs. Products: first and second quality of flour, 61·1 per cent.; meal and fine bran, 30·2 per cent.; bran and waste, 4·7 per cent.

Vermicelli Manufactory.—External diameter of edge runners, 66·93 inches; internal diameter of edge runners, 62·99 inches; number of revolutions of the arbour of the mill per minute, 4; pounds of paste prepared per hour, 77 lbs.; power expended, 2·95 horse-power.

Bean Mill.—Number of revolutions of the millstone per minute, 496; power expended per hour, 1·76 horse.

Oil Mill.—Weight of edge runners, 6,600 lbs.; number of turns of the vertical spindle per minute, 6; weight of seed introduced every ten minutes, 55 lbs.; weight of seed crushed daily, 3,300 lbs.; product in oil in 12 hours, 1,320 lbs.; power expended, 2·72 horses.

Saw Mill.—Weight of the saw frame, 842·6 lbs. When cutting dry oak 8.73 inches thick, with 1 blade in operation, the reciprocations or strokes of the saw were, 88 per minute, the surface cut, ·525 square foot, and the power expended 3·3 horses. When cutting the same wood with 4 blades in operation, the number of strokes of the saw per minute was 79; the surface cut by each per minute ·433 square foot, or 1·73 square foot per minute for the 4; and the power expended was 3·70 horses, which is equivalent to 28 square feet cut per hour by 1 horse-power. When cutting four-year seasoned oak, 12·4 inches thick, with 4 blades, making 90 strokes per minute, the surface cut by each blade was ·35 square foot, and the surface cut by the 4 blades, 1·41 square foot. When the saw was run along the middle of a cylindrical log of beech one-year cut, 23·6 inches diameter, the number of strokes of the saw per minute was 88; the surface cut per minute, ·968 square foot; and the power expended, 3 horses. In these experiments the breadth of the saw cut was ·157 inch, and the experiments show that it does not take more power to drive a frame with one saw than to drive a frame with four, the greatest part of the power indeed being consumed in giving motion to the frame. The common estimate in modern saw mills, when the frame is filled with saws, is, that to cut 45 superficial feet of pine, or 34 of oak per hour, requires 1 indicated horse-power. The crank, which moves the frame up and down, and which is usually placed in a pit under the machine, should have balance

weights applied to it, the momentum of which weights, when the saw is in action, will be equal to that of the reciprocating frame. In some cases the weight of the saw frame is borne by a vacuum cylinder, and with a 20-inch stroke it makes 120 strokes per minute.

Circular Saw.—Diameter of saw, 27·5 inches; thickness of oak cut, 8·73 inches; number of revolutions per minute, 266; surface cut per minute, 1·93 square foot; power consumed 3·55 horses. When set to cut planks of dry fir, 10·62 inches broad, and one inch thick, the number of revolutions made by the saw per minute was 244; surface cut per minute, 7·67 square feet; and the power expended, 7·35 horses. These results show that in sawing the smaller class of timber one circular saw will do at least as much work as four reciprocating saws, with the same expenditure of power. The surface cut is, in all these cases, understood to be the height multiplied by the length, and not the sum of the two faces separated by the saw. The speed of the circular saw here given is not half as great as that now commonly employed. Circular saws now work with a velocity at the periphery of 6,000 to 7,000 feet per minute, and band saws with a velocity of 2,500 feet per minute, and it is generally reckoned that 75 superficial feet of pine, or 58 of oak, will be sawn per hour by a circular saw for each indicated horse-power expended. Planing machine cutters move with a velocity at the cutting edge of 4,000 to 6,000 feet per minute, and the planed surface travels forward $\frac{1}{20}$th of an inch for each cut.

Reciprocating Veneer Saw.—Length of stroke of saw, 47·24 inches; thickness of the blade, ·01299 inch; breadth of saw cut, ·02562 inch; length of teeth for mahogany and other valuable woods, ·196 inch; pitch of the teeth, ·3939 inch; distance advanced by the wood each stroke, ·0196 to ·03937 inch; number of strokes of the saw per minute, 180; surface cut per hour counting both faces, 107·64 square feet; power expended 0·66 horses.

Sawing Machine for Stones.—Soft sandstone: breadth of saw-cut, ¼ inch; time employed to saw 10 square feet, 5 minutes 25 seconds; power expended 4·54 horses. Hard sandstone: breadth

of saw-cut, $\frac{1}{4}$ inch; time employed to cut 10 square feet, 1 hour 37 minutes; power expended 2 horses.

Sugar Mill for Canes.—A three-cylinder mill, with rollers $5\frac{1}{2}$ feet long, 30 inches diameter, and making $2\frac{1}{4}$ turns a minute, driven by an engine of 25 to 30 horse-power, will express the juice out of 130 tons of canes in 12 to 15 hours. An acre of land produces from 10 to 20 tons of canes, according to the age and locality of the canes. The juice stands at 8° to 12° of the saccharometer, according to the locality. The product in sugar varies from 6 to 10 per cent. of the weight of the canes, according to the locality and mode of manufacture. Well-constructed mills give in juice from 60 to 70 per cent. of the weight of the canes, and one main condition of efficiency is, that the rollers shall travel slowly, as with too great a speed the juice has not time to separate itself from the woody refuse of the cane, and much of it is reabsorbed. To defecate 330 gallons of juice 6 boiling-pans, or caldrons, are required, 4 scum presses, and 10 filters; and to granulate the sugar 2 vacuum pans, $6\frac{1}{2}$ feet diameter, are required, with 2 condensers, and it is better also to have 2 air-pumps. The steam for boiling the liquor in the vacuum pans is generated in three cylindrical boilers, each 6 feet in diameter. To whiten the sugar there are 10 centrifugal machines, driven by a 12-horse engine, which also drives a pair of crushing-rollers. The sugar in the centrifugal machines is wetted with syrup, which is driven off at the circumference of the revolving cylinders of wire gauze, carrying with it most of the colouring matter of the sugar, which to a great extent adheres to the outside of the crystals, instead of being incorporated in them, and may consequently be washed off. When the sugar is thus cleansed it is again dissolved, and the syrup is passed through deep filters of animal charcoal. Provision must be made to wash the charcoal, both by steam and by water, and two furnaces, to re-burn the animal charcoal, will be required.

The action of animal charcoal in bleaching sugar is not well understood. But it appears to be due to certain metallic bases in the bones, which by burning are brought to or towards the metallic state, from the superior affinity of the carbon present

for the oxygen in the base at the high temperature at which the re-burning takes place. When, however, the charcoal is mixed with the syrup, the metallic base endeavours to recover the oxygen it has lost, by decomposing the water, leaving thereby a certain quantity of hydrogen in the nascent state; and this hydrogen appears to dissolve the small particles of carbon in the sugar which detract from its whiteness, and to form therewith a colourless compound. When the metallic basis has recovered all its lost oxygen the charcoal ceases to act, and has to be re-burned; and, after numerous re-burnings, the charcoal appears to be all burned out of the bones, when re-burning ceases to be of service. But their efficacy might be restored by mingling portions of wood charcoal. The use of charcoal in sugar refining is not merely a source of expense in itself, but it occasions a loss of sugar, as, when the mass of charcoal becomes effete, it is left saturated with syrup, and the water with which it is washed has to be boiled down, to recover the sugar as far as possible. I consequently proposed several years ago a method of revivifying the charcoal without removing it from the filter. But the method has not yet been practically adopted.

The begass, or woody refuse of the cane, is usually employed to generate the steam in the boilers. But it is generally necessary to use coal besides.

Fans for blowing Air.—The indicated power required to work a fan may be ascertained by multiplying the square of the velocity of the tips in feet per second by the collective areas of the escape orifices in square inches, and by the pressure of the blast in pounds per square inch, and finally dividing the product by the constant number 62,500, which gives the indicated power required. The pressure in pounds per square inch may be determined by dividing the square of the velocity of the tips in feet per second by the constant number 97,300.

Cotton-spinning Mill.—Number of spindles, 26,000; power consumed, 110 horses; Nos. of yarn spun, 30 to 40; spindles with preparation driven by each horse, 237. It is reckoned that each machine requires 1 horse-power.

Another example of a Cotton Mill.—Number of spindles,

14,508; power required to drive them, 50·5 horses; Nos. of yarn spun, 30 to 40; spindles and preparation driven by each horse-power, 287.

Another example of a Cotton Mill.—Number of spindles, 10,476; Nos. of yarn spun, 30 to 40; spindles and preparation driven by each horse-power, 235.

Details of power required by each Machine in Cotton Mills.—One beater making 1,100 revolutions per minute, with ventilating fan making half this number of revolutions, cleaning 132 lbs. of cotton per hour, requires 2·916 horse-power. One beater making 1,200 revolutions per minute, with combing drum 1·23 feet diameter and 2·8 feet long, making 800 revolutions per minute, and preparing 132 lbs. of cotton per hour, requires 800 revolutions per minute and 1·767 horse-power. Power required to work the fluted cylinders and endless web of this machine, ·312 horse. Twelve double-casing cylinders, with eccentrics, requiring 2·697 horses, including the transmission of the motion, or per machine, ·225 horse. Transmitting the motion for 26 carding-machines requires 1·82 horse-power. One simple card, consisting of a drum 39·37 inches diameter and 19·68 inches long, making 130 revolutions per minute, and carding 2 lbs. of cotton per hour, requires ·066 horse-power, without reckoning the power consumed in communicating the motion. The same card working empty requires ·044 horse-power. One double-carding machine carding 4·18 lbs. of cotton per hour, requires ·207 horse-power. A drawing-frame drawing 119 lbs. per hour requires 1·835 horse-power. A roving-frame, with 60 spindles, with cards, making 525 revolutions per minute, and producing 42 lbs. of No. 7 rovings per hour, requires ·760 horse-power. One frame with screw-gearing, having 60 spindles, making 550 revolutions per minute, and producing 42 lbs. of No. 7 per hour, requires ·486 horse-power. Two frames with screw-gearing, each containing 96 spindles, making in one case 510 revolutions and the other 500 revolutions per minute, producing 28·6 lbs. of No. 2·75 to 3 per hour, requires 1·482 horse-power. Two frames with screw-gearing, one containing 78 spindles making 344 revolutions per minute, and the other 60 spindles making 260

revolutions per minute, and producing 57·2 lbs. of No. 8 per hour, requiring ·797 horse-power. One spinning-frame, with cards, having 240 spindles, making 5,000 revolutions per minute, and producing 1·65 lb. of yarn of No. 38 to No. 40 per hour, requires ·686 horse-power, and in another experiment ·648 horse-power. Three spinning-frames for weft, having each 360 spindles, making 4,840 revolutions per minute, and producing 8 lbs. of No. 30 to No. 40 yarn per hour, require 2·103 horse-power. One retwisting machine, with 120 spindles, making 3,000 revolutions per minute, requires 1·19 horse-power. One dressing machine for calico 35½ inches wide, with ventilator: speed of the principal arbor, 176 revolutions per minute; speed of the brushes, 45 strokes per minute; power required, ·735 horse. The same machine, with the ventilator not going, requires ·206 horse-power.

Power-loom Weaving.—To drive one power-loom weaving calico 35½ inches wide, and 82 to 90 picks per inch, making 105 strokes per minute, requires, taking an average of four experiments, ·1195 horse-power.

Another example of Power-loom Weaving.—Number of looms weaving calico driven by water-wheel, 260; dressing machines, 15; winding machines, 5; warping machines, 8; small pumps, 6; yards of calico produced per month, 283,392; power required to drive the mill, 25·6 horses; number of looms, with accessories, moved by 1 horse, 12.

Another example of Power-loom Weaving.—Total number of looms, 60; dressing machines, 5; warping machines, 3; winding machines, 2; monthly production of cotton cloth called 'Normandy linen,' 47½ inches wide, 360 pieces, each 396 yards long; power consumed, 8 horses; looms with their accessories moved by each horse-power, 7·8.

Wool-spinning Mill.—Machines driven: simple cards, 29; double cards, 2; scribbling beater, 1; mules of 240 spindles, 8; mules of 200 spindles, 4; lathes, 3; power consumed, 9·75 horses. Also in another experiment with 9 simple and 3 double-carding machines, 2 beaters, and 2 scribbling machines, the power consumed in driving was 3·5 horses.

Another example of a Wool-spinning Mill.—A wheel exerting 10 horse-power drives 6 mules of 240 spindles, 6 of 180, 2 of 192, 2 of 120, and 5 of 100, making in all 3,644 spindles; also 32 carding and 2 scribbling machines. Another wheel, also exerting 10 horse-power, drives 8 mules of 240 spindles, 4 of 120, and 7 of 180, making in all 3,660 spindles; also 31 carding and 2 scribbling machines, and 2 beaters. The spindles, numbering in all 7,304, make 5,000 revolutions per minute, and the cards 88 to 89, requiring a horse-power for 365 spindles. Product per day of 12 hours, 1,100 lbs. of yarn from No. 12 to No. 13.

Details of Power consumed in spinning Wool.—One winding machine with 16 bobbins, without counting the power expended in the transmission of the motion, requires to drive it ·259 horse; 3 winding machines with 64 bobbins in all, with power lost by transmission, 1·427 horse; one mule spinning No. 6 warp yarn, with 220 spindles, making 3,650 revolutions per minute, ·259 horse. One mule called 'Box-organ,' spinning No. 50 warp yarn with 300 spindles, making 3,200 revolutions per minute, requires 1·273 horse-power.

Mill for spinning Wool and weaving Merinos.—Nineteen machines to prepare the combed wool, having together 350 rollers; 16 mules with 3,400 spindles; one winding machine of 60 rollers to prepare the warp; 2 warping machines; 2 self-acting feeders; 100 power-looms; 2 lathes for wood and iron, and 1 pump, require in all 30 horse-power. Produce: 13,600 cops of woollen thread, of 45 cops to the lb., each measuring 792 yards. The looms make 115 revolutions per minute, and produce daily 4 pieces of double-width merino of 68 yards each, and 4 pieces of simple merino of 1·2 to 1·4 yard broad, and each 88 yards long.

Fulling Mill.—In fulling the cloths called 'Beauchamps,' each piece being 220 yards long, and ·66 yard wide, and weighing from 121 to 127 lbs., the fuller making 100 to 120 strokes per minute, each piece requires 2 hours to full it, and the expenditure of 2 horse-power during that time.

Flax Manufacture.—A machine for retting the flax, having

15 pairs of rollers with triangular grooves, requires 3·376 horse-power, and the heckles ·057 horse-power.

One fly breaking-card 12·59 inches diameter and 47·24 inches long, making 915 revolutions per minute, with a drum of 42·12 inches diameter, and 47·24 inches long, making 76 revolutions per minute; 4 distributing rollers, having a diameter of 4 inches and a length of 47·24 inches, making 380 revolutions per minute; 8 travellers, 5 inches diameter and 47·24 inches long, making 10 turns per minute, and one combing cylinder 15 inches diameter and 47·24 inches long, making 6 revolutions per minute, require together 1·939 horse-power, and produce 17 lbs. of carded flax per hour.

One finishing carding cylinder, 40 inches diameter and 47·24 inches long, making 176 revolutions per minute; 5 distributing rollers, 4 inches diameter, making 23 revolutions per minute; 4 travellers, 5 inches diameter, making 7·3 revolutions per minute; 1 combing cylinder, 15 inches diameter, making 3·4 revolutions per minute, together require ·811 horse-power, and produce 8¼ lbs. of carded flax per hour.

One spinning-machine, containing 132 spindles, making 2,700 revolutions per minute, spinning yarns from No. 7 to No. 9, requires 1·24 horse-power, and produces 3¾ lbs. of yarn per hour.

One spinning-machine, having 168 spindles, making 2,700 revolutions per minute, and producing 3 lbs. of No. 18 to 24 yarn per hour, requires 1·96 horse-power.

Wet spinning of flax: one drawing-frame drawing a sliver for No. 20 yarn, requires ·493 horse; drawing-frame drawing sliver for No. 50 yarn, requires ·487 horse; drawing-frame drawing sliver for No. 70 yarn, requires ·495 horse.

Second drawing-frame, drawing two slivers for yarns Nos. 20 and 30, requires ·68 horse; second drawing-frame, drawing two slivers for yarns Nos. 30 and 40, requires ·544 horse; second drawing-frame, drawing one sliver for No. 60 yarn and one for No. 70, requires ·617 horse.

Third drawing-frame, drawing two slivers for yarns Nos. 30 tc 60, requires ·69 horse.

Roving-frame of 8 spindles, preparing the flax for yarn No.

20, requires ·608 horse; roving-frame of 8 spindles, preparing the flax for No. 30 yarn, requires ·486 horse; frame of 16 spindles, preparing the flax for No. 40 yarn, requires ·987 horse-power.

Paper Manufacture.—In some cases the pulp, or stuff of which paper is made, is obtained by beating the rags by stampers; but more generally it is produced by placing the rags between revolving cylinders stuck full of knives. When produced by stampers, the proportions of the apparatus are as follows: weight of stampers, 220 lbs.; distance of the centre of gravity from the axis of rotation, 4 feet; rise of the centre of gravity each stroke, 3½ inches; number of stampers, 16; number of lifts of each stamper per minute, 55; weight of rags pounded in 12 hours by each stamper, 33 lbs.; weight of stuff produced in 12 hours by each stamper, 122 lbs.; power consumed, 2·7 horses.

Chopping-cylinders, for preparing the pulp: number of cylinders working, 2; number of turns of cylinders per minute, 220; weight of rags chopped and purified in 12 hours, 528 lbs.; power consumed, 4·48 horses.

In another instance, 10 cylinders for preparing the pulp, making 200 revolutions per minute, 1 paper-making machine, cutting-machines, pump, and accessories, consumed 50-horse power. The machine made 13 yards of paper per minute, and the produce was 1 ton of printing paper per day of 24 hours.

In another instance, 28 pulping-cylinders, and 3 paper-making machines produced 2 to 3 tons of paper per day of 24 hours, and consumed 113 horse-power.

Printing Machinery.—Printing large numbers is now performed by cylindrical stereotype plates, revolving continuously; and the 'Times' and other newspapers of large circulation are thus printed. The impressions are taken from the types in *papier maché*, and in twenty minutes a large stereotype plate is ready to be worked from. The power required to drive this machine varies with the number of impressions required in the hour. For 5,000 impressions per hour, the power required is 3·75 horses; for 6,000 impressions, 4·77 horses; 7,000 impressions, 5·9 horses; 8,000 impressions, 7·03 horses; 9,000 impres-

sions, 8·75 horses; and 10,000 impressions, 10·35 horses. The paper should be supplied to such machines in a continuous web, with a cutter to cut off the sheets at the proper intervals, and a steam cylinder to dry and press them. But this has not yet been done. The machine could also be easily made to perforate the paper along the edges of the leaves, and to fold each paper up and put a printed and stamped paper envelope around it, so as to be ready at once to put into the post-office or to distribute by hand. The most expeditious mode of stereotyping would be to use steel types set on a cylinder, against which another cylinder of type-metal is pressed, and the paper would then be printed in the same manner as calico.

Glass Works.—Mill to grind red lead: to grind 3 tons, the vertical arbor requires to make for the first ton 20 revolutions per minute, for the second 25, and for the third 40, consuming 5·28 horse-power. Vertical millstones, to grind clay and broken crucibles; diameter of the granite stones or runners, 3·7 feet; thickness, 1·4 foot; weight, 1 ton; distance of edge runners from central spindle, 4 feet; number of turns of the arbor per minute, 7½; power consumed 1·92 horse. In the 12 hours 6 or 8 charges of about 300 lbs. each of old glass pots are ground, and about 3 tons of dry clay. Wheels for cutting the glass, 170; lathes for preparing the cutting wheels, 5; lathes for metal, 2; power consumed, 17·9 horses; wheels driven by each horse-power, 9·5.

Iron-Works.—The weekly yield of each smelting furnace in Wales is from 100 to 120 tons; pressure of blast, 2½ to 3 lbs. per square inch; temperature of the blast, 600° Fahr.; yield weekly of each refining-furnace, 80 to 100 tons; of each puddling-furnace, 18 tons; of each balling-furnace for bars, 30 tons; of each balling-furnace for rails, 80 tons; iron rolled weekly by puddle rolls, 300 tons; by rail rolls, 600 tons; power required to work each train of rail rolls, 250 horses; to work puddle rolls and squeezer, 80 horses; small bar train, 60 horses; pumping air into each blast-furnace, 60 horses; into each refining-furnace, 26 horses; rail saw, 12 horses.

Weaving by compressed air.—In common power-looms, the

shuttle is driven backward and forward by a lever which imitates the action of the arm in the hand-loom. But it has long been obvious to myself and others that it might be shot backward and forward like a ball out of a gun, by means of compressed air. This innovation has now been practically carried out. But the benefits derivable from the practice have been much exaggerated, and a much more comprehensive improvement than this is now required. Indeed, reciprocating looms of all kinds are faulty, as they make much noise, consume much power, do little work, and cannot be driven very fast; and the proper remedy lies in the adoption of a circular loom in which the cloth will be woven in a pipe, and in which many threads of weft will be fed in at the same time.

Circular Loom.—The obvious difficulty in a circular loom, is to drive the shuttle round continuously within the walls formed by the warp. One mode of driving proposed by me, is by magnets or other suitable form of electro-motive machine, which does not require contact; and the shuttle should be a circular ring, with many cops placed in it, so that many threads might be woven in at once. The desideratum, however, is to weave a vertical pipe with the bobbins of the weft in the centre of the circle; and this may be done by depositing the thread between metallic points, like circular heckles, which points will change their positions inward or outward at each time a thread is deposited. These points would conduct the threads of the warp.

CHAPTER VII.

STEAM NAVIGATION.

STEAM navigation embraces two main topics of enquiry:—the first, what the configuration of a vessel shall be to pass through the water at any desired speed with the least resistance; and the second, what shall be the construction of machinery that shall generate and utilise the propelling power with the greatest efficiency. The second topic has, in most of its details, been already discussed in the preceding pages; and it will now be proper to offer some remarks on the remaining portion of the subject.

The resistance of vessels passing through the water is made up of two parts:—the one, which is called the bow and stern resistance, being caused partly by the hydrostatic pressure forcing back the vessel, arising from the difference of level between the bow and stern, and partly by the power consumed in blunt bows in giving a direct impulse to the water; while the other part of the resistance, and the most important part, is that due to the friction of the water on the sides and bottom of the ship. The bow and stern resistance may be reduced to any desired extent by making the ends sharper. But the friction of the bottom cannot be got rid of, or be materially reduced, by any means yet discovered.

When a vessel is propelled through water, the water at the bow has to be moved aside to enable the vessel to pass; and the velocity with which the water is moved sideways will depend upon the angle of the bow and the speed of the vessel. When

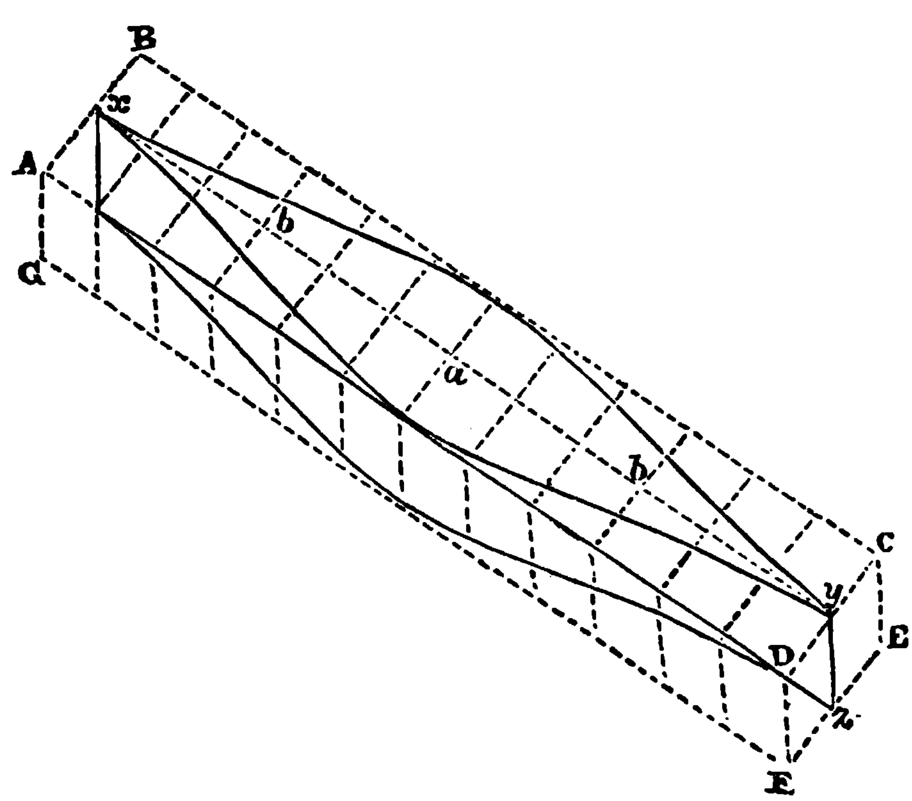
these elements are known it is easy to tell with what velocity the water will be moved aside; and when we know the velocity with which the water is moved, we can easily tell the power consumed in moving it, which power will, in fact, be the weight of the water moved per minute multiplied by the height from which a body must fall by gravity to acquire the same velocity. But as nearly all the power thus consumed in moving aside the water at the bow of a vessel is afterwards recovered at the stern by the closing in of the water upon the run, it is needless to go into this investigation further than to determine what amount of power is wasted by the operation, or in other words, what amount of power is expended that is not afterwards recovered.

If the vessel to be propelled is of a proper form, each particle of water will be moved sideways by the bow, in the same manner as the ball of a pendulum is moved sideways by gravity, so as to enable the vessel to pass; and when the broadest part of the vessel has passed through the channel thus created, each particle of water will swing backward again until it comes to rest at the stern. There will be no waste of power in this operation, except that incident to the friction of the moving water; just as in the swinging of a pendulum there is no expenditure of power beyond that which is necessary to overcome the friction of the air upon the moving ball. But as the movement of the vessel, however well she may be formed, will *somewhat* raise the water at the bow, and *somewhat* depress the water at the stern, there will be a certain hydrostatic pressure required to be continually overcome as the vessel advances in her course, which opposition constitutes the bow and stern resistance; and this, with the friction of the bottom, make up the whole resistance of the ship. Before, however, proceeding to investigate the amount of this hydrostatic resistance, it will be proper to show how accidental sources of loss may be eliminated from the problem by the introduction of that particular form of vessel which will make this resistance a minimum; and I will therefore first proceed to indicate in what way such form of vessel may be obtained.

If we take a short log of wood, such as is shown by the

dotted lines A B C D E F G, in the annexed figure (fig. 41), and if we proceed to enquire in what way we shall mould this log into a model which shall offer the least possible hydrostatic resistance in being drawn through the water, we have the following considerations to guide us in arriving at the desired knowledge: We shall, for the sake of simplification, suppose that the cross section of the completed model is to be rectangular, or in other words, that the model is to have vertical sides and a flat bottom; for although this is not the best form of cross section, as I shall

Fig. 41



afterwards show, the supposition of its adoption in this case will simplify the required explanations.

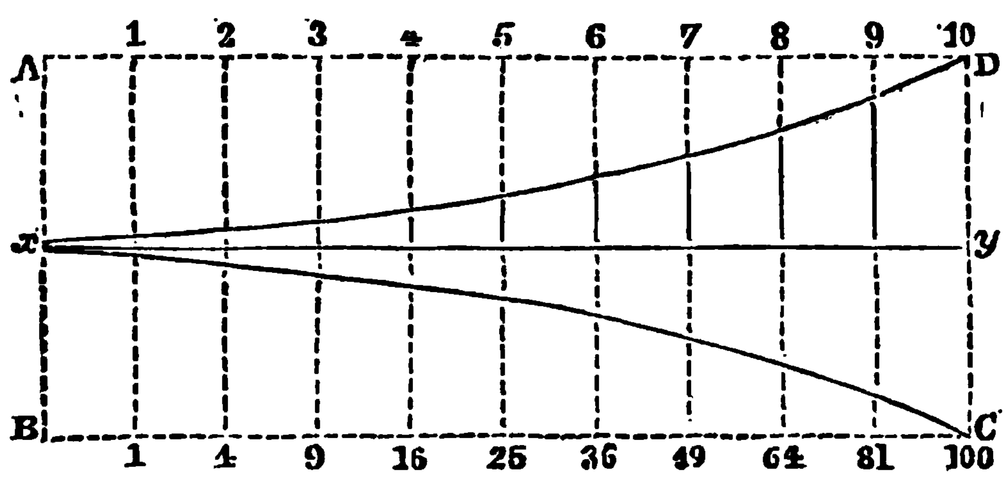
We first draw a centre line x y longitudinally along the top of the model from end to end, and continue the line vertically downward at the ends as at y z, which vertical lines will form the stem and stern post of the model. At right angles to the first line, and at the middle of the length of the model, we draw the line a, which answers to the midship frame; and midway between a and the ends we draw other two lines b b. We may afterwards draw any convenient number of equi-distant cross-

lines, or ordinates, as they are termed, that we find to be convenient. Now as, by the conditions of the problem, the particles of water have to swing sideways like a pendulum, in order that the resistance may be a minimum, the particle which encounters the stem at *x* must be moved sideways very slowly at first, like a heavy body moved by gravity, but gradually accelerating until it arrives at *b*, midway between *x* and *a*, where its velocity will be greatest; and this point answers to the position of the ball of the pendulum when it has reached the bottom of the arc, and has consequently attained its greatest velocity. Thereafter the motion, which before was continually accelerated, must be now continually retarded, as it is in any pendulum that is ascending the arc in which it beats, or in any ball which is projected upwards into the air against the force of gravity. When the particle of water has attained the position on the side of the model which is opposite to the midship frame *a*, it will have come to rest, this being the point answering to the position of the pendulum at the top of its arc, and when just about to make the return beat. Thereafter the particle which was before moved *outwards*, will now move *inward* with a velocity, slow at first, but continually accelerating, until it attains the position on the side of the model which is opposite to the frame *b*, when the velocity again begins to diminish; and the particle finally comes to rest at the stern. A particle of water that is moved in this way will be moved with the minimum of resistance; for since it retains none of the motion in it that has been imparted, but surrenders the whole gradually without impact or percussion, by the time it has come finally to rest, there can be no power consumed in moving it except that due to friction only. Wherever the water is not moved in this manner it will either retain some of the motion, which implies a corresponding waste of power, or heat will be generated by impact, which also involves a corresponding waste of power. That the water may be moved in the same manner as a pendulum is moved, is obviously possible, by giving the proper configuration to the sides of the model; and in fact, if an endless sheet of paper be made to travel vertically behind a pendulum, with a pencil or paint

brush stuck in the ball, the proper form for the side of the model will be marked upon the paper. The curve, however, which is a parabolic one, may be described geometrically as follows:—

If we compute the height through which a heavy body falls by gravity in any given number of seconds, we shall find that in the first quarter of a second it will have fallen through $1\frac{1}{192}$ foot, in the second quarter of a second $3\frac{4}{48}$ feet, in the third $9\frac{3}{4}$, in the fourth $16\frac{1}{12}$, in the fifth $25\frac{25}{192}$, in the sixth $36\frac{3}{16}$, in the seventh $49\frac{49}{192}$, in the eighth $64\frac{1}{3}$, in the ninth $81\frac{27}{64}$, and in the tenth quarter of a second $100\frac{25}{48}$. The height fallen through, therefore, or the space described by a falling body in a given time,

Fig. 42.



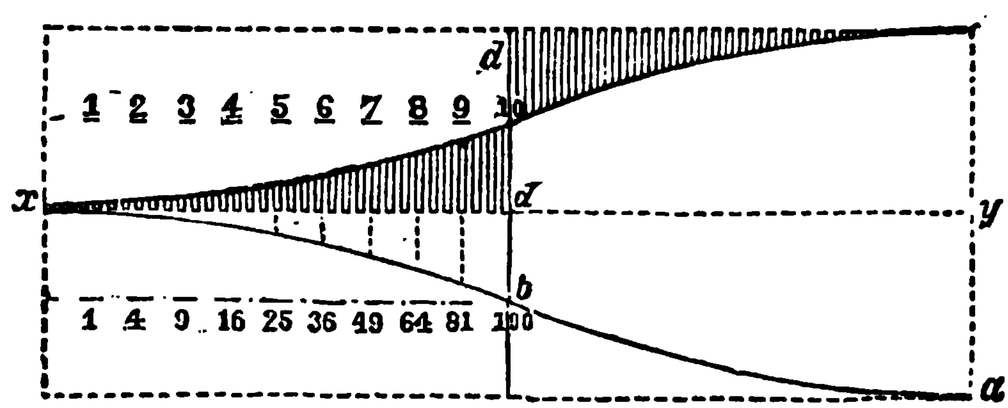
varies as the square of the time of falling; and any body which is to be moved in the same manner as a falling body is moved by gravity, must have the motion imparted to it gradually at the same rate of progression. If, then, we draw a line, $x\ y$ in fig. 42, and which line we may suppose to be the vertical plane of the keel, then if we form the parallelogram A B C D, with the line $x\ y$ passing through the middle of it, and make this parallelogram one-fourth of the length of the vessel and half the breadth, and divide the line $x\ y$ into any number of convenient parts or ordinates, say 10, by the vertical co-ordinates numbered from 1 to 10, then if we cause the lengths of these successive and equidistant co-ordinates, measuring from the line $x\ y$, to follow the same law of increase that answers to the height through which a body

falls by gravity in successive and equal portions of time, a line traced through the ends of these different lines will give the right form for the side of a vessel to have, in order that it may move the water sideways, in the same manner, or according to the same law, by which a heavy body falls vertically by gravity; and consequently such line is the proper water-line of a ship formed under the conditions supposed, in order that it may have a minimum resistance. The heights of the several vertical ordinates—which are drawn on a different scale from the lengths, marked on the line $x\ y$, are—1, 4, 9, 16, 25, 36, 49, 64, 81, and 100, which, it will be seen, are the squares of the horizontal ordinates 1, 2, 3, 4, 5, 6, 7, 8, 9, and 10; and the scale by which these vertical ordinates are measured is formed by dividing the distance y D, which represents one-fourth of the breadth of the vessel, into 100 equal parts. The ordinate y D is therefore equal to 100 of those parts, the next ordinate to 81 of them, the next to 64 of them, and so on, until the height vanishes at x altogether. We might have divided the line $x\ y$ into nine equal parts, or into 8, or 7, or any other convenient number. In such case the vertical line y D would have to be divided into 81 equal parts to obtain the vertical scale, or into 64, or into 49, according as 9, 8, or 7 had been the number selected; but the number of parts into which y D is divided must always be equal to the square of the number of ordinates, or the square of the number of parts into which the horizontal line is divided. As it is difficult to measure the hundredth part of such a small length as y D, we may call the number of parts 10 instead of 100, in which case the length of the next ordinate will be 8·1, of the next 6·4, of the next 4·9, and so on—the whole of the squares being divided by 10; which proceeding will in no way affect the result, as, in point of fact, the difference is only much the same thing as if we measured in inches instead of in feet.

In the figure, $x\ y$ is five times longer than y D, and $x\ y$ represents one-fourth of the length of the vessel, and y D one-fourth of the breadth. The curved line x D represents the proper form of the water-line of the front half of the fore body in the case of a vessel of these proportions, and with a rectangular cross-sec-

tion. The water-line of the second half of the fore body is formed by repeating the same curve, but inverted and reversed. this will be made obvious by an inspection of fig. 43, where the first half of the fore body is repeated on a smaller scale; and the second portion of the fore body is added thereto, thus continuing the water-line to the midship frame *a a*. Here the rectangle enclosing the water-line of the first half of the vessel is shown in dotted lines, as is also the rectangle enclosing the water-line of the first half of the fore body; and it is plain that the shaded space *a d* is the exact duplicate of the shaded space *x d*; so that if the figure *x d* has been obtained, we may obtain the figure *d a* by cutting out of the paper the figure *x d*, inverting it and re-

Fig. 43.



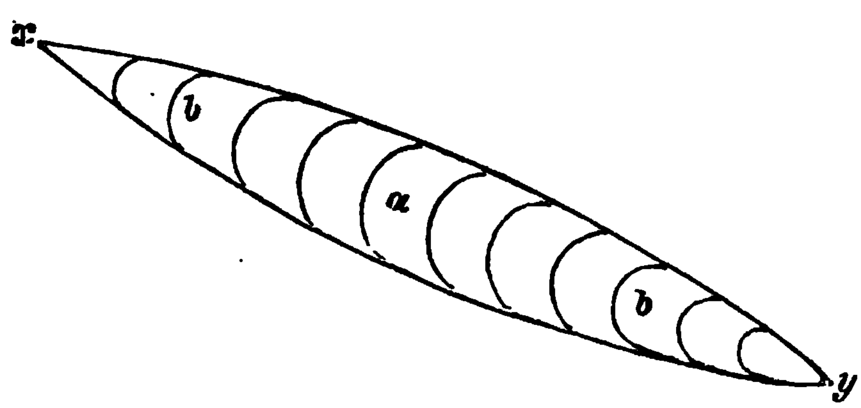
versing it, so that the line *x d* shall coincide with the line *d a*, and the point *x* with the point *a*; or the figure *d a* may be constructed by co-ordinates in exactly the same manner as the figure *x d*. If the vertical sides of the vessel be formed with the curve shown by the curve line *x a*, then it will follow that a particle of water encountering the stem at *x*, will be moved aside slowly at first, and with a rate continually increasing, like a body falling by gravity, until the frame *b* lying midway between the stem and the midship frame is reached, at which point the water will be moving sideways with its greatest velocity. Thereafter the vessel will not move the water, but merely follow up the motion already given to it, and as the water, when no longer impelled sideways by the vessel, will move slower and slower, and gradually come to rest, so the vessel will have less and less following up to do, until at the midship frame *a a*, the side motion of the water ceases

altogether. Thereafter the water begins to move in the opposite direction to fill up the vacuity at the stern left by the progress of the vessel. The water gravitates into the run slowly at first, and the velocity increases until the point midway between the midship frame and the stern is attained, at which point the velocity is greatest; and from thence the velocity of the water, flowing inward, continually diminishes, until it comes to rest at the stern.

A rectangular box, such as that shown by the dotted lines A B C D E F G, fig. 41, into which the model exactly fits, is called its *circumscribing parallelepiped*; and it will be at once apparent, on a reference to fig. 41, that the bulk or capacity of the model is exactly one-half of its circumscribing parallelepiped. The rectangle *x d* is equal to the rectangle *d y*, and the shaded space *x d* being equal to the shaded space *d a*, the area included between the water-line and the vertical plane of the keel, namely, the area *x y a*, is clearly equal to the rectangle *d d y a*. But that rectangle, and the rectangle standing beneath it, are equal to the whole area within the water-line of the fore body, and two similar rectangles are equal to the area within the water-line of the after body. As these four rectangles form just half the area of the circumscribing parallelogram, the total area within the water-line is equal to half the area of the circumscribing parallelogram. But the area multiplied by the depth gives the capacity, and as the depth of the model is the same as that of the box, or circumscribing parallelepiped, while the area of the circumscribing parallelogram is twice that of the area of the figure within the water line, it follows that the volume or bulk of the model is just one-half of the circumscribing parallelepiped. This forms a measure of sharpness which in no case it is useful to exceed, if the section be made rectangular, or, in other words, if the vessel be built with a flat bottom and vertical sides. But if the vessel be built with a rising floor the effect is equivalent to a reduction of the breadth, and the circumscribing parallelepiped would, in such case, be that answering to the equivalent breadth. Whatever be the form of the cross-section, however, the sectional area at each successive frame should be equal to that of a vessel with

a rectangular section having water-lines formed on the principle which has been here explained. There are other curves, no doubt, which equally with that described by a pendulum fulfil the indication of beginning and terminating the motion gradually so as to involve no loss of power, and any of these curves are eligible as the water-line of a ship. But the pendulum curve is the most readily understood, and the most conveniently applicable to practical uses, while it perfectly fulfis the required indications. If in any intended vessel we have a given form of cross-section, and a given ratio of length to breadth, we can easily determine the proper water-lines of such a vessel by taking the case of a hypothetical vessel of rectangular cross-section having

Fig. 44.



the same area of midship-section, and by forming the water-lines for this hypothetical vessel on the principle already explained. The area of cross-section at each successive frame of this hypothetical vessel, will be the proper area at each successive frame of the intended vessel. It is obvious that, according to the principle here unfolded, the form of water-line must vary with every alteration of the cross-section; and in some cases, although the same rate of displacement as that already indicated is preserved, the water-lines will cease to be hollow at any part. Thus the cylindrical solid, with pointed ends, shown in fig. 44, is virtually of the same form as that represented in fig. 41, since the area of each successive circular cross-section is the same as those of each rectangular cross-section in fig. 41. This solid is

supposed to be wholly immersed. It has, in some cases, been made an objection to the use of hollow water-lines for ships, that in the case of fishes, however fast swimming, no hollow lines are to be found in them. Fig. 44, however, which resembles the form of a fish, shows that fishes form no exception to the application of the law of progressive parabolic displacement already explained; and if a fast-swimming fish be cut across at equal distances, and the areas of these sections be computed and laid down with a rectangular outline of uniform depth, it will be found that the skin or covering placed over the ends of these sections or frames will assume the very form which has been delineated in the foregoing figures as that proper for a solid intended to pass through the water with the least amount of hydrostatic resistance.

In fig. 44, *x y* is the axis of the pointed cylindrical solid; and *a* is the circle or section which answers to the midship frame, and *b b* the sections answering to the frames lying midway between the centre frame and the ends. The other lines corresponding to those marked on the model shown in fig. 41, and the area of each successive circle is equal to the area of each successive rectangular section of the model delineated in fig. 41. The water consequently will be displaced *at the same rate* by one solid as by the other. For actual vessels, with rounded bilges and more or less rise of floor, the form of the water-lines will be neither that shown in fig. 41 nor fig. 44, but will be something intermediate between the two; but such, nevertheless, that the transverse sectional area of that part of the vessel beneath the water-line shall at each successive frame vary in the ratio pointed out.

As water is practically incompressible by any force which a ship can bring to bear upon it, the water which a ship displaces must find some outlet to escape; and it will escape in the line of least resistance, which is to the surface. A particle of water, therefore, on which a ship impinges, will have two kinds of motion—one a motion outwards and inwards, such as has been already described as resembling the motion of a pendulum, and the other a motion upwards and downwards, caused by the ne-

cessity of the particles beneath the surface rising up towards the surface to allow the vessel to pass, and afterwards of sinking down at the stern to fill the vacuity which the progress of the vessel would otherwise occasion. This last motion also resembles that of a pendulum, the particles of water at the stem rising up until they attain their greatest height at the midship frame, and then again subsiding towards the stern.

It is not difficult, from these considerations, to deduce the conclusion that the form of vessel with a flat floor is not the best which can be adopted, as will be more clearly understood by a reference to fig. 45, where the rectangle D E F G, represents the

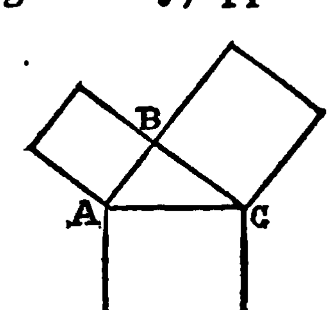
Fig. 45.

cross-section beneath the water-line of a flat-floored vessel at the point midway between the stem and the midship frame, while the triangle A B C is the cross-section of a sharp-floored vessel at the same point, and with the same sectional area. The draught of water in each case is 10 feet, represented by the figures 1 to 10; and the half breadth of the vessel with the rectangular cross-section at this point of the length is 5 feet, which also is one-fourth of the midship breadth. As the water has to be set back from the line of the stem to the line of the side, or in the case of the flat-floored vessel, through a distance of 5 feet, we may represent the power consumed in the operation by 5 feet multiplied by the mean hydrostatic pressure of the water on each square foot. The mechanical power required to be ex-

pended therefore in separating the water in the two sections will be as follows:—

Rectangular section.	Triangular section.
5 × 1 = 5	9 × 1 = 9
5 × 2 = 10	8 × 2 = 16
5 × 3 = 15	7 × 3 = 21
5 × 4 = 20	6 × 4 = 24
5 × 5 = 25	5 × 5 = 25
5 × 6 = 30	4 × 6 = 24
5 × 7 = 35	3 × 7 = 21
5 × 8 = 40	2 × 8 = 16
5 × 9 = 45	1 × 9 = 9
5 × 10 = 50	0 × 10 = 0
275	165

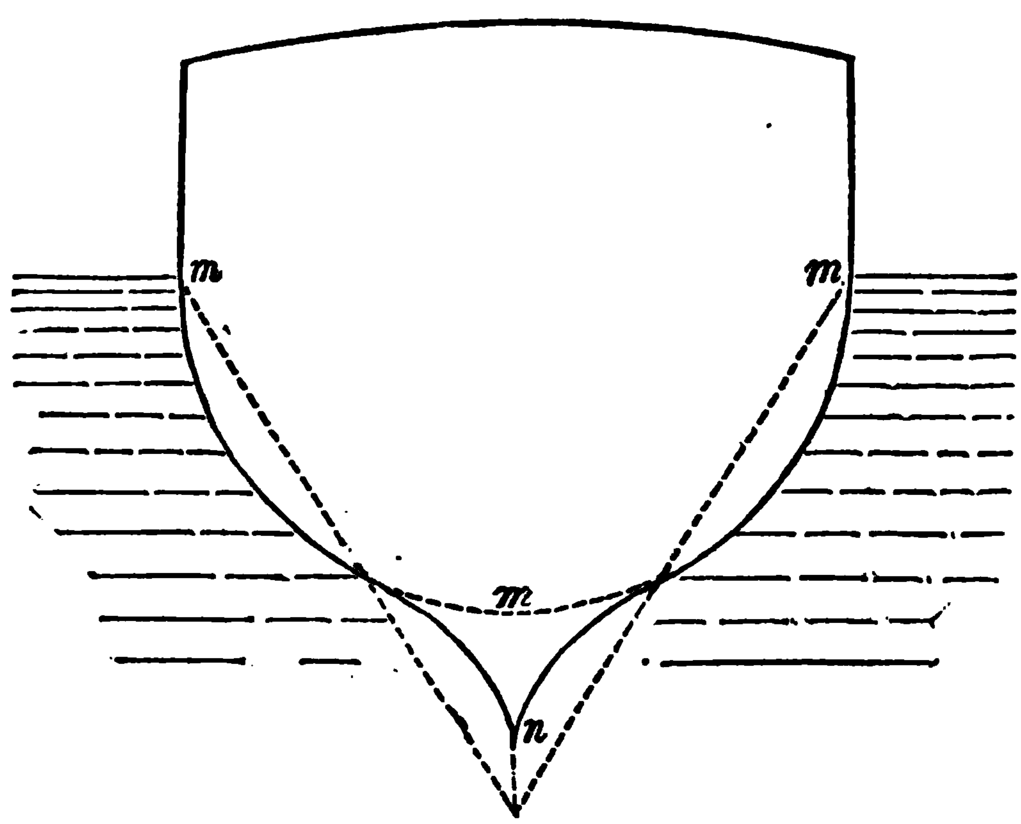
The area of the triangle A B C being equal to that of the rectangle D E F G, the weight of water displaced by a foot in the length of the vessel will be the same whichever form of cross-section is adopted; and as the areas of the shaded triangles A D x and B D x, or of the corresponding triangles B F x and C G x, are also the same, they represent equal amounts of outward motion of the water, and also equal amounts of displacement. In the one case, however, this motion is produced against a much greater hydrostatic pressure than in the other case; and as by shifting the triangle B F x into the position C G x—whereby we enable the vessel to move outward the same volume of water, but against a less hydrostatic resistance—we transform the rectangle H B F G into the triangle H B C, it follows that there is less resistance caused by the movement of the water in the case of triangular cross sections than in the case of rectangular. The rubbing surface too is less in the triangular section. By the principles of geometry, applicable to all right-angled triangles,* $(BF)^2 + (Fx)^2 =$



* This is proved by the 47th Proposition of the first book of Euclid, which shows that the area of the square described on the side A C, opposite to the right angle of a right-angled triangle is equal to the sum of the squares described on the other sides A B and B C.

$(B\,x)^2$. As $B\,F = 5$ feet and $F\,x$ also $= 5$ feet, then $(B\,F)^2 = 25$, and $(F\,x)^2 = 25$, and $25 + 25 = 50$, consequently $B\,x = \sqrt{50} = 7$ nearly. The length of the immersed triangular outline is consequently $7 \times 4 = 28$ feet, whereas the length of the rectangular outline $= 3 \times 10 = 30$ feet. As the resistance due to the friction of the bottom varies as the quantity of rubbing surface, it follows that, as regards friction, the triangular outline is also the more eligible. Instead, however, of a simple triangle, it is preferable

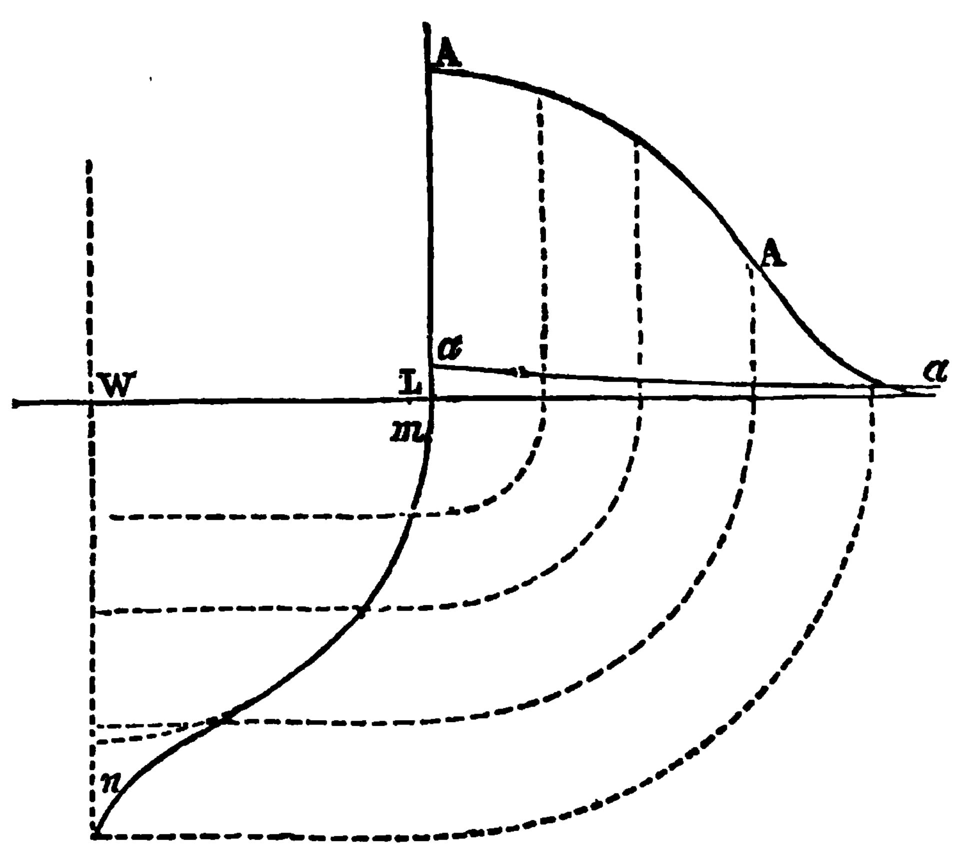
Fig. 46.



that the cross-section should be of the order of figure indicated as the best for the horizontal water-lines; and the same considerations which led to the conclusion that this form would offer the least resistance in the case of a body moving through stationary water lead also to the conclusion that it will offer the least resistance to water moving upwards past a stationary object—which a ship may be supposed to be relatively to the plane in which she floats. Such a figure is represented in fig. 46, in which the triangular section is shown in dotted lines, and the waving lines pass alternately without and within the dotted lines. The cross-section of the vessel is for the most part of the outline

a semi-circle *m m m*—a semicircle being the form which presents the smallest perimeter relatively with the immersed sectional area; but the triangular portion *m n* is added both to prevent the vessel from rolling inconveniently, and to bring the outline into the waving curve which other considerations point out as the most eligible. One of these considerations, as already mentioned, is that it best fulfils the condition of beginning the upward displacement slowly, and another is that it effects the least possible alteration in the shape of the displaced water. In

Fig. 47.



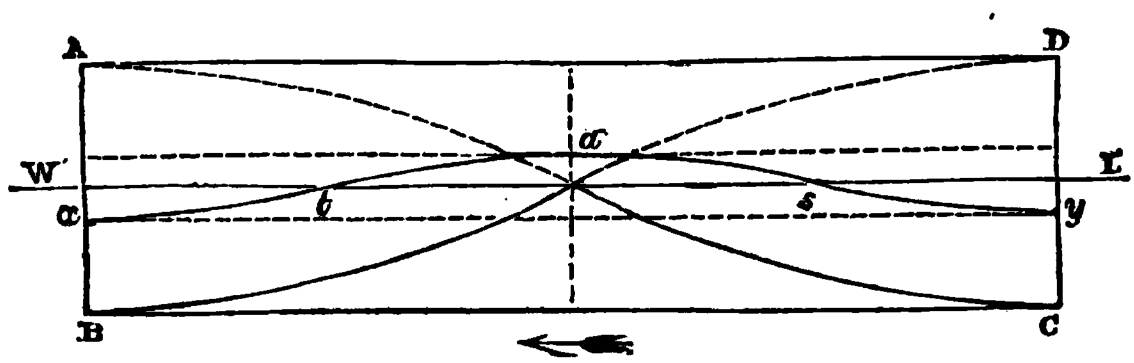
altering the form of a liquid, as in altering the form of a solid, there is a certain expenditure of force; and although this expenditure in the case of a liquid is relatively very small, it is large enough to be worthy of attention in a case where large amounts are consumed in giving motion to water. It hence becomes better, since the displaced fluids must assume the form of a wave, to effect the displacement so that this form shall be at once acquired, instead of some other form being first given to it which is subsequently changed by the action of other forces. This reasoning will be better understood by a reference to fig.

47, where W L is the water-level, *m n* the cross-section of half the vessel, and A A the wave which would be raised if there were no outward motion of the water, but only an upward motion. The outward motion reduces the altitude to some such small elevation as *a a*. Nevertheless it is advisable that the outline of the wave *a a* should be the same order of figure as the outline of the wave A A, only laterally extended. Such indeed is the shape it will necessarily assume; and there will be less change of shape and therefore less motion of the internal particles, if the wave *a a* is drawn out sideways from a block of water of the form A A, than if drawn out from a rectangular, triangular, or any other form of block. The dotted lines indicate the directions in which the pressure will be transmitted, and if we suppose these lines to be tubes, it will be obvious that the surface of the water in these tubes will only conform to the outline of a wave, if the side of the vessel has that outline. If we suppose the portions of those tubes rising above the water-line to be very much enlarged, then the height of the outline will fall from A A to *a a*, but the same order of figure will still be preserved, as it involves less expenditure of power to give this form at once than to give some other form which is afterwards reduced by the action of gravity to this one, so on this ground it is preferable to make the cross-section of the vessel of the form suggested. Taking all things into account, a curve of the same kind that has been shown to be the best for the water-lines, appears to be also the best for the cross-section; and the same ordinates which answer for the water-lines will answer for the cross-section, only in the latter case the ordinates must be placed closer together. If, for example, we have a vessel 200 feet long, and if the ordinates of the water-lines be 5 feet apart, there will be 40 ordinates; and if the vessel be supposed to draw 20 feet of water, the same ordinates placed 6 inches apart will give the proper form of the cross-section below the load water-line. The nearer the form of the cross-section approaches to a semicircle the less friction there will be in the vessel; and the proportions of the cross-section should in all cases, where practicable, approach to the proportions of a semicircle, or in other words the depth below

the water should be a little more than half the breadth at the water-line.

The ascending water will move more and more rapidly as it comes nearer to the surface, like the motion of a falling body inverted; and its momentum will carry it above the surface to a height equal to that which would generate the velocity. This motion of the water above the surface constitutes the second half of the beat of the pendulum which each ascending particle may be supposed to be—the motion of the particle from the keel to the water-line being the first half of such beat. But as, after passing the surface of the water, the particle has to encounter more of the power of gravity, whereas below the water line it is floated by the other contiguous particles, it will follow that the

Fig. 49.



motion of the particle above the surface will be smaller in the proportion of the greater retarding force it there has to encounter. This action will be better understood by a reference to fig. 48, where the parallelogram A B C D is supposed to be the side of a ship, W L is the surface of the water in which the ship swims, and the vertical dotted line at *a* shows the position of the midship frame. If we suppose a particle of water to be situated at *x* a little below the water-level at the bow, then as the vessel moves onward in the direction of the arrow, such particle will be moved upwards faster and faster, until midway between the bow and the midship frame, where its velocity upwards is greatest, it will rise above the surface of the water W L, and its own momentum and that of other ascending particles will carry it upwards until it reaches the position of the midship frame, when it will begin to sink, until at *y* it reaches the same level from

which it rose. The surface particles, no doubt, which terminate their motion at *y*, begin it at w and not at *x*, and to this circumstance we may trace the origin of the hydrostatic resistance of the bow. The depression at *y* will be as great below the mean water-level w L as the elevation at *a* is above it; and if the surface of the water at the stem stood at *x* instead of at w, the fore-body would be in equilibrium, seeing that the depression *t x* w would suck the vessel forward as much, or nearly so, as the protuberance from *t* to *a* would impede it. As the hydrostatic pressure from *a* to *s* pushes the vessel forward as much as the depression from *s* to *y* holds it back, the two portions of the after body will be in equilibrium; and the whole moving vessel would be in equilibrium if the surface of the water at the stem stood at *x* instead of at w. As, however, the water stands higher at the stem than at the stern, there will be a hydrostatic resistance to be encountered which is equal to the height of the wave midway between *a* and w, which will be $\frac{1}{2}a$, acting against the breadth of the ship. This will readily be understood by a reference to fig. 49½, which represents a horizontal slice of a floating body of the height of the wave which the body raises in passing through the water, and the form of the wave is represented by the triangular figure w *a* c, which is delineated on the plane surface formed by cutting away one-quarter of the model so as to clear the problem of the complication involved by the introduction of the curved form of the side. A transverse ordinate is drawn at *b*, and at the point *b b*, where this ordinate meets the side, a line is drawn parallel to the axis, intersecting the line *c e*. From the point of intersection a vertical line *b* is raised, on which is set off the height of the wave at *b b*, and by drawing any desired number of similar lines the wave w *a c* will be set off on the midship section in the form *c e d*, which figure represents the hydrostatic resistance of half the vessel. The area of the figure *c e d* is manifestly half the area of the parallelogram *a c e d*; and as there is a similar figure on the other side of the vessel, the total area representing the hydrostatic resistance will be equal to half the height of the wave acting against the breadth of the ship.

Supposing that no disturbing forces were in existence in in-

terfering with the upward and downward motion of the water, a particle of water at the forefoot B, fig. 48, would, as the vessel moved forward, follow the curved line B A; and if on rising above the line W L it had not to encounter more of the force of gravity, it would pursue its course along the dotted line *a* D.

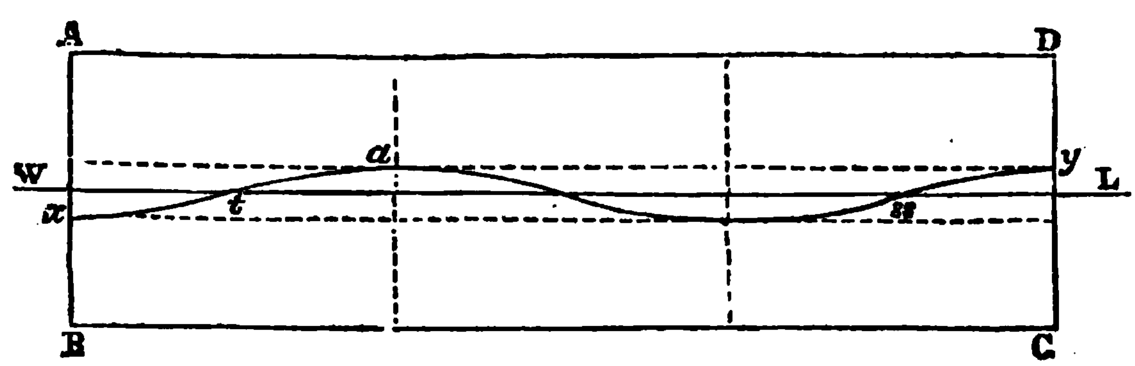
Fig. 49½.

As, however, as soon as the particle passes above W L, it has to encounter nearly the whole force of gravity, its momentum will not suffice to carry it up far, and it will proceed above the water level only to some such point as *a*, and will then immediately pass downward and astern in the track of the curved line *a* C. The whole of the ascending and descending particles will pursue courses nearly parallel to these tracks; and such lines might be drawn mechanically by a tracing point attached to a

pendulum in the manner already described, only that the half of the beat answering to the motion of the particle above the water-line, would be reduced in length by the ball being made in this part of its motion to compress a spring representing the increased power of gravity to which the particle is subjected during this part of its course.

Hitherto we have discovered no source of loss of mechanical power in the movement of the water by a vessel passing through it, except that involved by the necessity of overcoming a constant hydrostatic resistance in consequence of the difference in the level of the water at the bow and stern. There will, however, be the loss of the momentum left in the undulating mass of water. But this last loss will be diminished, if we shift the midship frame further forward, as say to *a*, fig. 50, which is one-third of the length from the bow, instead of half the length. For, although we have still the hydrostatic resistance equal to half the height of *a* above W L multiplied by the breadth of the vessel to encounter, yet if the after-body of the vessel be properly formed with diverging sides, the undulating mass of water will have surrendered most of its power to the vessel in aid of her propulsion before it leaves the stern at *y*. If we suppose the vessel to be cut off at the water line, we shall get rid of the question of the hydrostatic resistance, as the water rising above the water-level will in such case run over the deck; but the momentum of the undulating mass will remain, and the object to be attained is so to form the stern part of the vessel that the upward motion of the water above the water-line at the stern shall be resisted, whereby the mechanical power resident

Fig. 50.



in the heaving water will be communicated to the vessel. This is done at present practically by causing the stern part of the vessel to spread outwards near the load water-line, so that the ascending column of water is intercepted by it and gradually brought to rest.

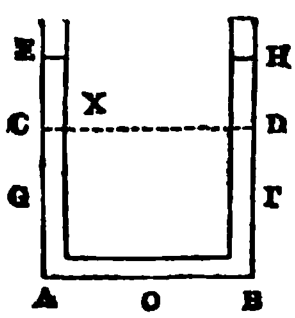
The rise of water at the bow, it will be observed, increases not merely the hydrostatic pressure against which the vessel has to force her way, but also the opposing area against which the pressure acts. In like manner the deficient height of water at the stern diminishes both the pressure and the pressed area. It is very important, therefore, that the difference of level at the bow and stern should be as small as possible. And although we have supposed that the height of the wave *a*, fig. 50, would only be the same if we shifted forward the centre frame, it would in point of fact be higher if the same speed of vessel were maintained. On this ground, therefore, it appears preferable to maintain the midship frame near the position shown in fig. 48, the more especially as the forward and ascending current due to the friction of the bottom of the vessel on the water has a tendency to bring the surface of the water relatively with the ship into the condition represented by the waving-line *x t a s y*. Before entering upon the consideration of the friction of the bottom, however, it may be stated that the hydrostatic resistance consequent on the increased elevation beginning at W instead of at *x* is not all loss. For while the height of the wave increases the pressure of the water beneath, it also helps to separate the water; and if the vessel be made without any straight part between the fore and after-bodies, a portion of the increased elevation which the mean water-line W L receives at the bow, will be retained to increase the elevation of the water at the stern, so that under certain conditions nearly the whole of the power expended in moving the water would be theoretically recoverable. In practice, however, such a result is never reached; and however perfect the arrangements for recovering the power may be made, yet a certain percentage of it is lost at every step; and the safest indication is to employ such a form of vessel as will disturb the water as little as possible. This will be a body of the form which I have indicated with a considerable

proportion of length to breadth, so that the vessel may be sharp at the ends. A length of 7 times the breadth is found to be a good proportion for such speeds as 15 or 16 miles an hour. But the proportionate length that is advisable, will increase with the intended speed.

It is not difficult when the intended speed of the vessel and also its length and breadth are determined, to find what the proper form of the vessel will be, and also the height of the wave which the vessel will raise at the midship frame by her passage through the water, one-half of which height multiplied by the breadth of the vessel will be the measure of the hydrostatic resistance. For as each particle of water at the stern has to describe the motion described by the ball of a pendulum which makes a double beat during the time that the vessel passes through her own length, the breadth of the arc will answer to half the breadth of the vessel, and the vertical height of the arc or the vertical distance fallen by the ball in passing from the highest to the lowest part of the arc, will be the height of the wave raised at the midship frame—that being the height necessary to give the velocity of motion, with which the particles of water must be moved sideways through half the breadth of the vessel, to enable the vessel to pass through in the prescribed time. If we suppose the ball of the pendulum to be replaced by a mass of liquid moving in a circular arc, the motion of this liquid will be the same—except in so far as it is affected by friction—as if it were frozen and suspended by a rod of the same radius as the arc; but if the mass of liquid be large so as to occupy any considerable part of the length of the arc, the motion will not be the same as that of a suspended point, as the whole of the particles will no longer rise and fall through the same height, while all of them will have still to be moved with the same velocity. So also if we have a tube open and turned up at both ends, and if we pour water into it and depress the water in one leg so as to disturb the equilibrium, the water when released will vibrate upward and downward like a pendulum. Such a tube is represented in fig. 51, where E A B H is the tube which is filled with water to the level of X. If the level in one leg be depressed from O to G, it will rise in the other leg

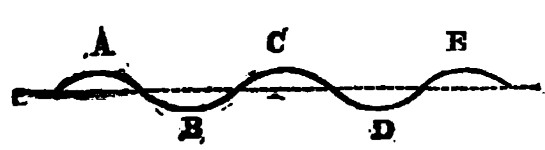
from D to H; and if the depressing force be now withdrawn, the water will fall from H with a velocity corresponding to its height above G, and will be carried by its momentum above C to E, just as the ball of a pendulum ascends in its arc by the momentum it possesses—and the water will continue to oscillate up and down like the ball of a pendulum, until it is finally brought to rest by friction. If the tube be of equal bore throughout and be bisected in O, then as the accelerating force is the difference in the masses of the two unequal columns divided by their sum, the accelerating force will be represented by E G divided by C A B D, or what is the same thing, by E A B F; or it will be proportional to the half of this, or to E C divided by C A O. The time of the oscillation or the time in which the surface of the water will fall from the highest to the lowest point, is equal to that in which a pendulum of the length C A O makes one vibration. Hence the time in which the surface will pass from the highest point to the lowest, and to the highest again, will be that in which a pendulum of the length C A O will make two vibrations, or it will be that in which a pendulum of four times that length makes one vibration, or a centrifugal pendulum of the height equal to C A O makes one revolution. These relations equally hold, if we suppose the same kind of motion which exists in the water to be produced by a piston at O; and the side of the ship may be supposed to be such a piston, and if properly formed, the ship will impart sideways to the water precisely the same kind of motion which exists in the case here illustrated.

Fig. 51.
(1.)



If a sheet of paper be drawn vertically behind a pendulum furnished with a tracing point, then if the pendulum be stationary, the tracing point will draw a straight line represented by the dotted line fig. 52. But if the pendulum be put into motion, then the tracer will describe the waving line A B C D

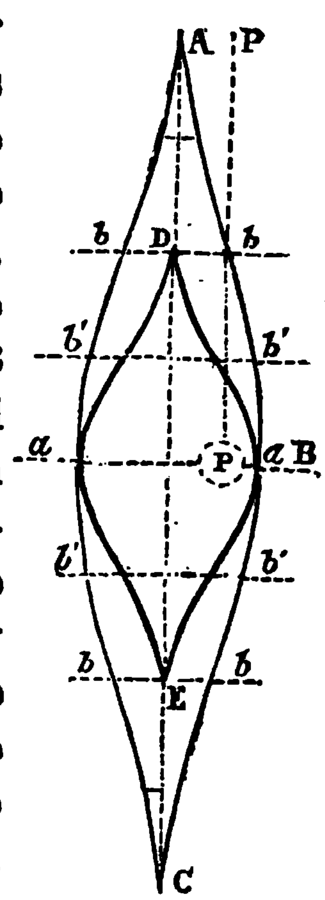
Fig. 52.
(2.)



where the point A answers to the stem of a ship, the point B to the midship frame, and the point C to the stern; and the paper will pass from A to C during the time the pendulum makes two oscillations. Since the pendulum has to make two oscillations while the vessel passes through a distance equal to her own length, the combined motions of the tracer and pencil will delineate the proper form for the side of the vessel; and if made in this form the particles of water will have the same motion as the ball of a pendulum, which motion enables the water to be moved with the minimum of loss. It will be useful, however, to take a particular case to show in what manner the proper form may be practically determined.

Suppose A C, fig. 53, to represent the keel of a vessel—which we may take at 200 feet long and 40 feet wide —and which is intended to maintain a speed of 10 statute miles per hour, or 880 feet per minute. Now as the vessel has to pass through her length, or from A to C, during the time that the pendulum P makes a double beat, or to pass from A to B, which is 100 feet, during the time the pendulum make a single beat, there will be 880 divided by 100, or 8·8 vibrations of the pendulum per minute; and the rod of the pendulum must be of such length as to produce that number of vibrations. Now to determine the length of the rod of a pendulum which shall perform any given number of vibrations per minute, we divide the constant number 375·36 by the number of vibrations per minute, and the square of the quotient is the length in inches. Hence 375·36 divided by 8·8 = 42·6, the square of which is 1814·76 inches or 151·23 feet, and a pendulum 151·23 feet long beating in an arc 20 feet long with the paper travelling at a speed of 880 feet per minute, will describe the line A B C, which will be the proper water-line for the side of a ship if the cross-section be rectangular; and whatever the form of cross-

Fig. 53.



section this figure will equally determine the proper area of cross-section at each successive frame. If instead of moving at 10 miles an hour, the vessel has only to move at the rate of 5 miles an hour, the figure described will be that represented by D *a* E, and the breadths *b b* in the longer figure and *b′ b′* in the shorter are the same, both being equal to half the breadth at *a a*. The rod of the pendulum P P passes through the point *b*, and the pendulum vibrates from the plane of the keel to the plane of the side, so that the chord of the arc in which the vibration is performed is equal to half the breadth of the vessel, while the versed sine or height through which the pendulum falls at each beat, will be equal to the height of the wave at the midship frame. To find the versed sine of the arc, we divide the square of half the chord by twice the length of the pendulum. The chord being 20 feet the half of it is 10 feet; and the pendulum being 151·23 feet long the double of it is 302·46 feet, and 100 divided by 302·46 = ·33 feet or 3·96 inches. The height of the wave at the midship frame, in a vessel formed in the manner indicated, will accordingly be 3·96 inches, or rather this would be the height if the water were moved without friction, so that practically the height will be somewhat greater than is here indicated.

If we increase the speed of the vessel, or increase the breadth, the hydrostatic resistance will increase very rapidly. Thus, if the speed of the vessel be increased to 20 miles an hour, or 1,760 feet per minute, the pendulum will require to make 17·6 beats per minute, and its length will be 375·36 divided by 17·6 = 21·3, the square of which is 453·69 inches, or 37·8 feet. Now, 100 divided by 37·8 = 2·6 feet, which will be the height of the wave at the midship frame in this case, and the hydrostatic pressure will be the half of this, or equivalent to 1·3 feet of water acting on the breadth of the vessel. In like manner, successive additions to the breath of the vessel without increasing the length add rapidly to the hydrostatic resistance, as they involve the necessity of the oscillating particles ascending higher and higher in the arc to enable the vessel to pass.

FRICTION OF WATER.

It remains to consider the friction of water upon the bottom of the vessel, and this is by much the most important part of the resistance which ships have to encounter. Beaufoy made a number of experiments to ascertain the amount of this resistance by drawing a long and a short plank through the water: and, by taking the difference of their resistances and the difference of their surfaces, he concluded that the friction per square foot of plank was, at one nautical mile per hour, ·014 lbs.; at two nautical miles per hour, ·0472 lbs.; at three, ·0948 lbs.; four, ·153 lbs.; five, ·2264 lbs.; six, ·3086 lbs.; seven, ·4002 lbs.; and eight, ·5008 lbs. At two nautical miles an hour, the force required to overcome the friction was found to vary as the 1·825 power of the velocity, and at eight nautical miles an hour as the 1·713 power. Other experimentalists have deduced the amount of friction from the diminished discharge of water flowing through pipes. If there were no friction in a pipe, the velocity of the issuing water should be equal to the ultimate velocity of a body falling by gravity from the level of the head to the level of the orifice.* But as the velocity is found by the diminished discharge to be only that due to a much smaller height, the difference is set down as the measure of the power consumed by friction. This mode of estimating the friction is not applicable to the determination of the friction of a ship; for, in the first place, the discharge is a measure not of the *maximum*, but of the *mean* velocity; and, in the second place, there is every reason to believe that the friction per square foot on the bottom of the ship is quite different near the bow from what it is near the stern. As the water adheres to the bottom there will be a film of water in contact with the ship, which will be gradually put

* There is sometimes misconception on this subject, arising from a neglect of the difference between the *ultimate* and *mean* velocities of a falling body. Thus, if water flows from a small hole in the side of a cistern, the water will issue with the *ultimate* velocity which a heavy body would acquire by falling from the level of the head to the level of the orifice, which, if the height be $16\frac{1}{12}$ feet, will be $32\frac{1}{6}$ feet per second. The *mean* velocity of falling, however, is only $16\frac{1}{12}$ feet per second, so that the ultimate velocity is twice the mean velocity.

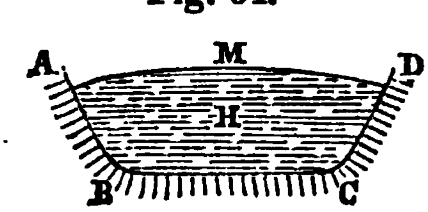
into motion by the friction; and the longer the vessel is the less will be the friction upon a square foot of surface at the stern—seeing that such square foot of surface has not to encounter stationary water, but water which is moving with a certain velocity in the direction of the vessel. The film of water moving with the vessel will become thicker and thicker as it passes towards the stern, and it will rise towards the surface by reason of the virtual reduction of weight consequent upon the motion. The whole of the power, therefore, expended in friction is not lost, as the power expended in the front part of the vessel will reduce the friction of the after part; added to which, the rising current which the friction produces may be made to aid the progress of the ship, if we give to the after-body of the ship such a configuration as to be propelled onward by this rising current. Finally, when the screw is the propelling instrument, the slip of the screw will be reduced, and may even in some cases be rendered negative, by the circumstance of the screw working in this current; and whatever brings this current to rest will use up the power in it, and so far recover the power which has been expended in overcoming the friction.

In my investigations respecting the physical phenomena of the river Indus in India, I observed that the water not only ran faster in the middle of the stream, but that it also stood higher in the middle, so that a transverse section of the river would exhibit the surface as a convex line. At the centre of the river the stream is very rapid, but it is slow at the sides, so that boats ascending the river keep as close as possible to either bank; and in some parts at the side there is an ascending current forming an eddy. I further observed, that not merely were there rapid and considerable changes in the velocity, which I imputed partly to the agency of the wind in deflecting the most rapid part of the current to the one side or the other of the river, but there were also diurnal tides; or, in other words, the stream ran more swiftly in the afternoon than in the early morning. This had been long before observed, and was imputed to the heat of the sun melting the snows in the mountains more during the day than during the night. But although such an effect might be

observable in a single feeder, the river is supplied from so many sources at different distances that such intermittent accessions would equalise one another. Moreover, the effect of the sun in the daytime in swelling the volume of the river, if acting without any equalising influence, could only produce a wave like a tidal wave in the river; and the increase of velocity would at some points take place at night and at some in the morning, whereas I found it to take place *everywhere* at the same time. I finally came to the conclusion that the phenomenon is caused by the influence of the sun in heating the water of the river, and thereby increasing its liquidity and its velocity throughout the whole length of the river. The temperature of the water in the river is commonly about 94° Fahr., but as the river is wide and shallow, it is rapidly heated and cooled, and there are several degrees difference between the temperature of the day and the night. In the early morning the river is coldest, and at that time also—other things being equal—its velocity is least. It may hence be concluded that any thing which gives more mobility to the particles of the water in which a vessel floats will diminish the friction of the bottom; and this end seems likely to be attained by the injection of air into the water at the stem and forefoot or front part of the keel.

It is not difficult to understand how it comes that the water in a river should stand higher at the middle than at the sides, as shown in fig. 54. If we hang a weight upon a spring balance we shall find the amount of the weight to be indicated on the scale or index; and this weight will continue to be shown so long as we hold the spring balance stationary. But if we allow it to move towards the earth with the velocity which a heavy body would acquire in falling by gravity, the index of the spring will show no tension at all—proving that with this amount of downward motion the body imparts no weight. If the spring is moved downward slower than a body falls by gravity, the spring will show that it is sustaining some weight; but at *any* velocity downward there will be a diminution in the

Fig. 54.

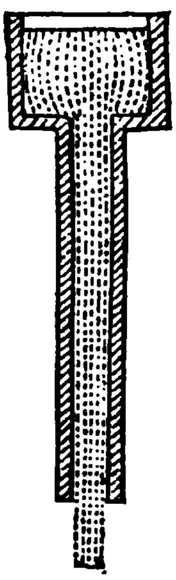


weight of the body answerable to that velocity. In two columns of water, therefore, moving at different velocities, the slower will exert most hydrostatic pressure on the pipe or channel containing it; and where two such columns are connected together sideways, as in a river, the faster must rise to a greater height to be in hydrostatic equilibrium sideways with the slower. The surface of the water consequently becomes convex, as shown at M in fig. 54, where H is the water and A B C D the bed.

It will be seen from these observations that there is a *hydraulic* as well as a *hydrostatic* head of water; and the hydraulic head is equal to the hydrostatic head, diminished by the height due to the velocity with which the water flows. This law is further illustrated by fig. 55, which represents a bulging vessel in which the water is maintained at a uniform height by water flowing into it at the top, while it runs out at E at the bottom. The velocity with which the water flows downward from A to E, varies with the amount of enlargement or contraction of the vessel; and the height of water which will be supported in the small pipes *b*, *c* and *d*, varies as the velocity of the water at their several points of insertion. Thus, the area at B, being greater than the area at A, the velocity will be less, and consequently the water will stand in the small pipe *b* at a point higher than the surface of A. The area at D being less than the area at A, the velocity will be greater; and the height of the water in the small tube *d* will not come up to the level of A. At C, the velocity of the water being very great, not only no height of column will be supported in the tube *c* there inserted, but the water will be sucked up through the inverted tube *e*, out of the small cistern F; and if there be no cistern air will be drawn through the tube. So also in fig. 56, if a pipe be led out at the bottom of a cistern of water, a

Fig. 55.

Fig. 56.



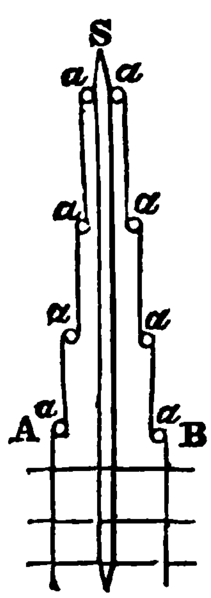
hole bored in any part of the pipe will draw air and not leak water, so long as the water is running out of the bottom of the pipe.

It follows, from these considerations, that the stratum of water put into motion by the friction of the vessel will rise to a higher level than the surrounding water, which is at rest; and advantage should be taken of this ascending current to aid in propelling the vessel, by spreading out the stern part so as to intercept and derive motion from the rising water. This is, to some extent, done in common vessels by the greater breadth which is given to the stern part near the water level; and although no very tangible reason is commonly adduced for the practice beyond that of affording greater accommodation for the cabins, the method of expanding the breadth at the stern is also useful in utilising the ascending current. The manner in which the ship acts upon the water in urging it into motion by friction is not known. But it is known that the vessel carries a film of water with it in the same manner as the belt-pump; and it is known that the particles of water nearest the vessel move with a velocity nearly the same as that of the vessel, and that the motion of each particle diminishes in amount the further it is from the vessel, until those particles are reached which are wholly at rest. The moving film may consequently be regarded as a roller interposed between the bottom of the vessel and the water; and such a roller would enable the vessel to move forward with twice the speed that the roller itself moves at. But before this roller can be set into motion, there will be a good deal of slip or pure friction, just as there is in the driving-wheel of a locomotive in starting the train. It is not known what length of vessel will suffice to move the film of water with the maximum velocity it can attain with any given speed of the ship; nor is it known what the maximum speed of the film is with any given velocity of the ship. The speed will always be less than the speed of the ship, but how much less is not known; and this speed, when once attained, will not be increased, as when it is reached the

power communicated by the friction of the bottom will be balanced by the power consumed in maintaining the motion among the internal particles. Up to a certain point, therefore, the friction upon a square foot of the ship's bottom will diminish with the distance from the stem; and the thickness of the moving film will also increase with that distance. But when that point of the length has been reached, the friction per square foot will become uniform, and there will be no further increase in the thickness of the film.

Instead, however, of supposing the film interposed between the stationary water and the moving bottom to be a single roller, it will be a nearer approximation to the truth if we suppose it to be composed of an infinite number of rollers, *a a a a* in fig. 57, where we may suppose S S to be the ship, while the line extending from roller to roller represents the amount of motion which the water receives from each successive length of the ship, and which diminishes as we recede from the stem until we reach the point A B, where the pure friction of the bottom upon the particles balances the power consumed in maintaining the internal motion of the water, and which power is ultimately transformed into heat. The whole power concerned in propelling the vessel is consumed either in moving the water or in heating it. The greater part of the power expended in moving the water aside at the bow, is recovered by the closing of the water at the stern; and most of that expended in friction in producing a rising current is recoverable by giving a proper configuration to the stern. Of the heat generated, the whole is not lost, as it will give greater mobility to the particles of the water, which will also be given by heating the bottom, as has been done in some steam-vessels, by converting the bottom into a refrigerating surface for condensing the steam; and by which arrangement the bottom itself has been heated to some extent. On the whole, however, that arrangement will be the most advantageous for reducing resistance by which the least motion is given to the

Fig. 57.



water, and the least heat generated in it; and the smoothness of the rubbing surface will somewhat affect that question. In pipes, it has been found that there is no increase of friction from increase of pressure. But it must not be therefore inferred that in vessels the friction per square foot is precisely the same at every point in the depth, any more than at every point of the length; for the moving water has to escape to the surface, and the difficulty of the escape will be the greater the further the surface is off. If we knew the ratio in which the resistance of a vessel increased with the length and with the depth, we should be able to tell what form the vessel should have, in order to offer the least resistance. But it is quite certain that the resistance per square foot of the bottom does diminish with the length in some proportion or other; and as the resistance also diminishes as the wetted perimeter, and as relatively with the sectional area, the wetted perimeter of large vessels is less than that of small, it is easy to understand how it comes that large vessels are swifter than small with the same proportion of propelling power. If we double the breadth and immersed depth of a vessel, we double the length of its perimeter. But we increase its sectional area fourfold; and as with any given length, and with equally fine ends, the wetted perimeter is the measure of the resistance, it follows that the large vessel will require less power per ton or per square foot of immersed section to maintain any given speed.

SPEED OF STEAM VESSELS OF A GIVEN POWER.

There were no accepted rules for ascertaining the speed that a steam vessel of a given type would probably obtain with engines of a given power, until the appearance of the first edition of my *Catechism of the Steam-Engine*, when I published the rule which had long been employed by Messrs. Boulton and Watt for determining this point. This rule was founded on a long-continued series of experiments on steam vessels of different types; and for *similar kinds of vessels* the results it gives have been found very nearly to accord with those subsequently ob-

tained by experiment. This rule, which proceeds on the supposition that the engine power required for the propulsion of a vessel varies as the area of the immersed midship-section, and as the cube of the speed, has been already referred to in page 77 as an example of the application of equations, and in algebraical language it is as follows:—

If S be the speed of the vessel in knots per hour, A the area of the immersed midship section in square feet, C a numerical coefficient, varying with the form of vessel and to be fixed by experiment, and P the indicated horse-power: then

$$P = \frac{S^3 A}{C}, \qquad C = \frac{S^3 A}{P} \qquad \text{and} \qquad S = \sqrt[3]{\frac{PC}{A}}$$

In words these rules are as follows:—

TO DETERMINE THE POWER NECESSARY TO REALISE A GIVEN SPEED IN A STEAM VESSEL BY BOULTON AND WATT'S RULE.

RULE.—*Multiply the cube of the given speed by the area in square feet of that part of the midship section of the vessel lying below the water-line, and divide the product by a certain coefficient of which there is a different one for each particular type of vessel. The quotient is the indicated power in horses that will be required to give the intended speed.*

Example.—The steamer 'Fairy,' with an immersed sectional area of 71½ square feet, and a coefficient of 465, attained on trial a speed of 13·3 knots per hour. What indicated power must have been exerted to attain this speed?

Here the cube of 13·3 is 2352·637, which multiplied by 71·5 = 168210·9, and this divided by 465 is equal to 363 horse-power, which was the power actually exerted in this case.

In the first edition of my *Catechism of the Steam-Engine* the coefficients of a number of steam-vessels were given, which had been ascertained experimentally by Boulton and Watt; and in the first edition of my *Treatise on the Screw Propeller*, published in 1852, I recapitulated a number of the coefficients of

the screw steamers of the navy, which had then been recently ascertained by the steam department of the navy, as also the coefficients obtained by multiplying the cube of the speed—not by the area of the midship section, but by the cube root of the square of the displacement—and dividing by the indicated power. The displacement of the 'Fairy' at the trial, at which the speed was 13·3 knots, was 168 tons. Now the square of 168 is 28224, the cube root of which is 30·45 nearly, and this multiplied by the cube of the speed 2352·637 and divided by the indicated power, 363 horses, gives 197 as the coefficient proper to be employed when this measure of the resistance is adopted. Neither the immersed section, however, nor the displacement, is the proper measure of the resistance in steam vessels; and I pointed this out in the first edition of my *Treatise on the Screw Propeller*, in 1852, and suggested the wetted perimeter as a preferable measure of the resistance; the perimeter being a measure of the friction; and nearly the whole of the resistance of well-formed ships being produced by friction. Under this view the velocity of ships with any given perimeter and propelling power would fall to be considered in much the same way as the velocity of the water flowing in rivers or canals, and in which the speed with any given declivity of the bed varies as the hydraulic mean depth, or in other words as the sectional area of the stream divided by the wetted perimeter. In such a comparison the engine power of the ship answers to the gravitation of the stream down the inclined plane of the bed, while the area of the transverse section of the ship beneath the water-line divided by the wetted perimeter constitutes the *hydraulic mean depth of the ship*. This measure of the resistance, however, though accurate enough for short vessels, is not applicable to long vessels without some allowance being made for the inferior resistance of long vessels of the same sharpness at the ends, in consequence of the proportion of power which long vessels recover, especially if propelled by the screw or any other propeller situated at the stern.

TO DETERMINE BOULTON AND WATT'S COEFFICIENT FOR ANY GIVEN VESSEL OF WHICH THE PERFORMANCE IS KNOWN.

RULE.—*Multiply the cube of the speed in knots per hour by the area in square feet of the immersed transverse section of the vessel, and divide the product by the indicated horse-power. The quotient will be the coefficient of that particular type of vessel.*

Example.—The steamer Fairy, with an area of immersed section of $71\frac{1}{2}$ square feet, and 363 indicated horse-power, attained a speed of 13·3 knots an hour. What is the coefficient of that vessel?

Here 13·3 cubed = 2352·637, which multiplied by 71·5 and divided by 363 horse-power = 465, which is the coefficient of this vessel according to Boulton and Watt's rule. A good number of coefficients for different vessels is given at page 77.

TO DETERMINE WHAT SPEED WILL BE ATTAINED BY A STEAM VESSEL OF A GIVEN TYPE WITH A GIVEN AMOUNT OF ENGINE POWER, BY BOULTON AND WATT'S RULE.

RULE.—*Multiply the indicated horse-power by the coefficient proper for that particular type of vessel, and divide the product by the area of the immersed transverse section in square feet. Extract the cube root of the quotient, which will be the speed that will be obtained in knots per hour.*

Example.—What speed will be obtained in a steamer of which the coefficient is 465, and which has an immersed section of $71\frac{1}{2}$ square feet, and is propelled by engines exerting 363 horse-power.

Here $363 \times 465 = 168795$, which divided by $71{\cdot}5 = 2360$. The logarithm of this is 3·372912, which divided by $3 = 1{\cdot}124304$, the natural number answering to which is 1331. Now the index of the divided logarithm being 1, there will be two integers in the natural number answering to it, which will consequently be 13·31, and this will be the speed of the vessel in knots per hour.

The coefficient of a steamer sometimes varies with the speed with which the vessel is propelled. If the vessel is properly formed for the speed at which she is driven, then her coefficient will not become greater at a lower speed; and if it becomes greater, the circumstance shows that the vessel is too blunt. When the 'Fairy' was sunk to a draught of 5 feet 10 inches, her speed was reduced to 11·89 knots, and her coefficient was reduced from 465 to 429, showing that she worked more advantageously at the higher speed and lighter draught. The 'Warrior,' which when exerting 5,469 horse-power attained a speed of 14.356 knots with a coefficient of 659, attained when exerting 2,867 horse-power a speed of 12·174 knots with a coefficient of 767; and when exerting 1,988 horse-power a speed of 11·040 knots with a coefficient of 825. This shows that the 'Warrior' is too blunt a vessel for a high rate of speed.

It will be satisfactory to ascertain the comparative eligibility of the forms of the 'Fairy' and the "Warrior,' which we may easily do by comparing the speed attained by each, with the speed which would be attained by an equal weight of water running in a river or canal, and impelled by an equal motive force. The rule for determining the speed of water flowing in rivers or canals of any given declivity is as follows:—

TO DETERMINE THE MEAN VELOCITY WITH WHICH WATER WILL FLOW THROUGH CANALS, ARTERIAL DRAINS, OR PIPES, RUNNING PARTLY OR WHOLLY FILLED.

RULE.—*Multiply the hydraulic mean depth in feet by twice the fall in feet per mile. Extract the square root of the product, which is the mean velocity of the stream in feet per minute.*

Now the 'Fairy,' when realizing a speed of 13·3 knots per hour with 363 horse-power, had a draught of water of 4·8 feet; a sectional area of 71·5 feet; a wetted perimeter of 24·7 feet, and a displacement of 168 tons. The hydraulic mean depth being the sectional area in square feet, divided by the length of the wetted perimeter in feet, the hydraulic mean depth will in this case be 71·5, divided by 24·7=2·9.

The engine made 51·6 revolutions per minute, and the screw

258 revolutions per minute, being five times the number of revolutions of the engines. The stroke is 3 feet, and the pitch of the screw 8 feet.

Now a horse-power being 33,000 lbs., raised 1 foot per minute, and as there were 363 horse-power exerted, the total effort of the engines will be 363 times 33,000, or 11,979,000 lbs., raised through 1 foot each minute. But the engine makes 51·6 revolutione each minute, and the length of the double stroke is 6 feet, so that the piston moves through 309·6 feet per minute; and the power being the product of the velocity and the pressure, the power 11,979,000 lbs. divided by the velocity of the piston, 309·6 feet per minute, will give the mean pressure urging the pistons, which will be 38,691 lbs. But the speed of the screw-shaft being five times greater than that of the engine-shaft, the pressure urging it into revolution must, in order that there may be an equality of power in each, be five times less; or it will be 7,538 lbs. moving through 6 feet at each revolution. Then the pitch of the screw being 8 feet, the thrust of the screw will be less than 7,538, in the proportion in which 6 is less than 8, or it will be 5,653, supposing that there is no loss of power by slip and friction. It is found on an average in practice, that about one-third of the power is lost in slip and friction; and the actual thrust of the screw-shaft will be about one-third less than the theoretical thrust, or in this case it will be 3,769 lbs. or 1·68 ton. Now, in order that 168 tons of water may gravitate down an inclined channel with a weight of 1·68 ton, the declivity of the channel must be 1 in 100. In 1 mile, therefore, it will be 52·80 feet. A cubic foot of salt-water weighs 64 lbs., so that there are 35 cubic feet in the ton, and in 168 tons there are 5,880 cubic feet. Dividing this by the sectional area 71·5 feet, we get a block of water 82·2 feet long, and with a cross-section of 71·5 feet, weighing 168 tons; and the wetted perimeter being 24·7 feet, and the length 82·2 feet, we get a rubbing area of 2020·34 feet; and as the friction on this surface balances the weight of 3,769 lbs., there will be a friction of 1·8 lb. on each square foot. If this block of water be supposed to be let down a channel falling 1 in 100, its velocity will go on increasing until

the friction balances the gravity, which, according to the rule given above, will be when the water attains a speed of 11 miles an hour, from whence we conclude that *the sum of the resistances of a well-formed ship are less than the friction alone of an equal weight of water of the same hydraulic depth, moved in a pipe or canal by an equal impelling force.* If instead of taking the declivity in 2 miles, as the rule prescribes, to ascertain the velocity of the water, we take the declivity in twice 2, or 4 miles, we shall arrive at a pretty exact expression of the speed of the vessel in this particular case. Taking the knot at 6,101 feet, 13·3 knots will be equal to 15·3 statute miles, and the declivity in 1 mile being 52·8 feet, the declivity in 4 miles will be 211·2 feet. Multiplying this by 2·9, the hydraulic mean depth, we get 612·48, the square root of which is 24·7, which multiplied by 55, gives the speed of the water in feet per minute= 1,358·5. This, multiplied by 60, gives 81,510 feet as the speed per hour, and this divided by 5,280, the number of feet in a statute mile, gives 15·4 as the speed in statute miles per hour.

The 'Warrior,' with a displacement of 8,852 tons, a draught of water of 25½ feet, an immersed midship section of 1,219 square feet, and 5,469 horse-power, attained a speed of 14·356 knots, or 16·6 statute miles. The number of strokes per minute was 34¼, and the length of the double stroke 8 feet, while the pitch of the screw was 30 feet. The wetted perimeter is 88 feet, which makes the mean hydraulic depth 13·8 feet. The power being 5,469 horses, 33,000 times this, or 180,477,000 lbs., will be lifted 1 foot high per minute. But as the piston travels 54·25 times 8 feet, or 434 feet each minute, the load upon the pistons will be 415,845 lbs. The pitch of the screw, however, being 30 feet, while the length of a double stroke is 8 feet, the theoretical thrust of the screw will be reduced in the proportion in which 30 exceeds 8, or it will be 110,892 lbs. If from this we take one-third, on account of losses from slip and friction, we get 73,928 lbs., or 33 tons, as the actual thrust of the screw.

Now 8,852, which is the displacement in tons, divided by 33 tons, which is the motive force in tons, gives 268, or, in other words, the declivity of the channel must be 1 in 268, in order

that 8,852 tons may press down the inclined plane with a force of 33 tons. This is a declivity of very nearly 20 feet in the mile, or 40 feet in two miles, or 80 feet in twice two miles. The mean hydraulic depth being 13·8 feet, 80 times this is 1,104, the square root of which is 33·2, which multiplied by 55=1,826 feet per minute, or multiplying by 60=109,560 feet per hour. Dividing by 5,280, we get the speed of 20 miles per hour, which ought to be the speed of the 'Warrior' if her form were as eligible as that of the 'Fairy.' The speed falls 3·4 miles an hour short of this, which defect must be mainly imputed to the deficient sharpness of the ends for such a speed and draught, and the increased resistance consequent on the greater depth.

In a paper by Mr. Phipps, on the 'Resistances of Bodies passing through Water,' read before the Institution of Civil Engineers in 1864, it was stated that these resistances comprised the Plus Resistance, or that concerned in moving out of the way the fluid in advance of the body; the Minus Resistance, or the diminution of the statical pressure behind any body when put into a state of motion in a fluid; and the Frictional Resistance of the surface of the body in contact with the water.

The Plus Resistance of a plane surface one foot area, moving at right angles to itself in sea water, was considered to be $R=\frac{64{\cdot}2\times v^2}{2g}$, and the Minus Resistance was one half the Plus Resistance.

For planes moving in directions not at right angles to themselves, the theoretical resistances were, for the Plus Pressure—

$$S=\frac{a}{r^2}, \text{ and } R=\frac{S64{\cdot}2v^2}{2g},$$

the Minus Pressure being one-half the above; where R was the resistance of the inclined plane; a, the area of the projection of the inclined plane upon a plane at right angles to the direction of motion; r, the ratio of the areas of the projected and the inclined planes; and S, the area of a square-acting plane of equivalent resistance with the inclined plane.

But, besides these theoretical resistances, the experiments of Beaufoy showed, that when the inclined planes were of moderate

length only, the Plus Resistance was considerably in excess of the above; so that when the slant lengths of the planes were to their bases in the proportion of

2 to 1, 3 to 1, 4 to 1, and 6 to 1,

the actual resistances exceeded the theoretical, as

1·1 to 1, 1·98 to 1, 3·24 to 1, and 6·95 to 1.

Mr. Phipps proposed a method of approximating to these additional resistances, by adding the constant fraction of $\frac{1}{7}$th of a square foot for every foot in depth of the plane to the quantity S previously determined, which empirical method he found to agree nearly with the results of Beaufoy's experiments.

The resistances of curved surfaces, such as the bows of ships, were adverted to, the method of treating them being to divide the depth of immersion into several horizontal layers, and then again into a number of straight portions, and to deal with each portion as a separate detached plane, according to the preceding rules.

The question of friction was then considered. The experiments of Beaufoy were referred to, giving 0·339 lb. per square foot as the co-efficient of friction for a plained and painted surface of fir, moved through the water at 10 feet per second, the law of increase being nearly as the squares of the velocities, viz., the 1·949 power. Mr. Phipps was, however, of opinion, that a surer practical guide for determining the coefficient of friction would be, by considering all the data and circumstances of a steam-ship of modern construction, moving through the water at any given speed. The actual indicated horse-power of the engines being given, the slip of the paddles being known, and the friction and other losses of power approximated to, it was clear that the portion of the power necessary to overcome the resistance of the vessel might be easily deduced. By determining approximately, by the preceding rules, the amounts of the Plus, the Minus, and the Additional Head resistances, and deducting them from the total resistance, the remainder would be the resistance

due to the friction of the surface. By this process, and taking as an example, the iron steam-ship 'Leinster,' when perfectly clean, and going on her trial trip 30 feet per second in sea-water, her immersed surface being 13,000 square feet, the coefficient of friction came out at 4·34 lbs. per square foot. Beaufoy's coefficient of 0·339 lb. per square foot at 10 foot per second would, according to the square of the velocities, amount to 3·051 lbs. at 30 feet per second. The difference between this amount and the above 4·34 lbs. might be accounted for by a difference in the degree of roughness of the surfaces.

Other methods for the determination of the coefficient of friction were then discussed. One, derived from the known friction of water running along pipes, or water-courses, was shown to be considerably in excess of the truth. It was founded upon the observed fact, that at a velocity of 15 feet per second, the friction of fresh water on the interior of a pipe was 25 oz.* per square foot. Applying this to the ship 'Leinster,' and increasing the friction as the square of the velocities up to 30 feet per second, the above friction would become 100 oz., or $6\frac{1}{4}$ lbs., per square foot, which, acting upon 13,000 square feet of surface, would absorb, at the above speed, no less than 4,395 H.P., whilst the total available power of the engines (after deducting from the indicated 4,751 H.P. $\frac{1}{10}$th for friction, working air-pumps, and other losses, and $\frac{1}{5}$th of the remainder for the observed slip), was only 3,421 H.P.; thus showing an excess of resistance equal to 974 H.P., without allowing any power to overcome the other resistances. The assumption of 25 oz. being the proper measure of the friction per square foot, at a velocity of 15 feet per second, upon the clean surface of an iron ship, seemed to have arisen from the opinion, very generally entertained, that there was no difference in the amount of friction in pipes and water-courses, whether internally smooth like glass, or moderately rough like cast-iron, and that the surfaces of ships were subject to the same action. The comparatively recent experiments, in France, of the late M. Henry Darcy were in opposition to the above view, and

* For sea water this quantity must be increased as the specific gravity, or as 62·5 to 64·2.

showed that the condition as to roughness of the interior of a pipe modified the friction considerably. Thus, with three different conditions of surface, the coefficients were:

A. Iron plate covered with bitumen made very smooth, 0·000432
B. New cast-iron . . . . 0·000584
C. Cast-iron covered with deposits . . . . 0·001167
The friction was, therefore, nearly as 1, $1\frac{1}{2}$, and 3.

As there appeared no reason to doubt the correctness of M. Darcy's experiments, even in pipes the notion of the friction being uninfluenced by the state of roughness of the interior could no longer be entertained. The 25 oz., previously mentioned as the measure of friction per square foot for the interior of pipes and water-courses, could not, therefore, be regarded as a constant quantity, applicable to all kinds of surfaces; but from Mr. Phipps' calculations, it appeared to come intermediately between the coefficients of the surface B and C, given in the above scale; as at 15 per second,

A	would give	$13\frac{1}{3}$ oz.	per square	foot
B	"	20	"	"
and C	"	40	"	"

Besides, there was another cause for an excess of friction in pipes and water-courses, over that upon ships, even when the surfaces were equally smooth. It arose from the circumstance, that where the velocity of the water in a pipe, or open water-course, was spoken of, the meaning was, its average velocity; whilst the velocity of a vessel through still water meant what the words implied, namely, the relation of the vessel's motion to the fluid at rest. If the case were taken of a water-course of such width, that the friction of the bottom only need be considered, with an average velocity of flow of 15 feet per second, the friction upon the bottom would be equal to 25 oz. per square foot; but according to the rules generally used, an average velocity of 15 feet per second corresponded to a surface velocity of 16·66 feet

per second, which was the velocity with which a vessel should pass through still water, to give an equal friction upon its sides. According to Beaufoy, the velocity of 16·66 feet per second would produce a friction of ·932 lbs. or 14·91 oz., where 15 feet would only give 12·2 oz. The difference between 14·91 oz. and 25 oz. (equal to 10·09 oz.) must, therefore, Mr. Phipps thought, be set down to the different degree of roughness of the surfaces in the water-course and the vessel.

Taking then 4·34 lbs. as the friction per square foot of a new iron ship, moving through the water at a speed of 30 feet per second, it would be found, Mr. Phipps considered, that this was equal to the $\frac{1}{207\cdot06}$ part of the plus resistance of a plane 1 foot square, moving through the water at right angles to itself at the above velocity. Also, as the resistance of both planes increased according to the same law of the square of the velocities, the ratio of 1 to 207·06 would subsist at all velocities.

$$\text{The ratio was } \frac{64\cdot2v^2}{2g} \text{ to } 4\cdot34 \text{ lbs.} = \frac{1}{207\cdot06}$$

Calling the ratio r, and the whole frictional surface in square feet s, and S, as before, the area of a square-acting plane of equivalent resistance, then

$$S = s \div r = s \div 207\cdot06.$$

As an example of the application of the previous deductions, the performance of the steam-ship 'Leinster,' on her trial trip, when going through sea-water at a speed of 30 feet per second, was referred to.

In this case—

m, the area of the immersed midship section was	336 sq. ft.
d, the draught of water	13 ft.
r, the reduced ratio of the slant length of the bow to the projection	10 to 1.
r', the same for the stern	10 to 1.
r'', the ratio of 1 square foot of square-acting plane, to 1 square foot of frictional surface	207·06 to 1.
v, the velocity in feet per second . . .	30

w, the weight of a cubic foot of sea-water . 64·2 lbs.
f, the area of the frictional surface . . . 13,000 sq. ft.

Calling P, the Plus, or head resistance; M, the Minus, or stern resistance; A, the Additional Head resistance; F, the Frictional, or surface resistance; S, the area of a square-acting plane having an equal resistance with each of the above; and R, the total resistance;

$$\text{Then, } P = \frac{m}{r^2} = S = \frac{336}{100} = 3·36 \text{ sq. ft.}$$

$$\text{"}\quad M = \tfrac{1}{2}\frac{m}{r^2} = S = \tfrac{1}{2}\frac{336}{100} = 1·68 \quad \text{"}$$

$$\text{"}\quad A = \frac{13}{7} = S = 1·86 \quad \text{"}$$

$$\text{"}\quad F = \frac{13000}{207·06} = S = 62·78 \quad \text{"}$$

$$S = 69·68 \quad \text{"}$$

$$R = 69·68 \times \frac{64·2v^2}{2g} = 69·68 \times 900 = 62{,}712 \text{ lbs}$$

$$H \text{ (Realized Power)} = \overset{\text{lbs.}}{62{,}712} \times 30 \div 550 = 3420·66 \text{ H.P.}$$

H' (Gross Power) including the slip and other losses, =

$$3420·66 \times \frac{100}{72} = 4751 \text{ H.P.}$$

Thus, by ascertaining the value of S for any vessel, which was entirely independent of velocity, it would be easy to determine the power necessary to propel it at any required speed, or the speed being given, to find the corresponding power.

$$\text{Generally } H = V S \frac{64·2\ V^2}{2\,g} \div 550 \qquad (1)$$

Or, because for sea water 64·2 was very nearly equal to 2 g,

$$H = \frac{V^3 S}{550} \qquad (2)$$

When the slip and other losses were in the same proportion as in the 'Leinster':

$$* \; H' = H\frac{100}{72} \qquad (3)$$

When the gross power was given, and the velocity was required

$$\dagger \; V = \left(\frac{\frac{72}{100}H' \times 550}{S\frac{64\cdot 2}{2g}}\right)^{\frac{1}{3}} \qquad (4)$$

Mr. Phipps then proceeded to examine the question of tho influence of form in reducing the resistance of vessels.

It was argued that, in vessels of similar type to the 'Leinster,' where $\frac{9}{10}$ths of the whole resistance was due to friction, and only $\frac{1}{10}$th to considerations involving the question of 'form,' no minor modifications of the latter could have much effect in diminishing the total resistance. The case of other vessels of different type, more bluff in the bows and not so fine in the run, was adverted to, and a particular instance was discussed, where the inertial resistance was supposed to be equal to $\frac{1}{5}$th of the total resistance, and the slant length of the bows to the base to be as 6 to 1. If such a vessel were altered, so as to make the above proportion $8\frac{1}{2}$ to 1, the improvement would only diminish the total resistance by $\frac{1}{10}$th.

The conclusion that the friction of ships constitutes the largest part of their resistances, was first pressed upon me in 1854, in which year I built two steamers with water-lines formed on the principle of imparting to the particles of water the motion of a pendulum, as already explained. I found, as I expected, that these vessels passed through the water with great smoothness, and without in any measure raising the water in a wave at the bow, as was a common practice in the older class of steamboats. Nevertheless I did not obtain a speed much superior to

* If for fresh water $H' \times 0\cdot 97$ = Gross H.P.
† If for fresh water $V \div 0\cdot 99$ = Velocity.

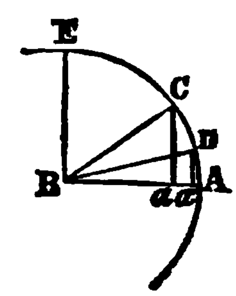
that of vessels less artistically formed; and the conclusion became inevitable—seeing that all other known causes of resistance had been reduced to a minimum without material benefit to the speed—that the friction, which alone remained unchanged, must constitute the main element of resistance; and other things being alike, the friction of a vessel, as of a river, would, in such case, be measurable by the wetted perimeter of the cross-section. It was further plain, that as there was not much difference between the resistance of a vessel formed with pendulum or wave curves, and that of well-formed vessels of the ordinary configuration, any mode of computing the resistance applicable in the one case would also be applicable without material error in the other. These conclusions, which I published in my 'Catechism of the Steam-Engine,' in 1856, are now very generally accepted; and when, in 1857, Mr. Rankine had to compute the probable speed of an intended vessel, he proceeded on the supposition that the resistance was due almost wholly to friction, and that the friction of a riband of the form of a trochoid or rolling wave, of the length of the ship and of the breadth of the wetted perimeter, would be an accurate measure of the resistance, the trochoid being the same order of curve as that which would be described by a pendulum. Since, however, a wave moves in different parts with different velocities, Mr. Rankine concluded that it would be proper to take this circumstance into account, and he therefore, instead of taking the actual surface of the vessel, took a surface so much larger, that its friction would produce a resistance equivalent to the increased friction caused by the varying velocities of the wave, and the hydrostatic pressure consequent upon the difference of level at the bow and stern, and which in a well-formed vessel is very small. This additional or hypothetical surface Mr. Rankine terms *augmented surface;* and by using this theoretical surface in his computations instead of the actual wetted surface of the ship, he deduces results singularly conformable to those obtained by actual experiment. The amount of the augmented surface will vary with the sharpness of the vessel—sharp vessels having the least augmentation;

and the sharpness is measured by the sines* of the angles of the water-lines at the bow and stern. I shall here introduce Mr. Rankine's able investigation, to which the only exception that can be taken, so far as I see, is that the resistance per square foot produced by friction in every part of the length of the vessel is not the same, but is more at the fore part, in consequence of the necessity of putting the water into motion; but after this has been done, the friction per square foot of the further length of the vessel will be uniform.

The Resistance due to Frictional Eddies remains alone to be considered. That resistance is a combination of the direct and indirect effects of the adhesion between the skin of the ship and the particles of water which glide over it; which adhesion, together with the stiffness of the water, occasions the production of a vast number of small whirls, or eddies, in the layer of water immediately adjoining the ship's sur-

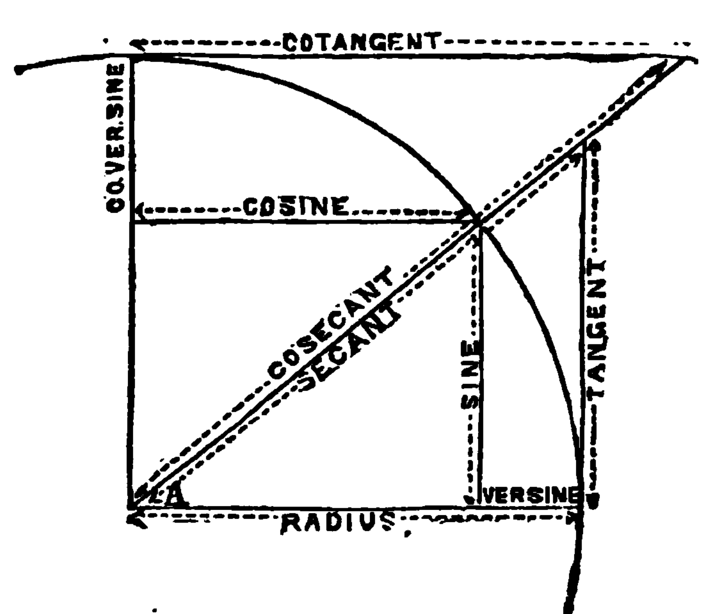
* A sine is one of the measures of an angle. Thus in the circle A D C E (fig. 58) the lines A B and A E are radii of the circle at right angles with one another, and C a is the sine of the angle C B A, and D a is the sine of the angle D B A. The circle is supposed to be divided into 360 degrees, so that a quadrant, or one-fourth of a circle, is 90 degrees.

Fig. 58.



In fig. 59 the various trigonometrical quantities relating to the angle A are graphically represented. The angle A is half a right angle, or 45 degrees, which is the eighth part of the whole circle of 360 degrees.

Fig. 59.



face. The velocity with which the particles of water whirl in those eddies, bears some fixed proportion to that with which those particles glide over the ship's surface; hence the actual energy of the whirling motion impressed on a given mass of water at the expense of the propelling power of the ship, being proportional to the square of the velocity of the whirling motion, is proportional to the square of the velocity of gliding; in other words, it is proportional to the *height due* to the velocity of gliding. The velocity of gliding of the particles of water over a given portion of the ship's skin, bears a ratio to the speed of the ship depending on her figure, and on the position of the part of her skin in question; and the height due to the velocity of gliding is equal to the height due to the speed of the ship, multiplied by the *square* of the same ratio. Further, the mass of water upon which whirling motion is impressed by a given part of the ship's skin while she advances through a unit of distance, is proportional to the area of that part of the skin, multiplied by the before-mentioned ratio which the velocity of gliding of the water past that part of the skin bears to the velocity of the ship.

Hence *the resistance to the motion of the ship, due to the production of frictional eddies by a given portion of her skin*, is the product of the following factors:—

I. The area of the portion of the ship's skin in question.

II. The *cube* of the ratio which the velocity of gliding of the particles of water over that area bears to the speed of the ship; being a quantity depending on the figure of the ship and the position of the part of her skin under consideration.

III. The height due to the ship's speed; that is,

$$\frac{(\text{speed in feet per second})^2}{64{\cdot}4}$$

$$\text{or, } \frac{(\text{speed in knots})^2}{22{\cdot}6}$$

IV. The heaviness (or weight of a unit of volume) of the water (64 lbs. per cubic foot for sea-water).

V. A factor called the *coefficient of friction*, depending on the material with which the ship's skin is coated, and its condition as to roughness or smoothness.

The sum of the products of the Factors I. and II. for the whole skin of the ship has of late been called her AUGMENTED SURFACE; and the Eddy-resistance of the whole ship may therefore be expressed as the product of her Augmented Surface by the Factors III. IV. and V. above mentioned.*

* In algebraical symbols, let $d\,s$ denote the area of a small portion of the ship's

The resistance thus determined, being deduced from the work performed in producing eddies, includes in one quantity both the direct adhesive action of the water on the ship's skin, and the indirect action, through increase of pressure at the bow and diminution of the pressure at the stern.

The existence of this kind of resistance has been recognised from an early period. Beaufoy made experiments on models to determine its amount; Mr. Hawksley and Mr. Phipps have included it in a formula for the resistance of ships; and Mr. Bourne pointed out that it must depend mainly on the ship's immersed girth. But the earlier researches, both experimental and theoretical, throw little light on the subject, and fail to give a trustworthy value of the coefficient of friction; because in them it was assumed that the frictional resistance was proportional to the *actual immersed surface* of the vessel, and the variations of the speed of the gliding of the water over different parts of that surface were neglected.

When the Editor of this treatise † (having occasion to compute, in 1857, the probable resistance at a given speed of a steam-vessel built by Mr. J. R. Napier), introduced for the first time the consideration of the *augmented surface*, he adopted, for the coefficient of friction, the constant part of the expression deduced by Professor Weisbach from experiments on the flow of water in iron pipes, viz.:

$$f = \cdot 0036;$$

and that value has given results corroborated by practice, for surfaces of clean painted iron. For clean copper sheathing, and for very smooth pitch, it appears probable that the coefficient of friction is somewhat smaller; but there are not sufficient experimental data to decide that question exactly. Experimental data are also wanting to determine the precise increase of the coefficient of friction produced by various kinds and degrees of roughness and foulness of the ship's bottom; but it is certain that that increase is sometimes very great.

The preceding value of the coefficient of friction leads to the following very simple rule for clean painted iron ships:—*At ten knots, the eddy-*

---

skin; q, the ratio which the velocity of gliding of the water over that portion bears to the speed of the ship; c, the speed of the ship; g, gravity; w, the heaviness of the water; f, the coefficient of friction; then

$$\text{Eddy-resistance} = f w \frac{c^2}{2g} \int q^3 \, ds;$$

$\int q^3 \, ds$ being the *Augmented Surface.*

† The treatise referred to is a 'Treatise on Shipbuilding,' by Mr. Rankine and other eminent authorities, in course of publication in 1865.

resistance is one pound avoirdupois per square foot of augmented surface; and varies, for other speeds, as the square of the speed.

COMPUTATION OF PROPELLING POWER AND SPEED.

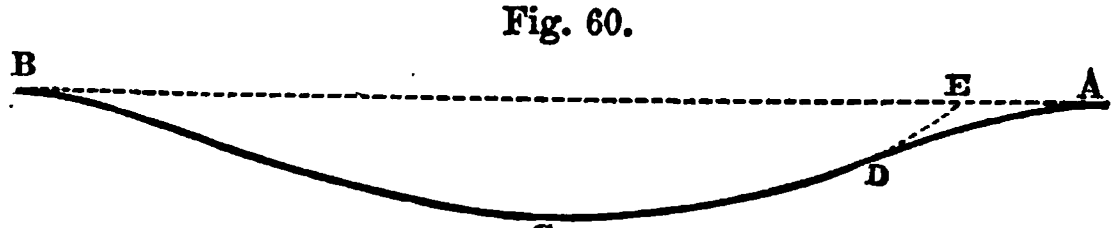
General Explanations.—The method of calculation now to be explained and illustrated was first practically used in 1857, under the circumstances stated. A very condensed account of it, illustrated by a table of examples, was read to the British Association in September, 1861, and printed in various mechanical journals for October of that year; and some further explanations appeared in a paper on Waves in the 'Philosophical Transactions for 1862.'*

The method proceeds by deducing the eddy-resistance from an approximate value of the augmented surface. It is therefore applicable to those vessels only in which eddy-resistance forms the whole of the appreciable resistance; but such is the case with all vessels of proportions and figures well adapted to their speed, as has been explained in the preceding sections; and as for misshapen and ill-proportioned vessels, there does not exist any theory capable of giving their resistance by previous computation.

Computation of Augmented Surface.—To compute the *exact* augmented surface of a vessel of any ordinary shape would be a problem of impracticable labour and complexity. The method employed, therefore, as an approximation for practical purposes, is to choose in the first instance a figure approximating to the actual figure, but of such a kind that its augmented surface can be calculated by a simple and easy process, and to use that augmented surface instead of the exact augmented surface of the ship; care being taken to ascertain by comparison with experiments on ships of various sizes and forms whether the approximation so obtained is sufficiently accurate.

The figure chosen for that purpose is the trochoid, or rolling-wave-curve, extending between a pair of crests, such as A and B in fig. 60;

Fig. 60.



for by an easy integration, published in the 'Philosophical Transactions for 1862,' it is found that the augmented surface of a trochoidal riband †

* A prediction of the speed of the 'Great Eastern,' with different amounts of engine-power, obtained by this method of calculation, was published in the 'Philosophical Magazine' for April, 1859.

† This is the species of curve that will be described by a pendulum, the surface

of a given length in a straight line, and of a given breadth, is equal to the product of that length and breadth, multiplied by the following *coefficient of augmentation*;—

1 + 4 (sine of greatest obliquity)^2 + (sine of greatest obliquity)^4; the *greatest obliquity* meaning the greatest angle, B E D, made by a tangent, D E, to the riband at its point of contrary flexure, D, with its straight chord, A B.

In approximating to the augmented surface of a given ship by the aid of that of a trochoidal riband, the following values are employed:

I. For the length, $\overline{AB}$, of the riband, the length of the ship on the plane of flotation.

II. For the total breadth of the riband, the *mean immersed girth*; found by measuring, on the body-plan, the immersed girths of a series of cross-sections, and taking their mean by Simpson's Rule, or by measuring mechanically with an instrument the sum of a number of girths, and dividing by their number.

III. For the *coefficient of augmentation*, the mean of the values of that coefficient as deduced from the greatest angles of obliquity of the series of water-lines of the fore-body, shown on the half-breadth plan. It is not necessary to measure the angles themselves, but only their sines.

The augmented surface is then computed by multiplying together those three factors.

The Computation of the Probable Resistance (in lbs.) at a given speed is performed according to the rule already stated, by *multiplying the augmented surface by the square of the speed in knots, and dividing by* 100 (for clean painted iron ships).

The process just described is virtually equivalent to the following:— An ocean wave is conceived (A C B in fig. 60), of a length, A B, equal to that of the ship on her water-line; and having its steepest angle of slope, B E D, such that the function of that slope, given in Article 162 as the coefficient of augmentation, shall be equal to the mean value of the same function for all the water-lines of the ship's bow. A solid of a breadth equal to the ship's mean immersed girth is then conceived to be fitted into the hollow, A C B, and to be moved along with the advance of the wave; and the resistance due to frictional action between that solid and the particles of water is taken as the approximate value of the resistance of the vessel.

In *Computing the Probable Engine Power required at a given Speed*, allowance must be made for the power wasted through slip, through wasteful resistance of the propeller, and through the friction of the en-

of which was shown by me in my 'Catechism of the Steam-Engine,' published in 1856, to be the measure of the resistance—a conclusion deduced by me from experiment several years before.

gine. The proportion borne by that wasted power to the effective or *net* power employed in driving the vessel, of course varies considerably in different ships, propellers, and engines; but in several good examples it has been found to differ little from 0·63; so that, as a probable value of the *indicated power* required in a well-designed vessel, we may take—

$$\text{net power} \times 1·63.$$

Now an indicated horse-power is 550 foot-pounds per second; and a knot is 1·688 foot per second; therefore an indicated horse-power is

$$\frac{550}{1·688} = 326 \text{ \textit{knot-pounds}, nearly;}$$

or 326 lbs. of gross resistance overcome through one nautical mile in an hour. If we estimate, then, the *net* or *useful* work done in propelling the vessel as equal to the total work of the steam divided by 1·63, we shall have

$$\frac{326}{1·63} = 200 \text{ knot-pounds}$$

of net work done in propulsion for each indicated horse-power. Hence the following

RULE.—*Multiply the Augmented Surface in square feet by the cube of the speed in knots and divide by* 20000; *the quotient will be the probable indicated horse-power.*

The divisor in this rule, 20000, *expresses the number of square feet of augmented surface which can be driven at one knot by one indicated horse-power:* it may be called the COEFFICIENT OF PROPULSION.

It is, of course, to be understood that the exact coefficient of propulsion differs in different vessels, according to the smoothness of the skin, the nature of its material, and the efficiency of the engines and propellers; being greatest in the most favourable examples.

In clean iron ships, with no evident fault in shape or dimensions, or in the propeller and engine, it has been found on an average to be somewhat above 20000; and the value 20000 may be taken as a probable and safe estimate of the coefficient of propulsion in any proposed vessel designed on good principles. In every instance in which that coefficient is materially less than 20000, the shortcoming can be accounted for by some fault, such as undue bluntness of the bow or stern.

In vessels sheathed with copper or coated with smooth pitch, the coefficient of propulsion is unquestionably greater; but in what precise proportion it is at present difficult to say, owing to the scarcity of experimental data.

Computation of Probable Speed.—When the augmented surface of a ship has been determined, her probable speed with a given power is computed as follows:—

Multiply the indicated Horse-power by the Coefficient of Propulsion (say for clean iron ships, 20000): *divide by the Augmented Surface, and extract tl cube root of the quotient for the probable speed in knots.*

EXAMPLE I.—Calculation of Probable Speed of H. M. S. 'Warrior.'

Displacement on Trial....................8997 tons

Draught of Water........... { Forward... 25·88 feet. Aft........ 26·75 "

Water-lines.	Sine of Obliquity.	Square of Sine.	4th power of Sine.
L.W.L	·370	·1869	·01874
2 W.L	·315	·0992	·00984
3 W.L	·290	·0841	·00707
4 W.L	·265	·0702	·00492
5 W.L	·235	·0552	·00304
6 W.L	·165	·0272	·00074
Keel	·000	·0000	.00000
Means		·0674	·00583

1 + (4 × ·0674) + ·00583 = 1·275, Coefficient of Augmentation

Half-girths from Body-plan Feet.	Simpson's Multipliers.	Products.
21·0	1	21·0
27·2	4	108·8
30·8	2	61·6
34·6	4	138·4
38·8	2	77.6
41·5	4	166·0
42·6	2	85·2
44·0	4	176.0
44·0	2	88·0
44·0	4	176.0
43·3	2	86·6
42·1	4	168·4
40·3	2	80·6
38·1	4	152·4
36·0	2	72·0
35·0	4	140·0
32·0	1	32·0

Divide by.......... .... 3)1830·6 Sum.

Divide by ⅓ number of Intervals.................. 8) 6·012

Mean Immersed Girth............................ 76·3
× Length.. 380

Product.. 28994
× Coefficient of Augmentation................. 1·275

Augmented Surface.............................. 36979 Square Feet.

Indicated Horse-power on Trial.................. 5471
× Coefficient of Propulsion 20000

Divide by Aug. Surface..................36979)109,420,000 Product.

Cube of Probable Speed............................ 2959

Probable Speed, computed......................... 14·356 Knots.
Actual Speed, on Trial............................ 14·354

Difference .... ·002

EXAMPLE II.—H. M. S. 'Fairy' will next be taken as an example, on account of the great contrast in size between her and the 'Warrior.'

Displacement .... 168 tons.
Draught of Water .... 4·88 feet.

Water-lines.	Sine of Obliquity.	Square of Sine.	Fourth power of Sine.
L.W.L	·23	·0529	·0028
2 W.L	·22	·0484	·0023
3 W.L	21	·0441	·0019
4 W.L	·17	·0289	·0008
Keel	0	0	0
Means		·0304	·0015

1 + (4 × ·0304) + ·0015 = 1·123, Coefficient of Augmentation.

Length on Water-line .... 144 Feet.
× Mean Immersed Girth (measured mechanically with an Instrument 19
× Coefficient Augmentation .... 1·123

Augmented Surface .... 3072 Square Feet.

Indicated Horse-power, on Trial 364
× Coefficient of Propulsion 20000

÷ Augmented Surface ....3072)7,280,000 Product.

Cube of Probable Speed 2370

Probable Speed, computed .... 13·333 Knots.
Actual Speed, on Trial .... 13·324

Difference .... ·009

The 'Fairy' occurs in the table of examples given in the paper of 1861, already referred to: in the present paper the measurements have been revised and improved in precision, especially as regards the coefficient of augmentation. The difference in the result is but small.

EXAMPLE III.—H. M. S. 'Victoria and Albert'—a wooden vessel, sheathed with copper, will now be employed, not to illustrate the computation of probable power at a given speed, or of probable speed at a given power; but to compute a value of the coefficient of propulsion for a copper-sheathed vessel.

Displacement on Trial Trip .... 1980 tons.
Draught of Water .... { Forward .... 13·8 feet. / Aft .... 14·0 "

Water-lines.	Sine of Obliquity.	Square of Sine.	4th power of Sine.
L.W.L	·19	·0361	·0013
2 W.L	·185	·0342	·0012
3 W.L	·17	·0289	·0008
4 W.L	·14	·0196	·0004
Keel	0	0	0
Means		·0252	·0008

1 + (4 × ·0252) + ·0008 = 1·102, Coefficient of Augmentation.

Length on Water-line..............................	300	Feet.
× Mean Immersed Girth (measured mechanically with an Instrument)...	40	
× Coefficient of Augmentation......................	1·102	
Augmented Surface..............................	13224	Square Feet.
× Cube of Speed in Knots.....................17^3 =	4913	
÷ Indicated Horse-power on Trial...... . .2980)	64,969,512	Product.
Coefficient of Propulsion..........	21,802	

Had the probable speed been computed with the coefficient of propulsion 20000, the result would have been 16·58 knots, instead of 17.

Proportions of Length to Breadth.—Principles which have been already explained fix the *least absolute length* suitable for a vessel which is to be driven at a given speed. But after that least length has been fixed, a question may arise as to whether that least length, or a greater length, is the most economical of power. That question is answered by finding the proportion of length to breadth, which gives the *least augmented surface* with the required displacement.

That proportion can be found in an approximate way only; because of the approximate nature of the process by which the augmented surface itself is found. The following are some of the results obtained in certain cases:—

I. When the proportion of breadth to draught of water, and the figure of cross-section, are fixed, so that the mean girth bears a fixed proportion to the breadth, it appears that the proportion of length to breadth which gives the least augmented surface for a given displacement, is about 7 to 1.

II. When the absolute draught of water is fixed, the proportion of length to breadth which gives the least augmented surface for a given displacement depends on the proportion borne by the draught of water to a mean proportional between the length and breadth, and on the figures of the cross-sections. The following are some examples for flat-bottomed vessels:—

$\frac{(\sqrt{\text{Length} \times \text{Breadth}})}{\text{Draught}}$ from	4 to 5;	7 to 10;	12 to 16;	17 to 23:
$\frac{\text{Length}}{\text{Breadth}}$	7;	8;	9;	10.

III. By cutting a vessel in two amidships, and inserting a straight middle body, the proportion borne by her resistance to her displacement is always diminished; because the midship section has a less mean girth in proportion to its area than any other cross-section of the ship; and therefore the new middle body adds proportionally less to the augmented surface than it does to the displacement.

IV. It does not follow, however, that a straight middle between tapering ends is the most economical form; for by adopting continuous

curves from bow to stern for the water-lines, instead of the lines compounded of curved ends and a straight middle, the same length, the same displacement, and almost exactly the same mean girth may be preserved, and the obliquity of the water-lines at the entrance diminished.

GENERAL CONCLUSIONS.

The principal conclusions to be drawn from the foregoing exposition are the following:—

1st. That the bulk of a ship should be equal to half the bulk of the circumscribing parallelepiped, supposing the areas of all the cross-sections have been translated into the form of a rectangle.

2d. That the sectional area of each successive frame should vary as the square of the distance from the stem or stern, until the points midway between the midship frame and the stem or stern have been reached, and that the areas at all the frames should vary in the manner already pointed out.

3d. That it is better to place the midship frame before the centre of the ship, in order that any wave raised at the stern may be sufficiently far forward to assist the propulsion.

4th. That the horizontal water-lines should be pendulum or trochoidal curves, or such equivalent curve as will enable the progressive displacement to follow the prescribed law, and that the transverse section should be formed with similar curves made as nearly as possible coincident with a semicircle.

5th. That nearly the whole of the resistance in a well-formed vessel is made up of friction, and that the friction per square foot of surface is less at the stern than at the bow, but that the law of variation is not known. Also, that at a certain point of the length the water adhering to the ship will attain its maximum velocity, and thereafter every foot of the length will have the same resistance.

6th. That the friction is diminished by making the bottom fair and smooth, and by coating it with a suitable lubricant, and that a portion of the power expended in friction may be recovered by making the stern part of the vessel to overhang near the water-line, so as to be propelled by the upward motion of the current which the friction generates, and also by placing the propeller in the stern or quarters instead of at the sides

7th. That both by Boulton and Watt's method and by Mr. Rankine's method the speed of a steamer may be accurately predicted. Boulton and Watt, by whom the screw engines of the 'Great Eastern' were made, predicted that the speed of the vessel would be 16·57 statute miles with 10,000 actual horse-power. Not more than 8,000 horse-power were actually generated, in consequence of a deficiency of steam. But on trial the speed was found to be as nearly as possible what it ought to be according to their rule with this proportion of power. The coefficient they employed for statute miles in making this computation was 900, which is also the coefficient they habitually use in the case of fast river boats of considerable size and good form.

8th. That any expedient for diminishing the resistance of well-formed vessels to be of material efficacy must have for its object the diminution of the friction of the bottom, either by reducing the adhesion of the particles of water to the ship, or to one another, or both; and also by making the adhering surface as small as possible.

EXAMPLES OF LINES OF APPROVED STEAMERS.

In fig. 61 we have the body-plan of H. M. screw yacht 'Fairy,' 144 feet 8 inches long between perpendiculars; and the hor-

Fig. 61.

BODY PLAN OF H.M.S. 'FAIRY.'

izontal water-lines can easily be constructed from the body-plan, by dividing the length by the number of vertical lines or frames, and by setting off at each division the given breadth of each wa-

ter line at that part. The 'Fairy' is 312 tons, 21 feet 1½ inch extreme breadth, and has 74·4 square feet of immersed section at 5 feet draught. She is propelled by two oscillating geared engines of 42 inches diameter of cylinder and 3 feet stroke, and has attained a speed of 13·3 knots per hour, exerting 363·8 indicated horse-power.

The 'Rattler' is 176 feet 6 inches long between perpendiculars, 32 feet 8½ inches extreme breadth, 888 tons burden, 894 tons displacement at 11 feet 5½ inches mean draught, 281·8 square

Fig. 62.

BODY PLAN OF H. M. S. 'RATTLER.'

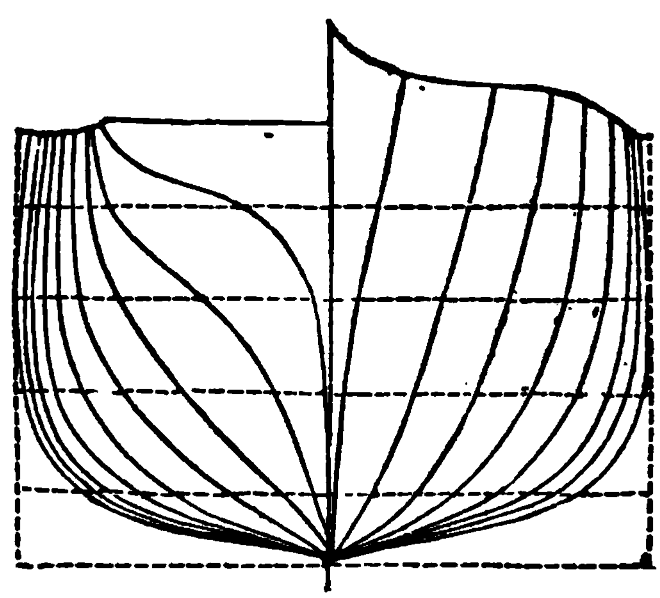
feet of immersed section, and is propelled by geared engines of 200 nominal horse-power. With 428 indicated horse-power she attained a speed of 10 knots—a high result, imputable partly to her good form for such speed, and partly to the smoothness of the copper sheathing—the 'Rattler' being a wooden vessel.

In the steamer 'Bremen,' which has given a very favourable result in working, the length of keel and fore-rake is 318 feet; the breadth of beam 40 feet; depth of hold 26 feet; tonnage, builder's measurement, 2,500 tons; power, two direct-acting inverted cylinder engines, with cylinders of 90 inches diameter and 8½ feet stroke. With a draught of 18½ feet the displace-

ment was 3,440 tons, the area of immersed section 606 square feet, and with the engines working to 1,624 horse-power the speed attained was 13·15 knots.

Fig. 64 is the body plan of the Cunard steamer 'Persia.' The vertical sections are 17½ feet from one another, and the breadth of the vessel is 45 feet. The engines are side lever; cylinders 100 inches diameter and 10 feet stroke, making 18 strokes per minute. The daily consumption of coals in eight boilers containing 40 furnaces is 130 tons, and the pressure of

Fig. 63.



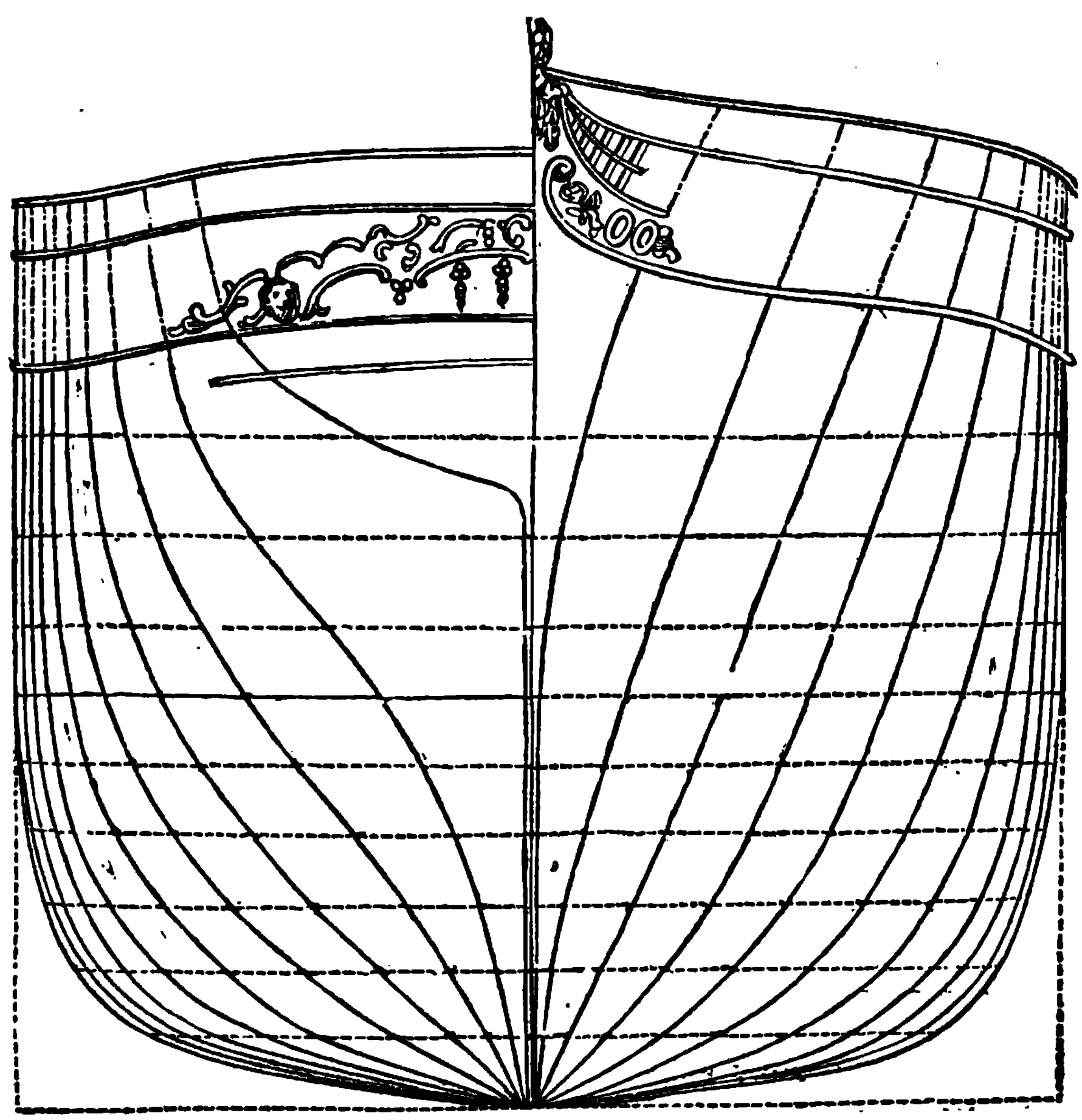
BODY PLAN OF SCREW STEAMER 'BREMEN.

the steam is 25 lbs. per square inch. The performance of the 'Persia' has been very satisfactory, except that she was not strong enough in the deck and had to be strengthened there, and she has a great deal too much iron in the shape of frames, which conduce to weakness rather than to strength. The paddle-wheels are 40 feet in diameter, and the floats are 10 feet long and 3 feet wide.

In fig. 65 the body plan of the iron-plated steamer 'Warrior' is given, and 66 is a transverse section of the same vessel, showing the guns. The 'Warrior' is an iron-clad steamer of 6,039 tons, 380 feet long, 58 feet broad, and 1,250 horse-power; and with the exertion of 5,460 actual horse-power, and at 26 feet draught, and with an area of immersed section 1,219 square feet,

she realized a speed of 14·3 knots. The utility of such vessels as the 'Warrior' does not promise to be considerable, and in fact the whole idea of constructing ships that would be impenetrable to shot turns out to be a complete delusion, as was plainly perceived by a number of competent observers would necessarily be

Fig. 64.



BODY PLAN OF PADDLE STEAMER 'PERSIA.'

the case before the expensive demonstrations were resorted to which the Admiralty has thought fit to institute. If there had been any natural law which restricted the penetrating power of ordnance to the narrow limits hitherto existing, there would have been some reason in the conclusion that by making the iron sides of ships very thick the shot would have been prevented

from penetrating them. But two facts were quite well known. first, that a steel punch may be made to pierce an iron plate however thick, if the diameter of the punch be equal to the thickness of the iron; and, secondly, that by increasing the dimensions of the gun an amount of projectile force could be obtained that would suffice for the punching through of any thickness whatever. The amount of this force, and of the dimensions of

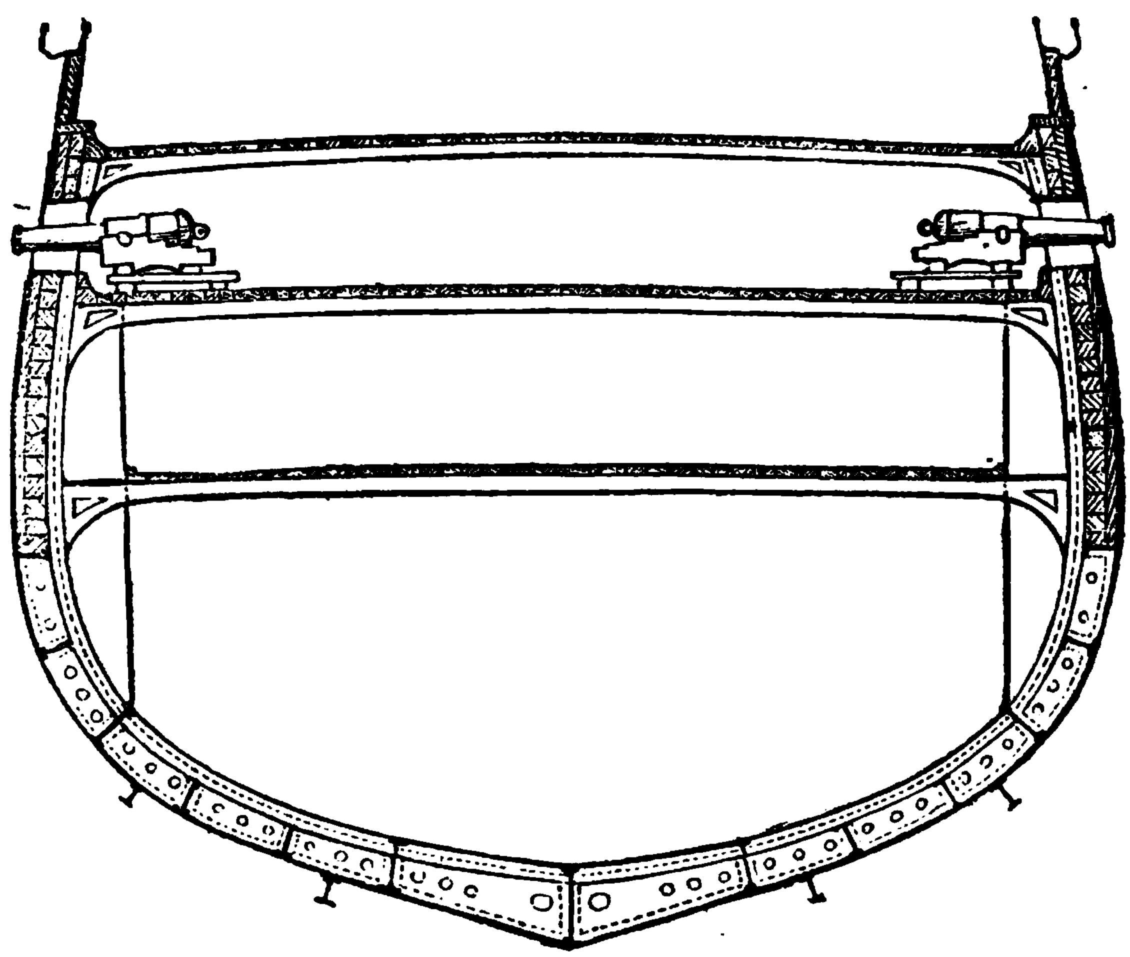
Fig. 65.

BODY PLAN OF H. M. S. 'WARRIOR.'

gun requisite to produce it, are of perfectly simple computation. The punching pressure is about the same as that required to tear asunder a bar of iron of the same sectional area as the surface cut by the punch, which is about 60,000 lbs. per square inch: and this pressure must act through such a distance as will suffice to overcome the continuity of the metal. The distance through which iron stretches before it breaks is quite well known, and this distance, multiplied by the separating pressure per square

inch, gives a measure of the power required. The velocity of cannon balls is also known, and, by the law of falling bodies, the height from which a body must descend by gravity to acquire that velocity can easily be determined; and the weight of the ball in lbs., multiplied by this height in feet, must always be greater than the punching pressure in lbs. multiplied by the distance in parts of a foot through which iron stretches before it breaks, else the ball will not penetrate. We have by no means

Fig. 66.



TRANSVERSE SECTION OF H. M. S. 'WARRIOR.'

reached the limit of the power of projectiles, nor is the exploration of those limits yet begun. Piston guns may be made in which the projectile would consist of a cigar-like body, or thunderbolt, with spiral fins supporting a wooden piston or wad, which would transmit to a projectile of small diameter the power generated in a cylinder of large diameter. A gun is virtually a cylinder, and the ball is the piston; and the power given to the ball will be represented by the pressure exerted by the ex-

ploding powder multiplied by the capacity of the gun. As, however, there are practical limits to the length of a gun, it may be advisable to increase the diameter, in order to get the requisite power. But this must be done without increasing the diameter of the ball, which would encounter greater resistance if made too large; and piston guns are the obvious resource in such a case—the piston being so contrived that it would be left behind by the ball so soon as it had left the mouth of the gun, and had acquired all the power which a piston could communicate. The projectile itself should have a sustaining power as well as a projectile one, to which end it should contain a certain quantity of rocket composition that would burn during the flight of the ball; and as the velocity of the ball would be high, the rocket gas would operate with little slip, and with much greater efficiency, therefore, than in rockets. The spiral feathers would cause the projectile to revolve in its flight, in the same manner in which a patent log is turned by the water; and any need for rifling the gun would thus be obviated, as the air would act the part of the rifle grooves. By these means far greater ranges and far greater accuracy of aim may be obtained than is at present possible, and it needs no great perspicacity to see that the success of maritime warfare will henceforth depend on the speed of the vessels employed, and the range, force, and accuracy of the projectiles. A small and very swift steamer with projectiles of the kind I have described would be able to destroy at her leisure a vessel like the 'Warrior,' while herself keeping out of range of the best existing guns which the assailed vessel could bring to bear against her opponent. With great accuracy of aim, and by choosing a position where the wind would have little disturbing influence, a large vessel could be struck at a distance at present deemed chimerical, and a few of such vessels as I have described, without any armour at all, would speedily disable any vessel which was not provided with the same species of projectile. Even if the large vessels, however, were to be armed with projectiles of equal range and power, the advantage would still be with the small vessels, as they would be more difficult to hit; and by taking up an external position and firing their guns in converg-

ing lines, of which the assailed object would be the focus, a great advantage would be given in the attack.

The vessels called Monitors, recently constructed in America, and which, I believe, owe their most valuable features to the talents of Ericcson, the eminent Swedish engineer—whose services were lost to this country through the incapacity of the Admiralty at the time of the introduction of the screw-propeller—are a very judicious embodiment of the leading principles of iron-clad vessels so as to secure the greatest possible efficiency. The constructors of those vessels saw that the thickness of the sides must be very much greater than it is in our iron-clads, to prevent heavy shot from going through them; and this thickness is reconciled with the usual buoyancy by making the sides of the vessel very low, so that only a small area has to be protected. Very powerful guns are employed in these vessels; and as it would be difficult to manœuvre such guns by hand, a steam-engine is introduced for this purpose, which gives great facility in the handling. To protect the guns and gunners from hostile shot, they are placed in towers of iron, the metal of which is 15 inches thick, and these towers are turned like a swing-bridge to enable the gun to be pointed; but the mechanism is so contrived, that the hand of a child acting on the engine will suffice to move the tower. Admiral Porter states that a Monitor of this construction would be able to cross the Atlantic, and attack and sink our iron-clads at her leisure, without being herself liable to injury; and I think he is right in his conclusion, though it was a most indelicate thing for him to have indicated such an occupation for this class of vessels. But persons who infer the helplessness of this country to resist such attacks, from the imbecility of the Admiralty, will find themselves mistaken; and there are obviously two ways in which such Monitors could be destroyed. Those vessels, though immensely strong above the water, are weak below, being there without armour, as they are protected from shot by the water. But a vessel like the 'Warrior,' if armed in a line with the keel—or a little above it—with a great steel blade or horn 40 or 50 feet long, would by running against a Monitor, break into the bottom and sink her. Such a

conflict would be like a sword-fish attacking a whale; and the horn or blade would in no way affect the steering of the vessel, as it would only virtually make her so much longer. Another way in which Monitors could be destroyed, is by running over them. As they are not many feet out of the water, to submerge them for a few feet more, by placing a corresponding weight upon their deck, would sink them altogether; and if we suppose a vessel with a very raking stem, and so trimmed by the stern as to bring the forefoot out of the water, to be run against a Monitor, it will be obvious, if the vessel be a large and heavy one, and the speed of propulsion be high, that she would run up on the deck of the Monitor, and sink her at once. The weight and speed of vessel that would work this catastrophe in the case of any given Monitor, is matter of simple calculation; and it is quite an error, therefore, to imagine that any Monitor yet constructed might not be promptly disposed of. Certainly they might be made tight, like diving-bells, so that even if sunk and ridden over, they would come up again. But this would be a difficult thing to do; and even if it were done, the next step would be, that the attacking vessel would not go over, but would stop upon them. No doubt the Monitor might as easily run into the attacking vessel as the attacking vessel into her, provided the Monitor had equal speed. But the construction of Monitors is not favourable for speed; and if speed is to settle the question, there is no need for iron plating. The fact is, such infallible recipes for victory as Monitors are supposed to constitute, almost always break down. I believe that such vessels may be made sea-worthy; they may be made impenetrable to any guns at present in our navy, and the guns they mount may be able to riddle our iron-clads like so many ships of card-board. All that I grant. But guns can be made to go through the towers and sides of Monitors, though twice as thick as they are all the existing Monitors can easily be outstripped in speed; and vessels with steel horns may rip up their bottoms, and vessels built with greatly slanted stems may be made to run over and sink them. It is true there are the guns of the Monitor to be encountered by the attacking vessel. But if that vessel has

Fig. 67.—American Iron-clad Ram 'Dictator.'
Length, 320 feet; breadth, 50 feet; depth, 22 feet; draft, 20 feet.

several decks, and if the deck over the main hold be made into a water-tank, with water-tight trunks communicating between the hold and the decks above, a shot between wind and water would not let water in, as the space is filled with water already; and the attacking vessel, therefore, could not be sunk by any fire the Monitor could bring against her, unless it could be made to pierce through the sea so as to enter the lower hold by which the flotation is given. With the low elevation of the Monitor turrets, however, this does not appear to be a probable contingency. Small rocket-vessels, propelled at a high speed by rocket gas issuing at the stern beneath the water, will probably be used in actual warfare for many purposes; and the same resource may be employed temporarily to increase the speed of large steamers. If, for example, the iron-clads of the 'Warrior' type had a tube opening beneath the water at each quarter, out of which rocket flame and gas were made to issue, the speed of the vessel, while the emission lasted, would be increased; and this temporary acceleration might suffice to give her a decisive superiority over an opponent.

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